# Neural Network Augmented Intelligent Iterative Learning Control for a Nonlinear System

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***Abstract*—An iterative learning controller (ILC) is an online method which exploits the information of past trials to improve the performance of the system. For a system controlled by ILC, the state, error, and ILC time histories for varying operating conditions can be recorded. This paper proposes an offline learning method using a neural network which exploits this data-set to approximate the converged ILC for a nonlinear system. The proposed method provides an approximate ILC for the first iteration based on the data collected thereby achieving a faster convergence. The efficiency of the method is tested for a nonlinear problem and results are presented.**

***Index Terms*—iterative learning control, neural network, offline learning**

## I. Introduction

Iterative learning control (ILC) method is based on the idea that the performance of a system that executes the same process multiple times can be improved by learning from the previous trials [1]. ILC applications are heavily designed for systems with same operating conditions. For such systems the tracking error on each trial is the same. ILC controller exploits the information of tracking error for previous trial to improve the control for subsequent trials there by improving the performance of the controller for the system. There are other learning controllers such as adaptive control, repetitive control and neural network (NN) control which have been developed and successfully applied to several systems defining their efficacy to improve the performance of the system.

In [1], Bristow et al gave an extensive study of iterative learning control focusing on the analysis and design. The survey describes the ideas, potential and limitations of ILC and provides a detailed description of the different design techniques. Since the development of ILC, it has been applied to several engineering problems, both linear and nonlinear. The paper by Liu et al [2], focused on application of ILC to linear time varying systems with high order internal model (HOIM). In this case, an ILC controller was designed for an iteratively varying reference trajectory where the HOIM was formulated using a polynomial between two consecutive trials. Another application of ILC for a linear time varying system was described in [3]. In this paper, the main focus was on the design of an ILC controller with input-output constraints. The application of ILC has been abundant in the area of robots which perform the same task. In the work by Barton et al [4], a norm optimal approach to time varying ILC system was detailed. The controller was used to design optimal learning filters based on design objectives. The efficacy of the proposed method was shown through its implementation for a MIMO motion system. ILC control has also been studied for nonlinear time varying systems. In the work by Chien [5], for a discrete MIMO nonlinear time varying system with initial state error, input disturbance, and output measurement noise, a discrete iterative learning controller was presented. The learning controller was updated with a higher order feed-forward learning algorithm.

ILC has been combined with other learning methods like neural network to update the learning control. In the works presented in [8] and [9], an online NN control was used to predict the learning control for the subsequent operating cycle. In [8] Chien et al, proposed a feed-forward neural network with sigmoid hidden units which was used to design a NN based ILC for a nonlinear system. The weights of the neurons were updated during each trial depending on the desired learning performance. A Lyapunov like analysis was utilized to determine the adaptive law for weights of neurons and analysis of learning performance. In the work presented in [10], [11], and [12], the authors presented a neural network based iterative learning controller as an alternative to conventional adaptive and ILC schemes. A neural network is used to learn the state space model of the unknown plant based on input/output data collected from closed loop control for an unstable/poorly damped system. Once a satisfactory neural model was obtained for the nonlinear process, an ILC update rule based on data driven neural network was used for the control synthesis. The resulting method is a learning process with adaptable training parameters. The authors claimed to achieve zero error convergence for both P and D type learning controllers.

In [13], Liu et al presented an adaptive terminal ILC to track iteration varying target points. This paper proposed a neural network-based state learning mechanism which relaxes the strict identical initial condition requirement for terminal ILC schemes. In addition to relaxed initial condition the desired terminal point was also assumed to be iteration varying. The neural network approximates the effect of varying initial states on the terminal output and the weights of the NN are

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identified iteratively. The learning and adaptation are restricted by a predefined bound on the tracking error. The authors claimed that the method was able to track the iteration varying desired points with specified accuracy. Another neural network based adaptive learning method was presented in [14]. In this paper an adaptive ILC method to mitigate the trajectory tracking errors for robot manipulators with arbitrary initial errors was proposed. To overcome the nonzero initial errors, time varying boundary layer are used and an adaptive learning neural network was used to approximate the uncertain robotic systems. The scheme used a saturation difference learning method to estimate the unknown optimal weights for the neural network and the upper limit of the approximation error. The paper claimed through results that with the adaptive scheme all signals of the closed loop system remained bounded and perfect tracking performance was achieved. In [15] by Chi et al, a data driven predictive ILC design is presented. The presented controller design only depended on the input/output data of the system eliminating the need for an explicit mathematical model. The performance of the controller was enhanced by more prediction information along the iteration axis. The algorithm is claimed to have achieved desired control effect.

So far, the applications of learning controllers have been fairly focused on their ability to learn online. ILC controller design is based on the past error information available on hand, to improve the system performance online. The main motivation of this paper is the utilisation of all the stored information based on which an ILC scheme is operated. This paper therefore focuses on predicting the end control of an ILC algorithm based on the data available for a particular system for all the operating conditions such that the computational effort is reduced. Based on the extensive data collected for a system that performs the particular task, an offline NN can be trained using carefully selected inputs against the desired control to predict the control requirement with minimal tracking error for a new operating condition for the same system. The proposed scheme is implemented and tested for its effectiveness and the results are presented.

Rest of the paper is constructed as follows. Section II gives a brief description of the nonlinear system dynamics. Section III describes the feedback and ILC controller design. Section IV focuses on the learning method for predicting the end controller input for the same system based on the data available followed by Section V in which the comparison study of the effectiveness of the proposed method is presented. The conclusion of the paper is given in section VI.

## II. Problem Formulation

### A. Dynamics of Earth Orbiter

The trajectory of an Earth orbiter in low altitude is significantly affected by the $J_2$ perturbation which arises due to the oblateness of the Earth. The effect of this equatorial bulge is significant when analyzing Earth's gravitational potential function [6].

The dynamics of a low altitude Earth orbiter can be modelled using the two body gravitational model including the $J_2$ effect as

$$\ddot{\mathbf{r}} = -\frac{\mu \mathbf{r}}{r^3} + J_{2,pert} \tag{1}$$

where $\mathbf{r} = \begin{bmatrix} x & y & z \end{bmatrix}^T$ and $r = \sqrt{x^2 + y^2 + z^2}$. The second term $J_{2,pert}$ in equation (1) represents the perturbation due to the equatorial bulge and is a highly nonlinear function defined by $J_{2,pert} = (\frac{3\mu {R_e}^2 (5z^2 - r^2) J_2}{2r^7})\mathbf{r}$. The dynamics can be rewritten in the state space representation with a state vector defined as $\mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{bmatrix}^T = \begin{bmatrix} x & y & z & \dot{x} & \dot{y} & \dot{z} \end{bmatrix}^T$. The equations of motion of the Earth orbiter is then represented in terms of $\mathbf{x}$ as

$$\dot{\mathbf{x}} = f(\mathbf{x}) + B\tilde{f}(\mathbf{x}) + B\mathbf{u}$$
$$= \underbrace{\begin{bmatrix} x_4 \\ x_5 \\ x_6 \\ -\frac{\mu x_1}{r^3} \\ -\frac{\mu x_2}{r^3} \\ -\frac{\mu x_3}{r^3} \end{bmatrix}}_{f(\mathbf{x})} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} \frac{3\mu {R_e}^2 x_1 (5x_3^2 - r^2) J_2}{2r^7} \\ \frac{3\mu {R_e}^2 x_2 (5x_3^2 - r^2) J_2}{2r^7} \\ \frac{3\mu {R_e}^2 x_3 (5x_3^2 - r^2) J_2}{2r^7} \end{bmatrix}}_{\tilde{f}(\mathbf{x})} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{B} \mathbf{u} \tag{2}$$

$$\mathbf{y} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}}_{C} \mathbf{x} \tag{3}$$

where $\tilde{f}(\mathbf{x})$ represents the $J_{2,pert}$ in terms of $\mathbf{x}$, $\mu$ is the gravitational constant of the Earth (3.986 $\times 10^5$ $km^3/s^2$), $R_e$ is the radius of the Earth($6378.14 km$) and $J_2 = 0.0010826$.

## III. Controller Design

The controller given in equation (2) has two components. One is the baseline feedback controller $\mathbf{u}_{fb}$ and the other is the ILC controller $\mathbf{u}_{ilc}$ [7]. It should be noted that since the orbiter revolves in the particular low altitude orbit it is launched to, an ILC scheme is an adequate control method that can be applied to this problem, as the trajectory remains the same. Because the orbiter and Earth rotate in the inertial frame the time period of the launched orbit is taken as the ILC operating cycle. It should also be noted that in each cycle the $J_2$ perturbation effect is the same which satisfies the repetitive disturbance in the system for an ILC problem. The desired trajectory for this problem is assumed to be the trajectory of an orbiter in the absence of the $J_2$ perturbation.

### A. Feedback Control Design

The feedback controller is designed to track the desired orbit and is based on the tracking error. The tracking error in position vector is defined as $\mathbf{e} = \begin{bmatrix} e_1 & e_2 & e_3 \end{bmatrix}^T$ where $e_i = x_i - x_{i,d}$ for $i = 1 : 3$ and velocity vector as $\dot{\mathbf{e}} = \begin{bmatrix} e_4 & e_5 & e_6 \end{bmatrix}^T$ where $e_i = x_i - x_{i,d}$ for $i = 4 : 6$. The feedback controller is designed such that the closed loop system is Hurwitz stable. The input/output feedback linearization method is used for feedback control design which states that, if the error dynamics of the system is Hurwitz stable, the tracking error should go to zero asymptotically [16].

The system dynamics defined by (2) and (3), in the absence of perturbation effect can be rewritten as

$$\begin{aligned}\dot{\mathbf{z}}_1 &= \mathbf{z}_2 \\ \dot{\mathbf{z}}_2 &= f(\mathbf{z}) + \mathbf{u}_{fb}\end{aligned} \tag{4}$$

$$\mathbf{y} = \mathbf{z}_1 \tag{5}$$

Where $\mathbf{z}_1 = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$, $\mathbf{z}_2 = \begin{bmatrix} x_4 & x_5 & x_6 \end{bmatrix}^T$ and $f(\mathbf{z}) = -\frac{\mu}{r^3}\mathbf{z}_1$. The i/o feedback linearization technique for class of nonlinear problems is utilised to define the feedback control $\mathbf{u}_{fb}$ as follows [16]:

A linear system representation can be obtained by redefining the input as

$$\mathbf{v} = f(\mathbf{z}) + \mathbf{u}_{fb} \tag{6}$$

so that,

$$\mathbf{u}_{fb} = -f(\mathbf{z}) + \mathbf{v} \tag{7}$$

giving,

$$\begin{aligned}\dot{\mathbf{z}}_1 &= \mathbf{z}_2 \\ \dot{\mathbf{z}}_2 &= \mathbf{v}\end{aligned} \tag{8}$$

which is equivalent to

$$\ddot{\mathbf{y}} = \mathbf{v} \tag{9}$$

For the above feedback linearized system, a PD tracking controller can be designed as,

$$\mathbf{v} = \ddot{\mathbf{y}}_d - A_1\dot{\mathbf{e}} - A_2\mathbf{e} \tag{10}$$

where $\mathbf{e}$ is the position tracking error and $\mathbf{e} = \mathbf{y} - \mathbf{y}_d$. Note that, $\mathbf{y} = \mathbf{z}_1 = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$. Substituting this control $\mathbf{v}$ into (9) yields the closed loop system,

$$\ddot{\mathbf{e}} + A_1\dot{\mathbf{e}} + A_2\mathbf{e} = 0 \tag{11}$$

which is equivalent in state space form as,

$$\frac{d}{dt}\begin{bmatrix} \mathbf{e} \\ \dot{\mathbf{e}} \end{bmatrix} = \underbrace{\begin{bmatrix} 0_{3\times 3} & I_{3\times 3} \\ -A_2 & -A_1 \end{bmatrix}}_{\tilde{A}} \begin{bmatrix} \mathbf{e} \\ \dot{\mathbf{e}} \end{bmatrix} \tag{12}$$

The tracking error convergence can be ensured by carefully choosing the feedback gains $A_1$ and $A_2$. Thus the final feedback control expression obtained from this method is,

$$\mathbf{u}_{fb} = -f(\mathbf{z}) + \ddot{\mathbf{y}}_d - A_1\dot{\mathbf{e}} - A_2\mathbf{e} \tag{13}$$

The equivalent feedback control for the earth orbiter in (2) and (3), in terms of $\mathbf{x}$ is then given by,

$$\mathbf{u}_{fb} = -\begin{bmatrix} -\frac{\mu x_1}{r^3} \\ -\frac{\mu x_2}{r^3} \\ -\frac{\mu x_3}{r^3} \end{bmatrix} + \ddot{\mathbf{y}}_d - A_1\dot{\mathbf{e}} - A_2\mathbf{e} \tag{14}$$

### *B. Iterative Learning Control Design*

The feedback control stabilises the system with respect to $J_2$ [7]. The contribution of ILC controller is to cancel the periodic perturbation effect caused by the oblateness of the Earth. The ILC controller is only needed once the orbiter is stabilised in the desired trajectory and revolves around the Earth. The ILC controller uses the control and tracking error from the previous cycle. It reduces the tracking error from the feedback controller and drives the error to zero. The ILC controller expression is given as,

$$\mathbf{u}_{ilc,j}(i) = \mathbf{u}_{j-1}(i) + L_q\mathbf{e}_{j-1}(i+1) \tag{15}$$

where $i$ denotes the time-step and $j$ denotes the operating cycle and $L_q$ is the learning gain for the tracking error. Thus, the total controller expression for the above problem is,

$$\mathbf{u}_j = \mathbf{u}_{fb,j} + \mathbf{u}_{ilc,j} \tag{16}$$

Note that for the first operating cycle, there exists only the feedback controller which stabilizes the closed loop system and the ILC is introduced to correct the tracking error after the feedback controller, due to the $J_2$ perturbation effect.
Consider the system defined as,

$$x_j(i+1) = Ax_j(i) + Bu_j(i) \tag{17}$$

$$y_j(i) = Cx_j(i) + d(i) \tag{18}$$

where $j$ is the iteration number, $i$ is the time-step, $q$ is the time shift operator and $d$ is the repetitive disturbance in the system response. From (17) and (18), $y_j(i)$ can be rewritten in the following form

$$y_j(i) = \underbrace{C(qI - A)^{-1}B}_{\text{P(q)}}\, u_j(i) + d(i) \tag{19}$$

The ILC controller for the above system is given by

$$u_j = u_{j-1} + L_q e_{j-1} \tag{20}$$

Then the error convergence for the above ILC controller can be proved as follows:
Consider the system dynamics in iteration domain given by (19). Let $r$ be the reference trajectory. Then the tracking error $e_j$ can be obtained as

$$e_j = r - y_j \tag{21}$$

Substituting (19) in (21) we get,

$$e_j = r - Pu_j - d \tag{22}$$

Rearranging the terms in (22) the controller expression can be obtained for the $j^{th}$ iteration as

$$u_j = P^{-1}(r - e_j + d) \tag{23}$$

and for the $j - 1^{th}$ iteration, it can be given as

$$u_{j-1} = P^{-1}(r - e_{j-1} + d) \tag{24}$$

substituting $u_j$ and $u_{j-1}$ in (20) and rearranging,

$$e_j = (I - PL_q)e_{j-1} \tag{25}$$

The above equation defines the closed loop error dynamics in the iteration domain. For such a system the asymptotic stability is defined in [1] by the following theorem.

*Theorem 1:* For a system defined in (25) the asymptotic stability exists if and only if

$$\rho(I - PL_q) < 1 \tag{26}$$

where the $\rho(A) = max_i|\lambda_i(A)|$ is the spectral radius of the matrix $A$ and $\lambda_i(A)$ is the $i^{th}$ eigenvalue of $A$. The learning gain $L_q$ is designed such that the above theorem is satisfied. The ILC scheme for the Earth orbiter problem is depicted in Figure 1.

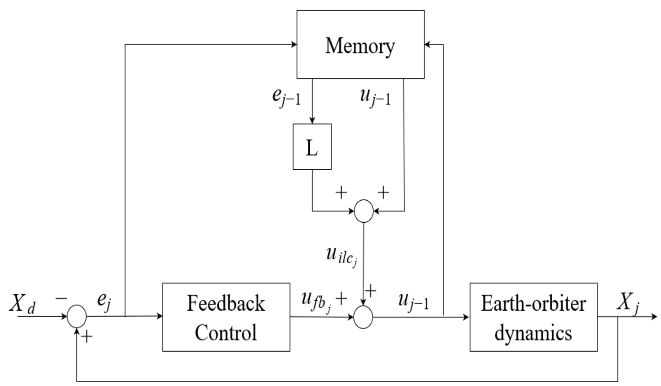


Figure 1. ILC scheme

## IV. Neural Network Augmented Intelligent Iterative Learning Control

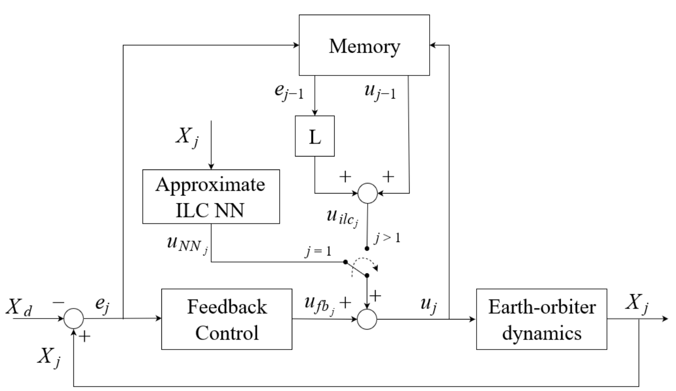


Figure 2. Neural Network scheme

This section defines how a learning mechanism like NN can be used to get an approximation of the controller for a system based on the data available for different operating conditions. For example, for the Earth orbiter problem defined in Section II, the ILC defined in Section III can be utilized to negate the error in tracking caused due to the $J_2$ perturbation. The performance of designed ILC is measured based on the decreasing norm of the tracking error with iterations. For the Earth orbiter problem the same control scheme can be used for controlling orbiters at different lower earth orbits. This implies that once the orbiter has been stabilised in each orbit the ILC will negate the effects of $J_2$ perturbation with required number of iterations to satisfy the specified convergence criteria.

The proposed scheme given in Figure 2, attempts to exploit the data collected for a system controlled by the ILC scheme with varying operating conditions. For the orbiter problem defined above, this data consists of the converged position history and the converged ILC history corresponding to a range of low earth orbits. An offline NN is used to learn the correlation between the converged position and the converged ILC. In the above case, a two layer NN which consists of 3 inputs, 30 neurons in the hidden layer, and 3 neurons in the output layer was implemented. The activation function used for the hidden layer is tangent sigmoid and the activation function used for the output layer is linear. The Bayesian regularization-based back-propagation algorithm is used to train the NN [17], [18]. The data-set used to train and validate the NN is randomised in order to improve the training of the NN. The NN is trained by using 70 percent of the data collected, and validated using the remaining 30 percent data.

This NN approximation based on available data can provide an approximate control effort at each time instant for the ILC to control the orbiter in a new orbit within the range of data collected. The performance of the proposed method is illustrated with simulation results in Section V. This NN augmented ILC technique can be applied to other systems for which there exist a relevant data-set. With the proposed method, in the first iteration itself, in addition to feedback control the NN provides an ILC control input. Thus faster convergence is achieved in terms of iterations, due to this additional control from NN.

## V. Simulation Results

The ILC control method with a stabilising feedback control is designed by the procedure presented in section III for the Earth orbiter problem defined in section II. For this study circular polar orbits are considered. For the desired trajectory the orbiter is assumed to begin orbiting at 45° latitude and 45° longitude . For the simulation study a range of altitudes from $100 - 1000$ km are considered. For the actual system with perturbation, the orbiter is assumed to begin orbiting at 30° latitude ,60° longitude, and the altitude is set 100 km higher than the desired orbit. The tuning parameters for this example are selected as,

$$\tilde{A} = \begin{bmatrix} 0_{3\times3} & I_{3\times3} \\ -3\times10^{-3}I_{3\times3} & -3\times10^{-1}I_{3\times3} \end{bmatrix}$$

and learning gain $L_q$ of

$$L_q = \begin{bmatrix} -9\times10^{-12}I_{3\times3} & -9\times10^{-9}I_{3\times3} \end{bmatrix}$$

The convergence criterion for the ILC method is designed such that the norm of the error of the state vector is less than a specified threshold. In this example the threshold is set at $||E|| < 10^{-2}$and $||\dot{E}|| < 5\times10^{-5}$, where $E = \begin{bmatrix} e_x & e_y & e_z \end{bmatrix}^T$ is the position error vector and $\dot{E} = \begin{bmatrix} \dot{e}_x & \dot{e}_y & \dot{e}_z \end{bmatrix}^T$ is the velocity error vector. The results for the ILC method for orbit at 200 km altitude are given below.

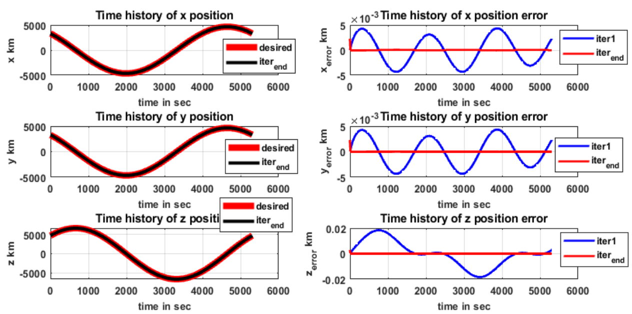


Figure 3. Position history vs time

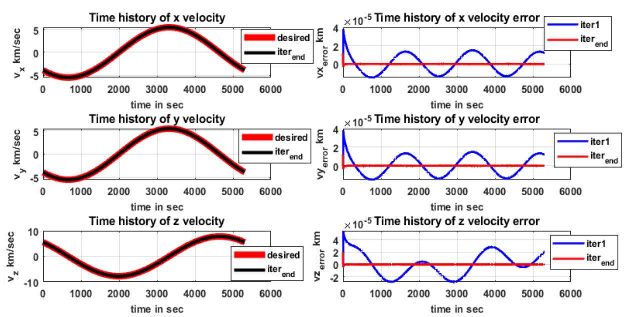


Figure 4. Velocity history vs time

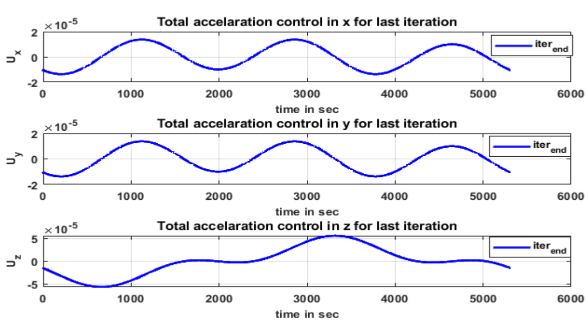


Figure 5. Total control history vs time for last iteration

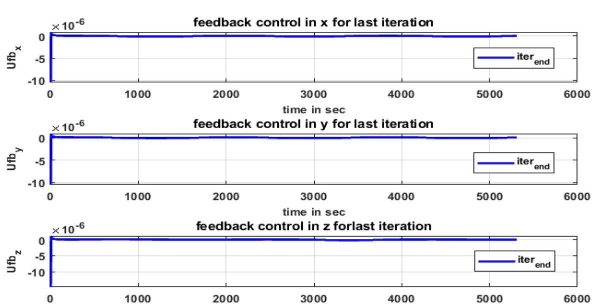


Figure 6. Feedback control history vs time for last iteration

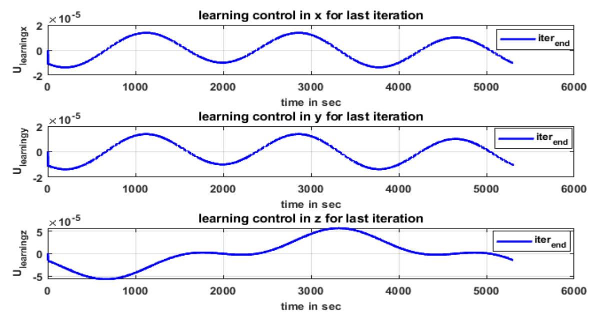


Figure 7. Learning control history vs time for last iteration

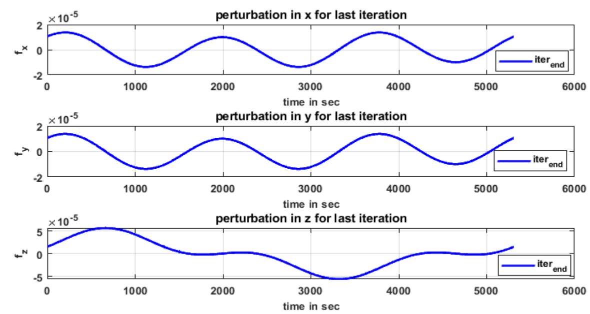


Figure 8. $J_2$ perturbation history vs time

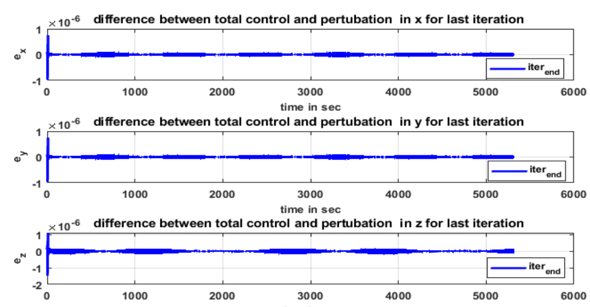


Figure 9. Error between final control and perturbation

From Figures 3 and 4, it can be seen that the actual states are following the desired states. Comparing the errors in position and velocity for the first and the last iterations, although the feedback control reduces the error to an order of $10^{-3}$, with more iterations the ILC control drives it to zero. This trend implies the asymptotic stability of tracking errors due to disturbances in the iteration domain. This shows the effectiveness of the selected learning gain $L_q$. From Figures 5-8, it can be seen that in the last iteration, the contribution to total control is dominated by the ILC control component and it can be seen to be almost equal in magnitude to the perturbation in $x, y \& z$. Figure 9, shows how accurately the ILC control compensates for the perturbation in the system. The values of $||E||$ and $||\dot{E}||$ for each iteration are given in

Table I. From this data it can be seen that the norm value of state errors decreases with increasing iterations.

Table I
NORM OF ERROR IN **X** FOR ORBIT AT 200 KM(ILC)

| | $\|\|E\|\|$ | | $\|\|\dot{E}\|\|$ | |
|---|---|---|---|---|
| axis | iter 1 | iter 2 | iter 1 | iter 2 |
| x | 1.0713 | 0.0063 | 0.0040 | 3.98E-05 |
| y | 1.0713 | 0.0063 | 0.0040 | 3.98E-05 |
| z | 3.7780 | 0.0095 | 0.0071 | 4.96E-05 |

The control histories and state histories for different altitudes of low Earth orbits were collected. The designed NN is trained using the position time histories against the converged ILC for each orbit, based on the data. To test the NN approximation, the trained NN is utilised to calculate the ILC equivalent control which is applied to the system along with the stabilizing feedback control, for the same 200 km orbit, for just one trial. The resulting norm of the state errors from this simulation are given in Table II.

Table II
NORM OF ERROR IN **X** FOR ORBIT AT 200 KM(NN)

| | $\|\|E\|\|$ | $\|\|\dot{E}\|\|$ |
|---|---|---|
| axis | iter 1 | iter 1 |
| x | 0.0034 | 0.00018 |
| y | 0.0034 | 0.00018 |
| z | 0.0057 | 0.00026 |

Comparing the norm of error values in Table I and II, it can be seen that the NN approximation with the state feedback has considerably better performance. Using the NN approximated control, it can be seen that the norm of state vector is less than the defined threshold after the first iteration which proves that the NN approximation is close to the desired ILC. The results of the proposed method for a new altitude within the range of the data collected (115 km) was tested and the results are depicted below.

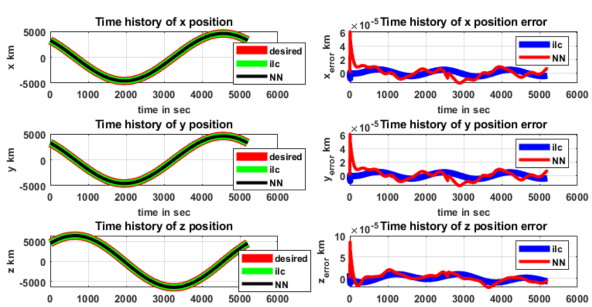


Figure 10. Position history vs time

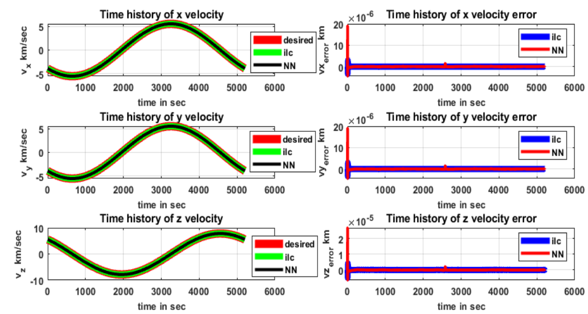


Figure 11. Velocity history vs time

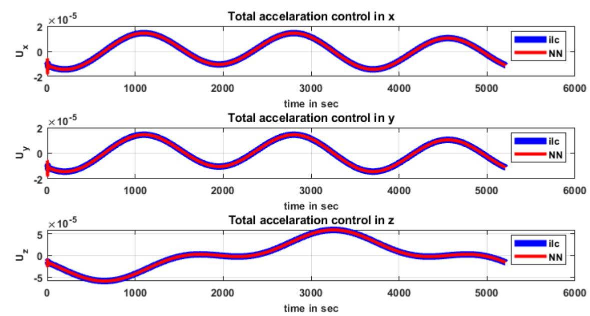


Figure 12. Total control time history for last iteration

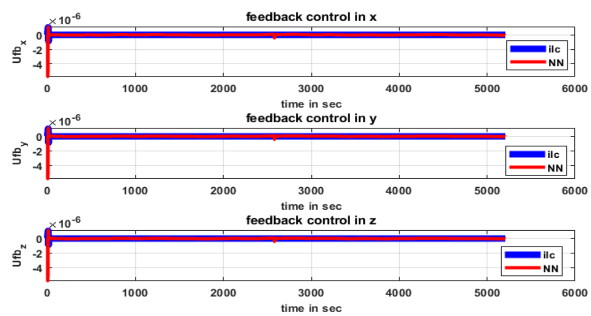


Figure 13. Feedback control history vs time for last iteration

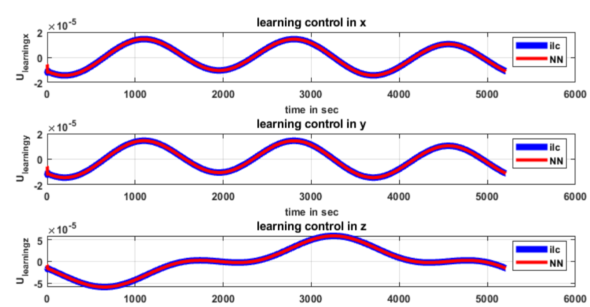


Figure 14. Learning control history vs time for last iteration

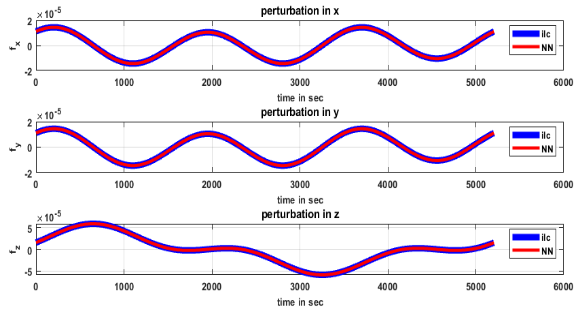


Figure 15. $J_2$ perturbation history vs time

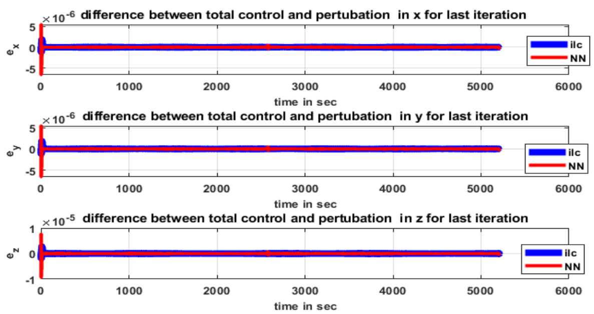


Figure 16. Error between final control and perturbation

From Figures 10-11, it can be seen that the results of NN scheme is close to the desired states and the corresponding results of the last iteration of the ILC method. The error values of states are also in the same range of order of magnitude, in both cases. Figures 12-14, show that the approximated control value from the NN even with only one trial is fairly close to the equivalent control in the ILC. The corresponding norm of error values for the ILC method and the NN predicted control are given in Table III and IV.

Table III
NORM OF ERROR IN X FOR ORBIT AT 115 KM(ILC)

| | $\|\|E\|\|$ | | | $\|\|\dot{E}\|\|$ | | |
|---|---|---|---|---|---|---|
| axis | iter 1 | iter 2 | iter 3 | iter 1 | iter 2 | iter 3 |
| x | 1.1152 | 0.0068 | 0.0013 | 0.0043 | 4.23E-05 | 3.22E-05 |
| y | 1.1152 | 0.0068 | 0.0013 | 0.0043 | 4.23E-05 | 3.22E-05 |
| z | 3.9388 | 0.0103 | 0.0023 | 0.0075 | 5.26E-05 | 4.71E-05 |

Table IV
NORM OF ERROR IN X FOR ORBIT AT 115 KM(NN)

| | $\|\|E\|\|$ | $\|\|\dot{E}\|\|$ |
|---|---|---|
| axis | iter 1 | iter 1 |
| x | 0.0030 | 1.53E-05 |
| y | 0.0030 | 1.53E-05 |
| z | 0.0041 | 2.17E-05 |

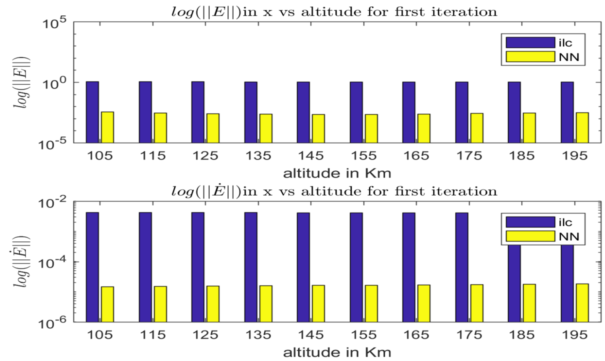


Figure 17. Variation of $||E||$ & $||\dot{E}||$ in x axis vs altitude.

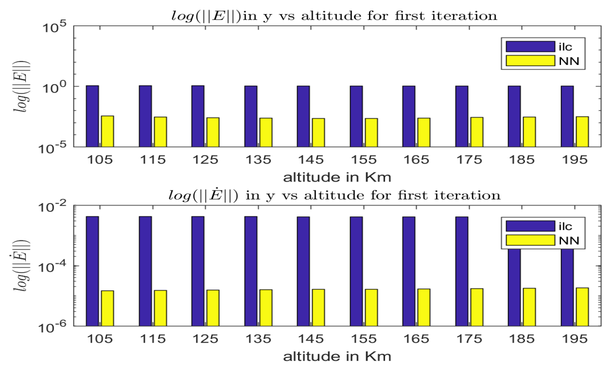


Figure 18. Variation of $||E||$ & $||\dot{E}||$ in y axis vs altitude.

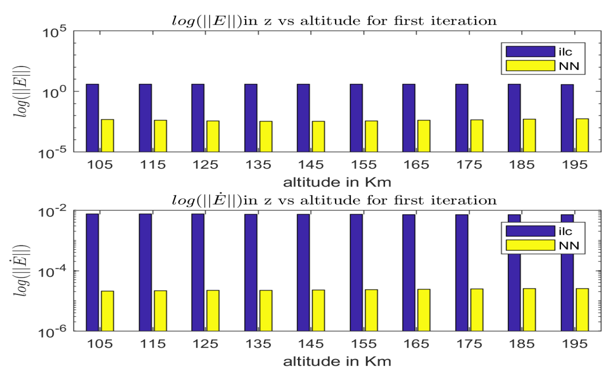


Figure 19. Variation of $||E||$ & $||\dot{E}||$ in z axis vs altitude

Even for a new altitude, the norm values of the proposed scheme is less than the values of second iteration in the traditional ILC and close to the last iteration in the traditional ILC. From Figures 17-19, it can be seen that for a set of new altitudes the norm of error value in all three axes are comparatively less for first iteration for the proposed method than the traditional ILC. This trend confirms that the NN approximation is accurate. Thus using the proposed method the online computational effort for an ILC is reduced as it gives a faster convergence.

## VI. Conclusions

An ILC improves the system performance based on past trials. However, even for a fine tuned learning gain the ILC scheme takes several iterations to converge. The NN based method presented in this paper provides an effective way to utilise the data collected for a learning control to be used for the enhancement of the controller performance. The proposed scheme thus attempts to give an approximate control for the first iteration based on the data collected for the system where an ILC is used. The data for a range of different operating conditions is embedded in an offline NN and is used for computing online control. Hence this technique reduces the computational effort and guarantees a faster convergence compared to the traditional ILC method. From the representative numerical results the proposed method seems to have a good potential for online use for typical application where the ILC is currently used.

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