Classifying Oscillatory Signatures of Expert vs NonExpert Meditators

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***Abstract*—EEG oscillatory correlates of expert meditators have been studied in the time-frequency domain. Machine Learning techniques are required to expand the understanding of oscillatory signatures. In this work, we propose a methodological pipeline to develop machine learning models for the classification between expert and nonexpert meditative state. We carried out this study utilizing the online repository consisting of EEG dataset of 24 meditators that categorized as 12 experts and 12 nonexperts meditators. The pipeline consists of four stages that include feature engineering, machine learning classifiers, feature selection, and visualization. We decomposed signals using five wavelet families consisting of Haar, Biorthogonal(1.3-6.8), Daubechies(orders 2-10), Coiflet(orders 1-5), and Symlet(2-8), followed by feature extraction using relative entropy and power. We classified the meditative state between expert and non-expert meditators employing twelve classifiers to build machine learning models. Wavelet coefficients d8 shows the maximum classification accuracy in all the wavelet families. Wavelet orders Bior3.5 and Coif3 produce the maximum classification performance with the detail coefficient d8 using relative power. We have successfully classified the meditative state between expert and non-expert with 100% accuracy using d5,d6,d7,d8,a8 coefficients. Multi-Layer Perceptron and Quadratic Discriminant Analysis attain the highest accuracy. We have figured out the most discriminating channels during classification and reported 20 channels involving frontal, central and parietal regions. We plot the high dimensional structure of data by utilizing two feature reduction techniques PCA and t-SNE.**

***Index Terms*—Meditation, Wavelet Decomposition, Feature Extraction, Classifiers, Machine Learning, Data Visualization**

I. INTRODUCTION

Over the past three decades, there has been considerable research examining structural, functional, and oscillatory correlates of meditation. Contemplative studies have produced a significant body of knowledge on the positive effects of meditation in enhancing various cognitive skills [1]–[3]. According to a research article [4], four hundred peer-reviewed scientific articles on meditation existed in 1990, and during 2018, a total count was more than four thousand papers. Progress in this field has been mostly because of two major reasons due to advancements in neuroimaging methods and due to incorporating meditation into therapeutic protocols in several medical practices [1], [5].

The important application of meditation is to reduce the episodes of mind wandering. Meditation practice enhances the major components of attention that involve the faster switching of attentional resources between tasks, robust cognitive flexibility, increased ability to sustain attention, awareness of automatic responding [6]. Attention is an essential component that is crucial for all aspects of higher-level processes and real-world activities [7]. The unprecedented advancement of digital technology and systems have made digital content and devices accessible more easily [8]. Younger generations have increased the usage of information technology and multitasking in their routine lives that promote numerous challenges to their attention [7].

Technological effects on the mental health of younger generations demand rigorous and rapid development in the field of digital healthcare [8]. It is needed to advance mental health monitoring tools to keep up with the pace of development in digital technology. Now, ease of availability of mobile EEG bands may help us to monitor the mental state activity quickly. Machine learning has been employed in various domains. We here propose to extract the features to classify the expert and nonexpert meditators. That will boost our understanding and contribute to our existing knowledge. This can be further deployed to understand the neural electrical activity of naive practitioners.

EEG has been a pivotal non-invasive technique in neuroscientific study meditation. This has been widely employed to examine the neural oscillatory changes during meditation. A majority of research papers on EEG studies conclude cognitive aspects and lack exploration insights to provide a concrete methodology to analyze signals and develop classifying models of meditation [9], [10]. EEG Pipeline such as PREP discusses the early stage processing and this paper describes pipeline which is majorly focused to build machine learning models using wavelet features [13]. The sole purpose of processing and analyzing electrical neural signals is to extract features and revealing the hidden patterns. Signals are decomposed to determine the features that are defined by linear expansion coefficients and the most popular linear expansion technique in the literature is Fourier transform [11]. Applying Fourier transform over the EEG signal doesn't yield the best result because of frequent characteristics of signals that are characterized by non-stationary time behavior. Wavelet transform has been an effective time-frequency analysis tool

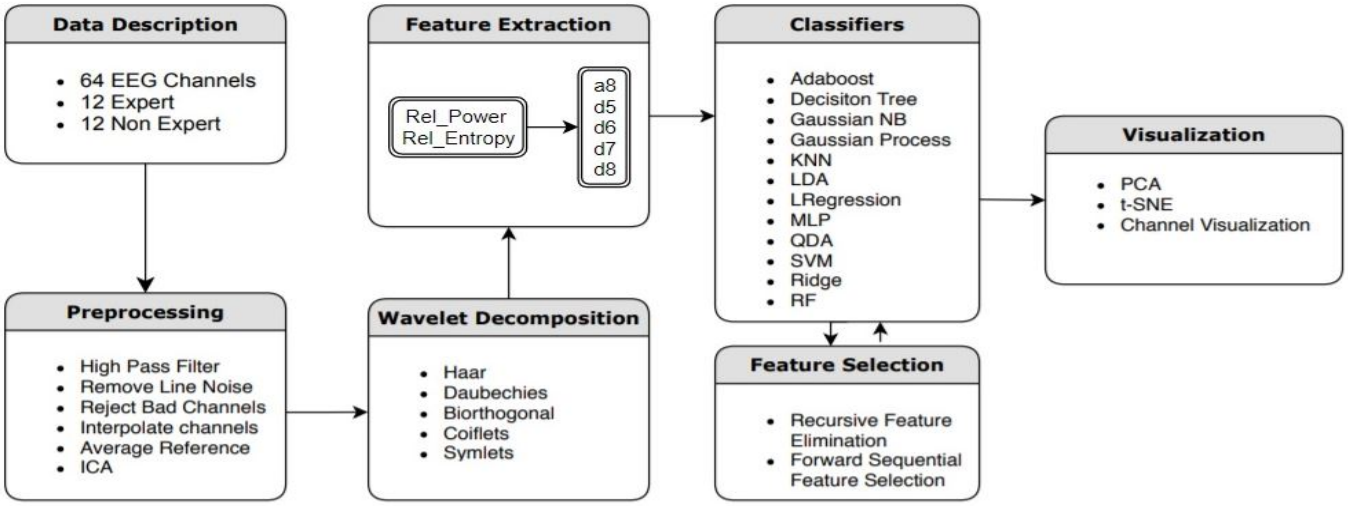

Fig. 1. A methodological Pipeline describes four stages that include feature engineering, machine learning classifiers, feature selection, visualization

for analyzing transient signals. In meditation studies, wavelet families have not been experimented to extract features for developing machine learning models of meditative state [12], [14], [15]. The research presented in the paper discusses the feature extraction using five wavelet families and build 12 classifiers models to classify the meditative state between expert and nonexpert meditators. This paper reports the most discriminating wavelet coefficient, top 20 channels and the best performing classifier.

II. DATA DESCRIPTION

Data were obtained from the git hub public repository [16]. EEG signals were captured using 64 channels Biosemi EEG system at the Meditation Research Institute(MRI) in Rishikesh, India. Detailed data description and experimental paradigm are mentioned in the paper [17] and accessed from there. Twenty-four meditators from the Himalayan Yoga tradition considered in this study, two groups were formed based on experience and hours of daily practice. The expert group involves the individuals who had practiced a minimum of 2 hours of daily meditation for 1 year or longer, the non-expert group includes those who were familiar with the meditation technique but irregular in their practice. All the necessary written consent and questionnaire were filled by participants and no report of medications that may affect the concentration. All participants were asked to meditate continuously and three questions were presented at random intervals to check the depth of meditation, mind wandering, and tiredness.

Data preprocessing was done using Matlab and EEGLAB software [18], [19]. Makoto's EEG pipeline was used in preprocessing the data [20]. Data were sampled at 256 Hz. A high pass filter was applied at 1 Hz, line noise frequencies of 50,100,150,200,250 were removed using CleanLine, bad channels were rejected using clean_rawdata and signals were corrected using Artifact Subspace Reconstruction. Spherical interpolation was done on all the removed channels. After this, average reference was applied on data. Adaptive Mixture Independent component analysis was applied to reject the artifactual components. Twenty second-epochs were segmented ranging from -20 seconds to -.05 seconds prior to the beginning of the question Q1. Finally, for an expert group, there were a total of 540 epochs(mean is 24.5, SD is 5.8) and 443 for a non-expert group(mean is 24.6, SD is 10.2).

III. PROPOSED METHODOLOGY

We propose a methodological pipeline that encompasses four stages - feature engineering which includes wavelet decomposition and feature extraction, learning classifiers, feature selection, and visualization. In Fig 1, a pipeline is defined.

A. Feature engineering using five wavelets families

The Fourier transform applies over the whole time axis which produces the frequency components of the signal but lacks the temporal information when a particular frequency occurs hence well suited for stationary signals. However, the nature of the EEG signal is non-stationary, time localization of the spectral components are required. Often time signals carry spectral information at any instant can be of significant interest, for such scenarios a transformation is needed to obtain the time-frequency representation. Short-Time Fourier Transform(STFT) works well to decompose the signals into time-frequency representation at any given interval of time but does not provide the capability to extract what spectral components exist at any given time instant. Wavelet transform (WT) was introduced to overcome the resolution problem faced during STFT. STFT computes fixed resolution at all times whereas WT generates variable resolution [21].

1) Wavelet Decomposition: The wavelet is smooth and rapidly vanishing oscillating mathematical function with significant emphasis on time and frequency localization. Dilations and translations are the two key parameters to form a wavelet

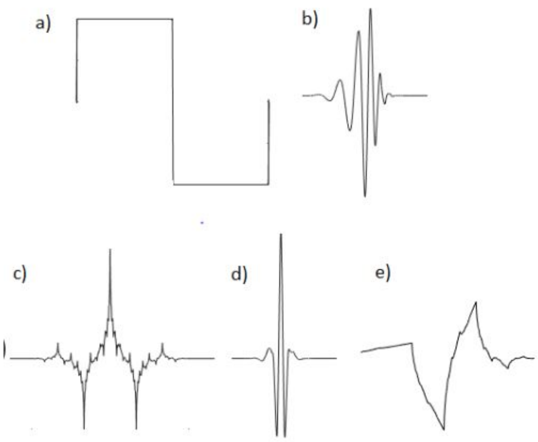

Fig. 2. Wavelet examples : a) Haar b) db8 c) bior2_2 d) coif5 e) sym2

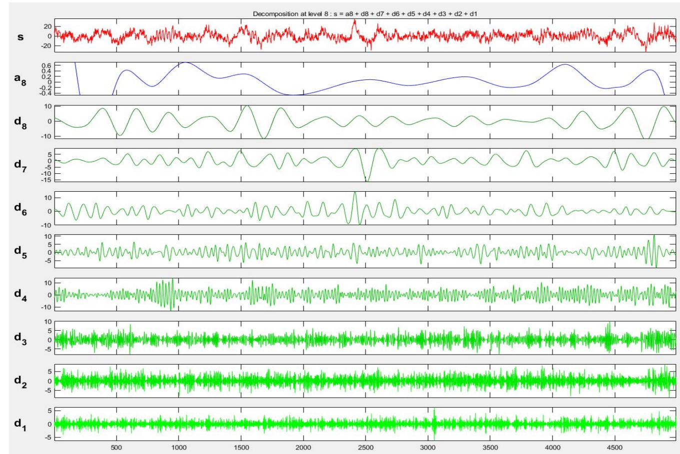

Fig. 3. A signal is decomposed into 8 levels using wavelet decomposition.

family [11] . The wavelet functions are generated using unique admissible mother wavelet $\psi(t)$.

$$\Psi_{x,y}(t) = \frac{1}{\sqrt{|x|}}\psi\Big(\frac{t-y}{x}\Big) \tag{1}$$

where t is the time and x,y $\in$ $R, x \neq$ 0, x,y are the scaling(dilation) and translation variables. The scale variable tunes the oscillatory frequency and length of the wavelet, the translation variable determines its shifting position.

There are five wavelet families considered for feature extraction with 8-levels of 1-D wavelet decomposition [22]:

1) Haar wavelet: This is the first and simplest orthogonal wavelet. The Haar wavelet is discontinuous and similar to a step function.
2) Daubechies: This is known as compactly supported orthonormal wavelets, which allows the practical application of discrete wavelets. There are 9 members of this family used, db_$\{2, 3, 4, 5, 6, 7, 8, 9, 10\}$.
3) Biorthogonal: This family of wavelets demonstrates the property of the linear phase which further advances to study signal and image reconstruction. There are 14 types of biorthogonal wavelets applied, bior_$\{1.3, 1.5, 2.2, 2.4, 2.6, 2.8, 3.1, 3.3, 3.5, 3.7, 3.9, 4.4, 5.5, 6.8\}$.
4) Coiflets: This family emphasizes to have scaling functions with vanishing moments. The wavelet and scaling functions have 2N and 2N-1 moments both equal to 0 and have a support of length 6N-1. Coiflets orders used: coif_$\{1, 2, 3, 4, 5\}$.
5) Symlets: This is the modification of the Daubechies family and represents nearly symmetrical wavelets. Symlets orders used: Sym_$\{2, 3, 4, 5, 6, 7, 8\}$. Few wavelets are shown as in Fig. 2.

2) Feature Extraction: Wavelet families decompose the signals to obtain the detail(d) and approximation vectors(a). In Fig. 3, a signal is decomposed into 8 levels of decomposition. The detail coefficients are used to extract features using Entropy [23] and Power.

1) Relative Entropy: Three steps were executed to compute the relative entropy of detail coefficients that we used as a feature vector.
   a) Shannon Entropy was computed using the below equation:

$$ENP(i,c) = -\sum_{j=1}^{N} d_{ij}^2 Log(d_{ij}^2) \tag{2}$$

   ENP gets the entropy value for each channel, where i represents decomposition level from 5 to 8, c is the channel ranges from 1 to 64 and j is the number of the coefficients of detail or approximation.
   b) Total Entropy was computed using

$$TOT\_ENP(c) = \sum_{i=5}^{8} ENP(i,c) \tag{3}$$

   c) Finally, Relative entropy was determined using (2) & (3).

$$RLTV\_ENP(c) = \frac{ENP(i,c)}{TOT\_ENP(i,c)} \tag{4}$$

2) Relative Power: Similarly as above, three steps were applied to compute relative power. Power was computed using the below equation.

$$POW(i,c) = \sum_{j=1}^{N} \frac{d_{ij}^2}{length(d_{ij})} \tag{5}$$

B. Learning Classifiers

After feature extraction, machine learning training was done in python using scikit learn package.

1) Model Development : There are 12 machine learning classifiers employed to develop the best performing model [24]. The table I lists the classifiers with some of their configurations, which may be tuned on the basis of data.

TABLE I
MACHINE LEARNING CLASSIFIERS

Classifier	Parameters
K-Nearest Neighbour(KNN)	leaf Size =30, neighbours =2
SVM	kernel = (linear,rbf)
Gaussian Process(GP)	optmzer= fmin_l_bfgs_b,kernel = rbf
Decision Tree(DT)	criterion='gini'
Random Forest(RF)	n_estimators=15
Multi Layer Perceptron(MLP)	hiddenlayer=[500,400,300,200,100]
AdaBoost	algorithm=SAMME.R
Gaussian Naive bayes(GNB)	smoothing=1e-09
Quadratic Discriminant Analysis(QDA)	threshold Rank Estimation = 0.0001
Logistic Regression(LR)	penalty = 'L2'
Linear Discriminant Analysis(LDA)	threshold Rank Estimation = 0.0001
Ridge	threshold Rank Estimation = 0.0001

2) K-Fold Cross Validation: This method is the statistical practice to divide the data into k small sets to validate the accuracy of the model. The original sample is partitioned into K subsets, one subset is used for validation and K-1 subsets are used for training. Each subset will get one chance to become the validation set. Hence there are K times training and testing involved. We applied 10 Fold Cross-Validation and reported the average accuracy.

C. Feature Selection Technique (FS)

Feature extraction was done on the time series and extracted features further refined using the feature selection methods. There are two major feature selection methods employed as follows -

1) Recursive Feature Elimination(RFE) - This algorithm ranks the features by associating the weights with features and prune the features as per the weights. It forms the smaller set after each iteration and terminates until the given (k) number of features is achieved [24].
2) Sequential Forward Feature Selection(SFFS) - A greedy search technique to reduce the dimensions of the feature vector from a d space to k space [25]. The concept behind this technique, let's suppose there is a bucket which initially contains zero feature, input vector is provided with n features, the algorithm will iterate till the bucket contains the specified no of features. At the first iteration, every feature individually is used for classification and a feature with the maximum classification performance would be selected in the bucket. In the next iteration, the selected feature present in the bucket will be used as prior and the remaining n-1 will pair up and select the next best performer in the bucket, and similarly iteration continues till the specified number of features selected.
   Base classifiers used for RFE and SFFS are Logistic Regression, Linear SVM.

D. Visualization

There is always a challenge to visualize the structure of the high dimensional dataset and mostly the dataset contains a large no of features and practically impossible to visualize it, so the dimensions of the dataset will be reduced to the lower dimensions from n variables to 3 or 2 variables while minimizing the information loss [26]. To reduce the dimensions, there are two techniques employed as follows -

1) Principal Component Analysis : It is one of the oldest and widely used dimensionality reduction techniques. It transforms the correlated to the new uncorrelated variables that successively maximize variance. To create new uncorrelated variables, the principal components, reduces to finding the eigenvalues/eigenvectors of the covariance matrix [27].
2) t-Distributed Stochastic Neighbouring Entities (t-SNE) : This is notably significant for high-dimensional data that present on various different, but related, low-dimensional manifolds, such as images of objects from multiple classes viewed from multiple viewpoints. It employs a Student-t distribution to compute the similarity between two points in the low-dimensional space [28].

IV. RESULTS AND DISCUSSION

Meditation data consists of session 1 to at most 3 for each subject, it depends on the practitioners who were willing and able to sit comfortably for longer periods of time, samples were preprocessed and extracted the 20 seconds epoch prior to stimuli. Each epoch consists of 64 channels and 4992-time points. There are a total of 943 epochs that comprise of 540 and 443 from expert and non-expert. To obtain the feature vector, first wavelet decomposition was performed and secondly, entropy and power were computed. Every channel represents one feature hence the feature matrix was of shape 943*64. We had successfully classified between mental states of expert and non-expert meditators with 100% accuracy, which indicates the characteristics of two classes and can be visualized using a dimensionality reduction technique.

We applied 8 levels of decomposition of five wavelet families and performed feature extraction on one approximation and four detail coefficients that include a8,d5,d6,d7,d8 and remaining d1 to d4 were considered as noise. There were a total of 32 distinct feature matrix classified that include wavelet functions from Haar, Daubechies, Biorthogonal, Coiflets, and Symlet. We first classified the feature vectors consist of only one detail or approximation coefficient. We had classified the

TABLE II
CLASSIFICATION ACCURACY: EXPERT AND NONEXPERT WITHOUT FEATURE SELECTION(FS)

Feature	Wavelet	Coefficient	Classifier	Accuracy(%)
Power				
	Bior3.5	d8	MLP	82.96
	Bior3.5	d8	QDA	82.82
	Bior6.8	d8	MLP	82.05
Entropy				
	Coif5	d8	MLP	82.46
	db2	d8	MLP	81.80
	Bior3.5	d8	GP	81.69

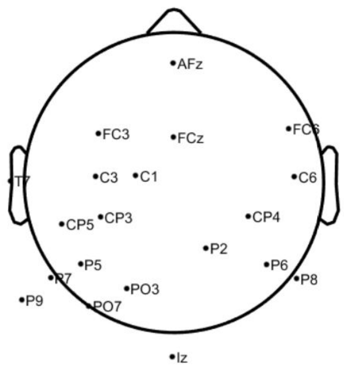

Fig. 4. Top 20 Channels are shown that are found to be most discriminating channels for classification.

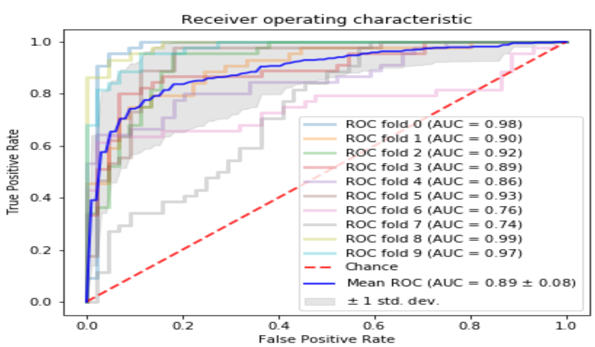

Fig. 5. ROC curve depicts the performance of a MLP classifier using Bior3.5 and d8 coefficient on 10 Fold cross validation.

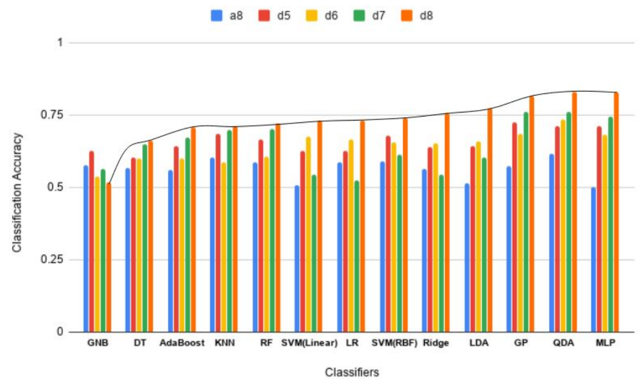

Fig. 6. Detail coefficient d8 outperforms all other coefficients.

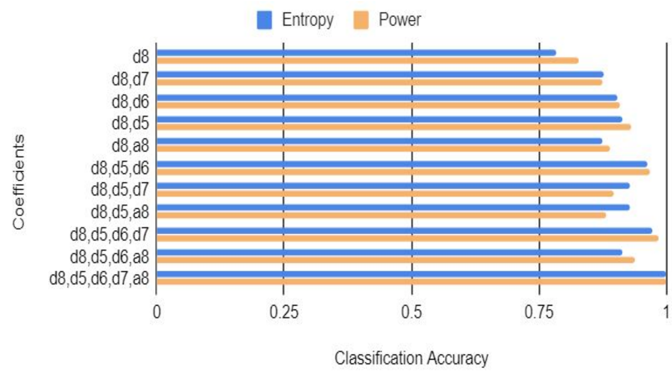

Fig. 7. Classification performance of wavelet coefficients using features extracted from entropy and power.

features obtained from relative power and achieved an accuracy of 82.96% using Multi Layer Perceptraon and 82.82% using the Quadratic Discriminant Analysis with wavelet Bior3.5 at decomposition level 8, top-performing mother wavelets with classifier are shown in Table II.

We experimented to find out the significant channels involved in classifying mental states of expert and non-expert meditators. We applied recursive feature elimination (RFE) and forward sequential feature selection(FSFS) techniques to select the top 20 channels that improved the accuracy from 82.96% to 88.57% as shown in Table III. We tried 5 different values to figure out the top discriminating channels, five values tested were [10,20,30,35,40] channels. Discriminating Regions involved during the classification of two mental states were frontal, central and parietal as shown in Fig 4.

Receiver Operating Characteristics (ROC) and Area Under the Curve(AUC) have utilized to visualize the classification model's performance on 10 Fold cross-validation using coefficient d8 as shown in Fig 5. Higher the AUC is better the performance, we found mean AUC is 0.89. We have found that there is a strong correlation between the decomposition level(DL) and the discriminating characteristics of EEG signals, mostly all classifiers produced maximum accuracies using d8. In Fig 6, we have shown wavelet Bior3.5 with all classifiers at different levels.

Biorthogonal and Coiflets families have produced the highest discriminating characteristics. Without feature selection, Bior3.5 has shown the maximum classification performance with 82.96% accuracy and with feature selection, Coif5 has produced the classification accuracy of 88.57%. Relative power yields the maximum classification in both the cases. Once we observed level d8 performs best, we further examined to concatenate levels using Sequential Forward Feature Selection technique, here we mean decomposition level as a feature. We initiated it with a selection of level 8 as it shows the highest accuracy among all levels, and then iterated it with

TABLE III
CLASSIFICATION ACCURACY: EXPERT AND NONEXPERT WITH FEATURE SELECTION(FS)

Feature	Wavelet	Channels	FS(RFE)	Coefficient	Classifier	Accuracy(%)
Power						
	Coif3	20	SVM	d8	GP	88.57
	Coif5	35	LR	d8	GP	87.33
	Coif5	30	SVM	d8	GP	87.32
Entropy						
	Bior3.7	35	SVM	d8	GP	86.07
	Bior3.7	30	SVM	d8	GP	86.41
	Bior3.5	30	SVM	d8	MLP	85.86

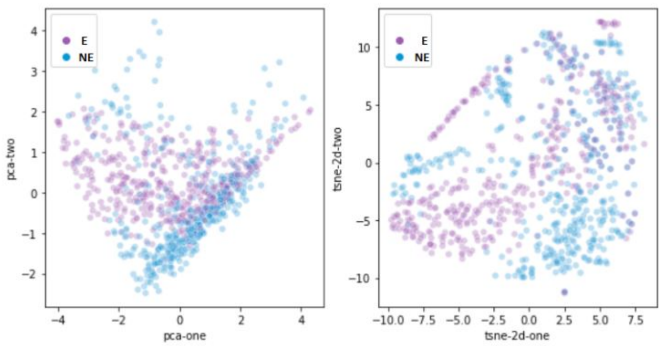

Fig. 8. Expert(E) and NonExpert(NE) data points are shown using PCA(left) and t-SNE(right)

other levels and chose the best discrimination combination and repeat it till we consumed all levels. In Fig 7, it depicts the accuracy achieved during the different combinations applying QDA. We have achieved 100% accuracy if we concatenate all the levels in all the families. We observed an essential observation that if we remove the approximation coefficient a8, an error rate of classification increases from 0 to 3%, and keeps on escalating if we sequentially remove the levels. It suggests the impact of each coefficient which contributes to enhancing the discriminatory property. QDA outperforms all the classifiers, so to figure out the possible reason, we employed PCA and t-SNE to visualize the structure of the high dimensional feature matrix. In Fig 8, features were reduced from 320 to 2 variables. The visualization suggests the quadratic behavior of the matrix.

V. Conclusion

In this work, we have demonstrated a methodological pipeline to develop the computational models for classifying the meditative state between expert and nonexpert meditators. This study explains the importance of detail coefficients extracted from five wavelet families using relative power and entropy as feature extraction techniques. We have successfully classified the meditative state between expert and non-expert meditators. We observed detail coefficient d8 has the highest accuracy when compared with a8,d5,d6,d7 coefficients. Bior3.5 and Coif3 have produced maximum classification characteristics. We have achieved 100% accuracy using features from a8,d5,d6,d7,d8 coefficients in all wavelet families. Relative power has shown better performance than relative entropy. Multi-Layer Perceptron and QDA outperforms all the mentioned machine learning classifiers. This work demonstrates computational models for meditation.

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