Distributed Fault Accommodation for a Class of Interconnected Nonlinear Systems with Event-Triggered Inter-Communications

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***Abstract*—In this paper, the distributed fault accommodation problem is studied for a class of interconnected nonlinear systems with event-triggered control and inter-subsystem communications. Each subsystem is subject to potential faults resulting from the local dynamics and/or transmitted from neighboring subsystems. The time periods before and after the fault detection in each subsystem are considered and corresponding event-triggered controllers are proposed, where the neural network based adaptive approximation technique is used to estimate the fault effect online. The closed-loop stability of the interconnected system with the proposed event-triggered fault accommodation controllers is rigorously analyzed.**

***Index Terms*—fault accommodation, nonlinear systems, neural network, event-triggered control**

I. Introduction

To enhance the reliability and safety of interconnected systems, various fault-tolerant control and fault accommodation methods have been reported under different fault and system scenarios. To mention a few, the cyclic-small-gain methodology is used in [1] to form a decentralized fault accommodation scheme, the backstepping method with adaptive approximation is employed to deal with the delayed process fault [2], the dead zone phenomenon and bias actuator fault are considered in [3], a neural network-based decentralized fault accommodation is proposed in [4, 5], fuzzy approximation-based actuator fault-tolerant is proposed in [6], and partial and coordinated communication based fault accommodation schemes are proposed in [7, 8], respectively. Note that the above mentioned results are all based on continuous inter-subsystem measurement and control implementation.

For the distributed control scheme of the interconnected system, inter-subsystem communication is essential. As each subsystem may not be collocated, the inter-subsystem communication burden may increase significantly with increasing the scale of interconnected systems. Moreover, the actuator has to respond to the control signal updating all the time. To reduce communication burden and extend the lifetime of the actuators, there is a need to balance the control performance, communication resources, and signal quality. To address this problem, the event-triggering approach provides a promising alternative since signals are aperiodically sampled and transmitted when needed, instead of all the time. The basic stability and triggering scheme dynamics (nonzero inter-event time or exclusion of Zeno behavior) of event-triggered control have been studied, e.g., see [9–12].

In this study, we focus on balancing the communication burden, control performance, and, most critically, fault tolerance; thus, an event-triggered fault accommodation scheme for a class of nonlinear uncertain systems is first proposed. In contrast to [11, 13], both the inter-subsystem communication and the controller to actuator communication are event-triggered. Each subsystem is subject to modeling uncertainty, interconnections with neighboring subsystems, and potential process faults arising from the local dynamics or transmitted from a neighboring subsystem. A full lifecycle of fault accommodation is designed, where the time periods before the occurrence of a fault, after the detection of a fault, and within the occurrence time and detection time of a fault (fault detection delay) are considered. An event-triggered nominal controller is proposed for the time period before the detection of a fault, and the event-triggered fault accommodation control scheme is proposed for the time period after the detection of a fault, where the neural network based adaptive approximation technique is employed to approximate the unknown fault effect with the event-triggered inter-subsystem measurement. The fault accommodation system stability during the overall lifecycle is rigorously analyzed. The main contribution of this work is the fault accommodation strategy design and analysis for the interconnected system with event-triggered inter-subsystem and control signal communications.

The rest of the paper is organized as follows. The system description, triggering scheme, and problem to be solved are presented in Section II. The event-triggered nominal controller design and analysis are given in Section III, and the details of fault accommodation scheme are given in Section IV. Concluding remarks are given in the last section.

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II. Problem Formulation

We consider a class of nonlinear systems, comprised of m interconnected subsystems. The i-th subsystem, $i \in M = \{1, ..., m\}$, is described by:

$$\begin{cases} \dot{x}_{ij} = x_{i(j+1)}, \qquad j = 1, ...n_i - 1, \\ \dot{x}_{in_i} = f_i(x_i) + g_i(x_i)u_i + \eta_i(x_i, t) \\ \qquad + \gamma_i(\bar{x}_i) + \beta_i(t - T_0^i)h_i(\bar{x}_i^a), \end{cases} \tag{1}$$

where $x_i = col\{x_{i1}, ...x_{in_i}\} \in R^{n_i}$ and $u_i \in R$ are the state and input of the i-th subsystem, respectively. Let M_i be the index set containing all the neighboring subsystems affecting the i-th subsystem, then $\bar{M}_i = M_i \cup \{i\}$ is the index containing all the subsystems affecting the i-th subsystem. Based on the index sets, $\bar{x}_i = col\{..., x_j, ...\} \in R^{\bar{n}_i}$ is the state of the neighbors of the i-th subsystem, where $j \in M_i$; $\bar{x}_i^a = col\{..., x_l, ...\} \in R^{\bar{n}_i^a}$ is the augmented neighboring state of the i-th subsystem, where $l \in \bar{M}_i$. The functions $f_i : R^{n_i} \mapsto R$ and $g_i : R^{n_i} \mapsto R$ are continuously differentiable functions representing the nominal dynamics of the i-th subsystem ($g_i \neq 0$), while $\eta_i : R^{n_i} \times R^+ \mapsto R$ is a continuous function representing the unknown and unstructured modeling uncertainty of the i-th subsystem; $\gamma_i : R^{\bar{n}_i} \mapsto R$ is a continuous function characterizing the interconnections between the i-th subsystem and its neighbors; $h_i(\bar{x}_i^a) : R^{\bar{n}_i^a} \mapsto R$ represents the fault effect in the i-th subsystem resulting from its local dynamics and/or neighboring dynamics due to the interconnections; and scalar function $\beta_i(t - T_0^i)$ characterizes the time profile function for the potential system fault in the i-th subsystem and T_0^i is the fault occurrence time in the i-th subsystem. In this study, we consider abrupt faults, where $\beta_i(t - T_0^i) = 0$ when $t < T_0^i$ and $\beta_i(t - T_0^i) = 1$ when $t \geq T_0^i$.

In this paper we consider the fault accommodation design problem, which is initiated after a fault is detected. Let the fault detection time be T_d^i for the i-th subsystem. The following assumptions will be used for control design.

Assumption 1. For $i \in M$ and $T_0^i \leq t \leq T_d^i$, it holds that $|h_i(\bar{x}_i^a)| \leq \varepsilon_{f,i}$, where $\varepsilon_{f,i} \in R^+$ is an unknown constant.

Assumption 2. For $i \in M$, $\bar{x}_i \in R^{\bar{n}_i}$, and $\hat{\bar{x}}_i \in R^{\bar{n}_i}$, it holds that $\left|\gamma_i(\bar{x}_i) - \gamma_i(\hat{\bar{x}}_i)\right| \leq l_{\gamma,i} \left\|\bar{x}_i - \hat{\bar{x}}_i\right\|$, where $l_{\gamma,i} \in R^+$ is the known Lipschitz constant.

Assumption 3. For $i \in M$, $x_i \in R^{n_i}$, and $t \geq 0$, it holds that $|\eta_i(x_i, t)| \leq \bar{\eta}_i(x_i, t)$, where $\bar{\eta}_i(x_i, t)$ is a known continuously differentiable bounding function.

Assumption 1 means that the magnitude of the possible fault in each subsystem remains bounded before the fault is detected. This is a reasonable assumption for control design as in the literature [14]. Assumption 2 presents a general requirement about the structure of the interconnections. To focus on the fault accommodation controller design and save computation resources, the global Lipschitz condition is assumed; however, the piecewise Lipschitz condition also works for the proposed controllers as can be seen in the following. Assumption 3 characterizes the upper bound of the local modeling uncertainty for the i-th subsystem.

Considering the distributed location of each subsystem, the event-triggered scheme for the inter-subsystem information transmission is adopted to save the communication resources. Thus, for the local controller, its neighbors' state information is not continuously accessible, but determined by an event-triggered scheme. To ensure the control performance, the local measurement is available to the local controller. At the same time, to reduce the updating of the actuator and extend the lifetime of actuators, an event-triggered control signal transmission/updating scheme is considered. To characterize the event-triggered signal transmission, two monotonically increasing sequences for the i-th subsystem are introduced:

$$\tau_{s,k}^i = \begin{cases} 0, & k = 0, \\ \inf\{t > \tau_{s,k-1}^i | S_s^i\}, & k \in N^+, \end{cases} \tag{2}$$

$$\tau_{c,k}^i = \begin{cases} 0, & k = 0, \\ \inf\{t > \tau_{c,k-1}^i | S_c^i\}, & k \in N^+, \end{cases} \tag{3}$$

where $\tau_{s,k}^i$ is the time sequence for the state information transmission and $\tau_{c,k}^i$ is the time sequence for the control signal transmission. S_s^i and S_c^i are Boolean variables denoting the corresponding triggering conditions: $S_s^i \triangleq \|x_i - \hat{x}_i\| \geq w_s^i \|\hat{x}_i\| + z_s^i$ and $S_c^i \triangleq \{\|u_i - \hat{u}_i\| \geq w_c^i \|u_i\| + z_c^i \vee t = \tau_{s,l}^j\}$, where $l \in N^+$, $j \in M_i$, $\hat{x}_i$ is the received state by the controller of the connected subsystems with the i-th subsystems, $\hat{u}_i$ is the received control signal by the i-th subsystem (actuator output or system input of the i-th subsystem), and u_i since here is the controller output as the event-triggered control implementation is used. Nonnegative scalars w_s^i, z_s^i, w_c^i, and z_c^i are the event-triggering parameters and $w_c^i \in [0, 1)$.

Based on (2) and (3), for the i-th subsystem with $j \in M_i$, we obtain:

$$\hat{x}_j(t) = x_j(\tau_{s,k-1}^i), \qquad t \in \left[\tau_{s,k-1}^i, \tau_{s,k}^i\right), \tag{4}$$

$$\hat{u}_i(t) = u_i(\tau_{c,k-1}^i), \qquad t \in \left[\tau_{c,k-1}^i, \tau_{c,k}^i\right). \tag{5}$$

The definitions in (3) and (5) imply two triggering scenarios of control signal transmission in the local controller. The first one is triggered by the neighboring system state information updating in the local controller, and the second one is triggered by the local controller output evolution. For the considered distributed control scheme, the neighboring system state and the local control signal are the input and output of the local controller, respectively. Hence, both the input and output changing of the local controller can trigger events for local control signal transmission. This triggering scheme design is motivated by the distributed control scheme and the dynamics of the controller.

In this study, an event-triggered fault accommodation strategy will be proposed to cover three time periods for each subsystem: fault-free period ($t < T_0^i$), fault detection delay period ($T_0^i \leq t < T_d^i$), and post-fault detection period ($t \geq T_d^i$). For the event-triggered fault accommodation strategy, a nominal controller will be designed for each subsystem for the period before the detection of any fault. Based on the adaptive approximation approach, an event-triggered fault accommodation law will be initialized to reconfigure the nominal controller after a fault is detected in the local subsystem. The stability under event-triggering properties of the overall closed-loop system will be presented for each of the three time periods of control.

III. Event-Triggered Nominal Controller

In this section, an event-triggered control design is proposed for each subsystem in the time period before any fault is detected ($t < T_0^i$ and $T_0^i \leq t < T_d^i$). The closed-loop stability analysis will be established as well.

A. Event-Triggered Nominal Controller Design

For the i-th subsystems, where $i \in M$, the original system description is rewritten as:

$$\begin{aligned}\dot{x}_i = & Ax_i + B[f_i(x_i) + g_i(x_i)\hat{u}_i + \eta_i(x_i,t) \\ & + \gamma_i(\bar{x}_i) + \beta_i(t - T_0^i)h_i(\bar{x}_i^a)],\end{aligned} \tag{6}$$

where

$$A = \begin{bmatrix} 0_{(n_i-1)\times 1} & I_{n_i-1} \\ 0 & 0_{1\times(n_i-1)} \end{bmatrix}, \quad B = \begin{bmatrix} 0_{(n_i-1)\times 1} \\ 1 \end{bmatrix}.$$

Note that $\hat{u}_i$ is utilized in (6), due to the event-triggered control implementation.

For the event-triggered controller design, the event-induced discrepancies in the controller input and system input for each subsystem will be analyzed. Based on (2), we have

$$\|x_j - \hat{x}_j\| \leq w_s^j \|\hat{x}_j\| + z_s^j, \quad j \in M_i. \tag{7}$$

Based on (7), the overall event-induced discrepancy in the input of the i-th local controller is characterized as

$$\left\|\bar{x}_i - \hat{\bar{x}}_i\right\| \leq \sum_{j\in M_i} (w_s^j \|\hat{x}_j\| + z_s^j). \tag{8}$$

Regarding the control input, based on the definition of S_c^i in (3), we obtain that

$$|u_i - \hat{u}_i| \leq w_c^i |u_i| + z_c^i, \quad i \in M, \tag{9}$$

where the "or" logic operation in S_c^i ensures the existence of the upper bound for $|u_i - \hat{u}_i|$. Based on (9), we have

$$\hat{u}_i = (1 + \delta_1^i w_c^i)u_i + \delta_2^i z_c^i, \tag{10}$$

where $\delta_1^i, \delta_2^i \in [-1,1]$. Here, the unknown parameters δ_1^i and δ_2^i are introduced to denote the discrepancy in system input, compared to the corresponding controller output. Resulting from the relative and constant triggering portion design in S_c^i, both multiplicative and additive discrepancy portions can be found for system input $\hat{u}_i$. Thus, taking (8) and (10) into consideration, a nominal controller for the i-th subsystem is designed as follows:

$$\begin{aligned}u_i(t) = & u_{o,i} \\ = & -\frac{1}{g_i(x_i)(1-w_c^i)}\Bigg\{ \left[K_i x_i + f_i(x_i) + \gamma_i(\hat{\bar{x}}_i)\right] \\ & \times \tanh\left[e_i \frac{K_i x_i + f_i(x_i) + \gamma_i(\hat{\bar{x}}_i)}{\omega_{i,1}}\right] \\ & + \bar{\eta}_i(x_i,t)\tanh\left[e_i\frac{\bar{\eta}_i(x_i,t)}{\omega_{i,2}}\right] \\ & + l_{\gamma,i}\sum_{j\in M_i}(w_s^j\|\hat{x}_j\| + z_s^j) \\ & \times \tanh\left[e_i \frac{l_{\gamma,i}\sum_{j\in M_i}(w_s^j\|\hat{x}_j\| + z_s^j)}{\omega_{i,3}}\right] \\ & + z_c^i g_i(x_i)\tanh\left[e_i\frac{g_i(x_i)}{\omega_{i,4}}\right]\Bigg\}.\end{aligned} \tag{11}$$

where $e_i = B^T P_i x_i$, $\omega_{i,l} \in R^+$ ($l = 1,2,3,4$) are the controller parameters, and K_i is the stabilization gain such that $A - BK_i$ is Hurwitz. For a given $Q_i > 0$, positive definite P_i is the solution of $(A - BK_i)^T P_i + P_i(A - BK_i) = -Q_i$.

B. Event-Triggered Nominal Controller Analysis

In this section, the stability of the considered interconnected nonlinear system will be analyzed.

Consider the Lyapunov function $V_i = \frac{1}{2}x_i^T P_i x_i$ for the i-th subsystem. First, we will focus on the time period $t < T_0^i$ for the i-th subsystem. Based on the actuator output and the system dynamics in (6), the time derivative of V_i is deduced as

$$\begin{aligned}\dot{V}_i = & -\frac{1}{2}x_i^T(A^T P_i + P_i A)x_i + e_i[f_i(x_i) \\ & + g_i(x_i)\hat{u}_i + \eta_i(x_i,t) + \gamma_i(\bar{x}_i)].\end{aligned} \tag{12}$$

The following lemma will be used in the sequel.

Lemma 1 [15]. For any $a \in R^+$ and $b \in R$, it holds that

$$|b| - b\tanh\left(\frac{b}{a}\right) \leq ca, \tag{13}$$

where c satisfies that $c = e^{-(c+1)}$ ($c \approx 0.2785$).

Based on the designed event-triggered nominal controller in (11) and Assumption 2, the following inequalities can be obtained:

$$\begin{aligned}\dot{V}_i \leq & -\frac{1}{2}x_i^T[(A - BK_i)^T P_i + P_i(A - BK_i)]x_i \\ & + e_i[K_i x_i + f_i(x_i) + \gamma_i(\hat{\bar{x}}_i) + g_i(x_i)\hat{u}_i \\ & + \eta_i(x_i,t) + \gamma_i(\bar{x}_i) - \gamma_i(\hat{\bar{x}}_i)] \\ \leq & -\frac{1}{2}x_i^T Q_i x_i + e_i[K_i x_i + f_i(x_i) \\ & + \gamma_i(\hat{\bar{x}}_i) + g_i(x_i)\hat{u}_i] \\ & + |e_i\eta_i(x_i,t)| + l_{\gamma,i}|e_i|\left\|\bar{x}_i - \hat{\bar{x}}_i\right\|.\end{aligned} \tag{14}$$

Next, we replace $\hat{u}_i$ in (14) by (10). Since the parameters δ_1^i and δ_2^i are unknown in (10), we will quantify their effect first.

Now, let's check the term $e_i g_i(x_i)(1+\delta_1^i w_c^i)u_i$. Recalling the property that "$tanh$" is an increasing function with $tanh(0) = 0$, we have that $e_i \tanh(e_i) \geq 0$. Since $w_c^i \in [0,1)$, based on (11), we obtain

$$e_i g_i(x_i) u_i \leq 0. \tag{15}$$

Thus, we have

$$\begin{aligned}\dot{V}_i \leq & -\frac{1}{2}x_i^T Q_i x_i + e_i[K_i x_i + f_i(x_i) + \gamma_i(\hat{\bar{x}}_i) \\ & + g_i(x_i)(1+\delta_1^i w_c^i)u_i + g_i(x_i)\delta_2^i z_c^i] \\ & + |e_i \eta_i(x_i,t)| + l_{\gamma,i}|e_i| \left\|\bar{x}_i - \hat{\bar{x}}_i\right\| \\ \leq & -\frac{1}{2}x_i^T Q_i x_i + e_i[K_i x_i + f_i(x_i) + \gamma_i(\hat{\bar{x}}_i) \\ & + g_i(x_i)(1-w_c^i)u_i] + |e_i \eta_i(x_i,t)| \\ & + l_{\gamma,i}|e_i| \left\|\bar{x}_i - \hat{\bar{x}}_i\right\| + |e_i g_i(x_i)| z_c^i. \end{aligned} \tag{16}$$

Based on Lemma 1, the controller given in (11), the event-induced discrepancy in (7), and Assumption 3, the following result can be obtained from (16):

$$\dot{V}_i \leq -\frac{1}{2}x_i^T Q_i x_i + c\omega_i, \tag{17}$$

where $\omega_i = \omega_{i,1} + \omega_{i,2} + \omega_{i,3} + z_c^i \omega_{i,4}$. For the overall interconnected system in the fault-free case, let $V = \sum_{i\in M} V_i$, then

$$\begin{aligned}\dot{V} &\leq \sum_{i\in M}\left(-\frac{1}{2}x_i^T Q_i x_i + c\omega_i\right) \\ &\leq \sum_{i\in M}(-\delta_{p,i}V_i + c\omega_i),\end{aligned} \tag{18}$$

where $\delta_{p,i} = \frac{\sigma_{\min}(Q_i)}{\sigma_{\max}(P_i)} > 0$. Based on (18), we know that the overall interconnected system is uniformly ultimately bounded for the time period when all the subsystems are fault-free and controlled by the nominal controllers given in (11), since $c\omega_i$ is bounded.

Now, considering the fault detection delay case for the i-th subsystem ($T_0^i \leq t < T_d^i$), where $\beta_i(t-T_0^i)h_i(\bar{x}_i^a) \neq 0$. Based on (17), the time derivative of V_i satisfies that

$$\dot{V}_i \leq \frac{1}{2}[-x_i^T Q_i x_i + 2c\omega_i + 2e_i h_i(\bar{x}_i^a)]. \tag{19}$$

Referring to Assumption 1, we obtain

$$2e_i h_i(\bar{x}_i^a) \leq \chi_i x_i^T x_i + \chi_i^{-1}\varepsilon_{f,i}^2 B^T P^2 B, \tag{20}$$

where $\chi_i > 0$. Note that, as $Q_i > 0$, $\exists \chi_i > 0$ such that $Q_{\chi,i} = Q_i - \chi_i I > 0$, for example, $\chi_i < \sigma_{\min}(Q_i)$. Thus, combining (19) with (20) yields

$$\dot{V}_i \leq -\delta_{\chi,i}V_i + c\omega_{\chi,i}, \tag{21}$$

where $\delta_{\chi,i} = \frac{\sigma_{\min}(Q_{\chi,i})}{\sigma_{\max}(P_i)}$ and $c\omega_{\chi,i} = c\omega_i + \frac{1}{2}\chi_i^{-1}\varepsilon_{f,i}^2 B^T P^2 B$.

Since the fault occurrence time and fault detection time among subsystems are different, the case considered here is to verify the stability of the overall interconnected system, where some subsystems are subject to fault detection delay and controlled by the corresponding event-triggered nominal controllers as in (11). As some subsystems are faulty, based on (18), we have

$$\dot{V} \leq \sum_{i\in M}(-\delta_{\chi,i}V_i + c\omega_{\chi,i}). \tag{22}$$

Comparing (22) and (21) with (18) and (17), respectively, one knows that the bounded stability conclusion for the interconnected system during the fault-free time period holds before any fault is detected, but may correspond to a larger state upper bound.

We summarize the above discussion of closed-loop stability for the interconnected nonlinear system by the following Theorem.

Theorem 1. Let Assumptions 1-3 hold. The event-triggered nominal controllers in (11) with the triggering scheme (2)-(5) guarantees that the state of the interconnected nonlinear system (1) is uniformly ultimately bounded, prior to the detection of a fault in any subsystems.

IV. Event-Triggered Fault Accommodation Controller

In this section, the event-triggered fault accommodation controller design and analysis will be presented. Adaptive approximation approach with the event-triggered measurement will be used to deal with the fault effect for any the i-th subsystem after $t \geq T_d^i$, where $i \in M$.

A. Event-Triggered Fault Accommodation Controller Design

For the i-th subsystem, to get sufficient information of the occurred fault after $t \geq T_d^i$, we employ the following neural network-based approximation model:

$$\hat{h}_i(\bar{x}_i^a, \theta_i) = \theta_i^T \pi_i(\bar{x}_i^a), \tag{23}$$

where $\hat{h}_i$ is the approximation of h_i, $\theta_i \in R^{n_{\theta,i}}$ is the neural network weight vector, and π_i is the preselected Radial Basis Function vector.

To measure the performance of the adaptive approximation based on (23), an optimal weight vector θ_i^* is introduced:

$$\theta_i^* = \arg \inf_{\theta_i \in R^{n_{\theta,i}}} \left\{ \sup_{\bar{x}_i^a \in X_i^a} \left|\hat{h}_i(\bar{x}_i^a, \theta_i) - h_i(\bar{x}_i^a)\right| \right\}, \tag{24}$$

where $X_i^a \subset R^{\bar{n}_i^a}$.

To facilitate the fault accommodation controller design, based on the optimal approximation weight vector, the approximation error or parameter uncertainty is further characterized by the following assumption.

Assumption 4. For $\bar{x}_i^a \in X_i^a$, $t \geq T_d^i$, and $i \in M$, it holds that $\left|\hat{h}_i(\bar{x}_i^a, \theta_i^*) - h_i(\bar{x}_i^a)\right| \leq \rho_i$, where $\rho_i \in R^+$ is an unknown bounding parameter.

Regarding the unknown bounding parameter ρ_i and the optimal weight vector θ_i^*, their estimation $\hat{\rho}_i$ and $\hat{\theta}_i$ are introduced for a feasible fault accommodation controller design, respectively. Due to the online estimation of these parameters, the fault accommodation controller will be dynamic, which is different from the nominal controller.

Taking the even-induced discrepancies in (8) and (9) and the fault approximation in (23) into consideration, the event-triggered adaptive approximation-based fault accommodation control law is designed as follows:

$$
\begin{aligned}
u_i =& u_{o.i} + u_{f,i}, \\
u_{f,i} =& -\frac{1}{g_i(x_i)(1-w_c^i)}\Bigg\{\left[\hat{\theta}_i^T\pi(\hat{\bar{x}}_i^a) + \hat{\rho}_i s(e_i)\right] \\
&\times \tanh\left[e_i\frac{\hat{\theta}_i^T\pi(\hat{\bar{x}}_i^a) + \hat{\rho}_i s_i(e_i)}{\omega_{i,5}}\right]\Bigg\},
\end{aligned} \tag{25}
$$

$$
\left\{\begin{array}{l}
\dot{\hat{\theta}}_i = \Pi_i\pi_i(\hat{\bar{x}}_i^a)e_i - \kappa_\theta^i\Pi_i\hat{\theta}_i, \\
\dot{\hat{\rho}}_i = \Gamma_i e_i s_i(e_i) - \kappa_\rho^i\hat{\rho}_i,
\end{array}\right. \tag{26}
$$

where $u_{o.i}$ is given as in (11) and $s_i(e_i) = \tanh\left(\frac{e_i}{\omega_{i,6}}\right)$. $\omega_{i,5}$, $\omega_{i,6}$, κ_θ^i, and κ_ρ^i are the positive controller parameters. $\Pi_i > 0$ and $\Gamma_i > 0$ are the learning rate for $\hat{\theta}_i$ and $\hat{\rho}_i$, respectively.

It can be seen that u_i in (25) is modularized, where $u_{o.i}$ is used to deal with the general dynamics of the i-th subsystem and $u_{f,i}$ is designed to deal with the general fault effect.

B. Event-Triggered Fault Accommodation Controller Analysis

In this section, the stability analysis will be presented for the faulty interconnected nonlinear system.

With (23) and (24), we can rewrite (6) by using the approximation model as

$$
\begin{aligned}
\dot{x}_i =& Ax_i + B[f_i(x_i) + g_i(x_i)\hat{u}_i + \eta_i(x_i,t) \\
&+ \gamma_i(\bar{x}_i) + h_i(\bar{x}_i^a) - \hat{h}_i(\bar{x}_i^a,\theta_i^*) + \hat{h}_i(\bar{x}_i^a,\theta_i^*)].
\end{aligned} \tag{27}
$$

For the i-th subsystem after $t \geq T_d^i$, consider a Lyapunov function:

$$
V_{f,i} = V_i + \frac{1}{2}\tilde{\theta}_i^T\Pi_i^{-1}\tilde{\theta}_i + \frac{1}{2\Gamma_i}\tilde{\rho}_i^2, \tag{28}
$$

where $\tilde{\theta}_i = \hat{\theta}_i - \theta_i^*$, $\tilde{\rho}_i = \hat{\rho}_i - \rho_i$, and V_i is defined in Section III. Taking (27) with (10) into (28), the time derivative of $V_{f,i}$ is obtained as

$$
\begin{aligned}
\dot{V}_{f,i} =& -\frac{1}{2}x_i^T(A^TP_i + P_iA)x_i + e_i[f_i(x_i) + g_i(x_i) \\
&\times (1+\delta_1^i w_c^i)u_i + g_i(x_i)\delta_2^i z_c^i + \eta_i(x_i,t) \\
&+ \gamma_i(\bar{x}_i) + h_i(\bar{x}_i^a) - \hat{h}_i(\bar{x}_i^a,\theta_i^*) + \hat{h}_i(\bar{x}_i^a,\theta_i^*)] \\
&+ \dot{\hat{\theta}}_i^T\Pi_i^{-1}\tilde{\theta}_i + \frac{1}{2\Gamma_i}\tilde{\rho}_i\dot{\hat{\rho}}_i.
\end{aligned} \tag{29}
$$

Based on (16) and the conclusion for $u_{o,i}$ in (18), (29) can be deduced as

$$
\begin{aligned}
\dot{V}_{f,i} \leq& -\frac{1}{2}x_i^T(A^TP_i + P_iA)x_i + e_i[f_i(x_i) + g_i(x_i) \\
&\times (1-w_c^i)u_i + \eta_i(x_i,t) + \gamma_i(\bar{x}_i) \\
&+ h_i(\bar{x}_i^a) - \hat{h}_i(\bar{x}_i^a,\theta_i^*) + \hat{h}_i(\bar{x}_i^a,\theta_i^*)] \\
&+ |e_i g_i(x_i)|\, z_c^i + \dot{\hat{\theta}}_i^T\Pi_i^{-1}\tilde{\theta}_i + \frac{1}{2\Gamma_i}\tilde{\rho}_i\dot{\hat{\rho}}_i \\
\leq& -\frac{1}{2}x_i^TQ_ix_i + c\omega_i + e_i[g_i(x_i)(1-w_c^i)u_{f,i} \\
&+ h_i(\bar{x}_i^a) - \hat{h}_i(\bar{x}_i^a,\theta_i^*) + \hat{h}_i(\bar{x}_i^a,\theta_i^*)] \\
&+ \dot{\hat{\theta}}_i^T\Pi_i^{-1}\tilde{\theta}_i + \frac{1}{2\Gamma_i}\tilde{\rho}_i\dot{\hat{\rho}}_i.
\end{aligned} \tag{30}
$$

Substituting $u_{f,i}$ in (25) with (26) into (30) yields

$$
\begin{aligned}
\dot{V}_{f,i} \leq& -\frac{1}{2}x_i^TQ_ix_i + c\omega_i + e_i[g_i(x_i)(1-w_c^i)u_{f,i} \\
&+ \pi_i^T(\bar{x}_i^a)\theta_i^* + \hat{\rho}_i s_i(e_i)] + e_i\pi^T(\hat{\bar{x}}_i^a)\hat{\theta}_i \\
&- e_i\pi^T(\hat{\bar{x}}_i^a)\theta_i^* - \kappa_\theta^i\hat{\theta}_i^T\tilde{\theta}_i \\
&- \frac{\kappa_\rho^i}{\Gamma_i}\tilde{\rho}_i\hat{\rho}_i + c\omega_{i,6}\rho_i \\
\leq& -\frac{1}{2}x_i^TQ_ix_i + \|e_i\theta_i^*\|\,\bar{\pi}_i + c\omega_i - \kappa_\theta^i\hat{\theta}_i^T\tilde{\theta}_i \\
&- \frac{\kappa_\rho^i}{\Gamma_i}\tilde{\rho}_i\hat{\rho}_i + c\omega_{i,5} + c\omega_{i,6}\rho_i,
\end{aligned} \tag{31}
$$

where $\bar{\pi}_i \in R^+$ is the maximum of π_i (it can be known that $\bar{\pi}_i \geq \left\|\pi_i(\bar{x}_i) - \pi_i(\hat{\bar{x}}_i)\right\|$). Note that the term $\|e_i\theta_i^*\|\,\bar{\pi}_i$ is resulted from the event-induced state discrepancy for the local controller.

Similar to (20), we have

$$
\|e_i\theta_i^*\|\,\bar{\pi}_i \leq \frac{\bar{\pi}_i\varpi_i}{2}x_i^Tx_i + \frac{\bar{\pi}_i}{2\varpi_i}\sigma_{\max}(B^TP^2B)\,\|\theta_i^*\|^2, \tag{32}
$$

where ϖ_i is a positive scalar such that $Q_{\varpi,i} = Q_i - \bar{\pi}_i\varpi_i I > 0$ (e.g., $\varpi_i < \frac{\sigma_{\min}(Q_i)}{\bar{\pi}_i}$). By completing the squares for the cross terms in (31) and based on (32), we have

$$
\dot{V}_{f,i} \leq -\delta_{h,i}V_{f,i} + c_{h,i}, \tag{33}
$$

where

$$
\begin{aligned}
\delta_{h,i} =& \min\left\{\frac{\sigma_{\min}(Q_{\varpi,i})}{\sigma_{\max}(P_i)}, \frac{\kappa_\theta^i}{\sigma_{\max}(\Pi_i^{-1})}, \kappa_\rho^i\right\}, \\
c_{h,i} =& \left(\frac{\kappa_\theta^i}{2} + \frac{\bar{\pi}_i}{2\varpi_i}\sigma_{\max}(B^TP^2B)\right)\|\theta_i^*\|^2 \\
&+ \frac{\kappa_\rho^i}{2\Gamma_i}\rho_i^2 + c\omega_i + c\omega_{i,5} + c\omega_{i,6}\rho_i.
\end{aligned}
$$

For the case that some subsystems are controlled by the event-triggered fault accommodation controller as in (25), combining results in (17) and (21) with (33) implies that the overall interconnected system is uniformly ultimately stable (following from the conclusion for (22)). Note that, due to the asynchronous fault accommodation controller switch among different subsystems, the event-triggered nominal controller and fault accommodation controller may be used simultaneously for the interconnected system. Comparing (33) with (21) and (17), it can be known that the controller switch will not destabilize the corresponding subsystem and also the overall interconnected system. The difference is that the residual $c\omega_{\chi,i}$ may be unbounded without the fault accommodation after the detection of any fault, and its effect can be decreased by using the fault accommodation scheme.

The following theorem provides a summary for the event-triggered fault accommodation controller.

Theorem 2. Let Assumptions 2-4 hold. The event-triggered nominal controllers in (25) with (11) under the triggering scheme (2)-(5) guarantees that the state of the nonlinear interconnected system (1) is uniformly ultimately bounded, after the detection of a fault in any subsystems.

V. Conclusions

The event-triggered fault accommodation for interconnected nonlinear systems has been studied. The inter-subsystem and controller to actuator communications are determined by the predefined events. The target control schemes for fault-free, fault detection delay, and post-fault detection time periods have been proposed, and uniformly ultimately bounded control performance for the overall interconnected system can be guaranteed.

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