GLOBAL ASYMPTOTIC VELOCITY OBSERVATION OF NONLINEAR SYSTEMS

Application to a Frictional Industrial Emulator

R. Guerra[‡], C. Iurian[*], L. Acho[‡*]

[‡]*Centro de Investigación y Desarollo de Technología Digital (CITEDI-IPN), Mexico*
[*]*Universitat Politècnica de Catalunya, Matemàtica Aplicada III, EUETIB, Barcelona, Spain*
inge_guerra@hotmail.com, {claudiu.iurian, leonardo.acho}@upc.edu

F. Ikhouane[†] and J. Rodellar[§]

[†]*Universitat Politècnica de Catalunya, Matemàtica Aplicada III, EUETIB, Barcelona, Spain*
[§]*Universitat Politècnica de Catalunya, Matemàtica Aplicada III, Barcelona, Spain*
{faycal.ikhouane, jose.rodellar}@upc.edu

Abstract: Development of velocity observers for mechanical systems with friction deserves a special emphasis, because as evidenced in numerical and experimental tests when a state-of-the-art observer is armed, friction can induce high-frequency oscillations in the estimated signal. In this short paper, two new velocity-observation algorithms are designed, based on this previously reported observer, that eliminate the high-frequency oscillations noted. Numerical and experimental performance comparisons are carried through making use of LuGre model and a mechanical PID control system that incorporates the estimated velocity into the feedback loop.

1 INTRODUCTION

Velocity-dependent control laws such as PD, PID, and most robust control laws, among many others, theoretically require direct access to velocity. In reality, there are many applications in which this is not available either due to considerable manufacturing savings in cost, weight, and volume, or because the velocity measurements are highly contaminated with noise. In the latter case, for instance when measuring robot joint velocities, it may not even be desirable to do so (Arteaga and Kelly, 2004). Consequently, if the full-state information is missing, it is necessary to estimate the unmeasurable velocity through the use of an observer and feed it back into the controller. Such is the case with the frictional industrial emulator ECP model 220 used in our experiments. It has been documented when studying mechanical closed-loop control systems, that friction causes tracking errors, limit-cycles, and stick-slip motions, among other difficulties and usually unwanted phenomena (Armstrong-Hétlouvry et al., 1994). As evidenced in (Canudas de Wit and Fixot, 1991), (Canudas de Wit and Fixot, 1992), (Berghuis and Nijmeijer, 1993), and (Arteaga and Kelly, 2004) velocity observer design is a topic that has been and continues to be ex-

tensively studied. State observation of nonlinear inexact plants has been treated by utilizing discontinuous observers (Choi et al., 1999), (Xiong and Saif, 2001), and (Xian et al., 2004). However, little research focused on velocity observation of mechanical systems with friction at low velocities which, when incorporating existing observers, exhibit high-frequency oscillations in the velocity-estimation signal. A state-of-the-art, globally asymptotic, discontinuous velocity-estimation scheme for second-order mechanical systems has been presented in (Xian et al., 2004). Though very reliable, this observer is not specialized for mechanical systems in the presence of friction. Thus, a high-frequency component of the velocity observation is detected when numerical simulations and experimental testing are performed. The two newly proposed observers claimed here are an attempt to alleviate this unwanted oscillatory effect and try offer an increase in the observation reliability.

The remainder of this document proceeds as follows: Section 2 postulates two modified observers based on the velocity estimator stated in (Xian et al., 2004). The next section presents numerical simulations evidencing the performance of the observers. Section 4 begins with a description of the frictional experimental testbed, the ECP industrial emulator

(ECP, 1995), and illustrates the results obtained when implementing of the original observer as well as the two modifications proposed in the PID mechatronic system. Finally, conclusions are drawn in Section 5.

2 VELOCITY OBSERVATION

Consider a class of mechanical systems expressed by (Xian et al., 2004)

$$\ddot{x} = h(x,\dot{x}) + G(x,\dot{x})u, \qquad (1)$$

where $x \in \mathbb{R}$ is the system output, $u(t) \in \mathbb{R}$ is the control input, and $h(x,\dot{x}) \in \mathbb{R}$ as well as $G(x,\dot{x}) \in \mathbb{R}$ are nonlinear functions[1]. The system (1) satisfies the following assumptions (Xian et al., 2004).

Assumption A1. Both $h(x,\dot{x})$ and $G(x,\dot{x})$ are C^1 functions.

Assumption A2. The control input is a C^1 function and $u(t), \dot{u}(t) \in L_\infty$.

Assumption A3. The system state is bounded for all time; *i.e.*, $x(t), \dot{x}(t) \in L_\infty$.

The goal of the velocity observer is to estimate the unmeasurable velocity signal $\dot{x}(t)$ using only-position measurement and assuming that $h(x,\dot{x})$, $G(x,\dot{x})$ and $u(t)$ are unknown (Xian et al., 2004). Let $\hat{x}(t) \in \mathbb{R}$ be the estimated velocity and $\tilde{x} = \dot{x} - \hat{x}$ the velocity estimation error. Then, the objective of the velocity observer is to ensure that $\dot{\tilde{x}}(t)$ converges to zero as the time tends to infinity.

Consider the following velocity observer (Xian et al., 2004)

$$\begin{aligned} \dot{\hat{x}} &= p + k_0\tilde{x}, \\ \dot{p} &= k_1 sgn(\tilde{x}) + k_2\tilde{x}, \end{aligned} \qquad (2)$$

where k_0, k_1, and k_2 are positive constants, and $sgn(\cdot)$ is the signum function.

Remark 1. *System (2) can be expressed as*

$$\begin{aligned} \dot{y} &= f(y,x), \quad y \in R^2, \quad x \in R \\ z &= g(y,x), \quad z \in R \end{aligned}$$

where $y = [\hat{x} \quad p]^T$, $f(y,x) = [p + k_0\tilde{x} \quad k_1 sgn(\tilde{x}) + k_2\tilde{x}]^T$, $g(y,x) = p + k_0\tilde{x}$, *and* $z = \hat{x}$ *is the output velocity estimation.*

To state the main result in (Xian et al., 2004), let

$$N_0(x,\dot{x},t) = h(x,\dot{x}) + G(x,\dot{x})u(t).$$

---

[1] Without loss of generality, we have assumed a one-degree-of-freedom (DOF) mechanical system.

Theorem 1 (Xian et al., 2004). *The velocity observer (2) ensures global asymptotic regulation of* $\dot{\tilde{x}}(t)$ *(i.e.,* $\dot{\tilde{x}}(t) \to 0$ *as* $t \to \infty$*) provided that* k_1 *satisfies*

$$k_1 > ||N_0(x,\dot{x},t)||_\infty + ||\dot{N}_0(x,\dot{x},t)||_\infty.$$

For detailed proof, see Theorem 2 in (Xian et al., 2004). Let us now put forward the following observer

$$\begin{aligned} \dot{\hat{x}} &= p + k_0\tilde{x}, \\ \dot{p} &= -k_1 sgn(\tilde{x}) + k_2\tilde{x}. \end{aligned} \qquad (3)$$

Theorem 2. *The velocity observer (3) ensures global asymptotic regulation of* $\dot{\tilde{x}}(t)$ *(that is,* $\dot{\tilde{x}}(t) \to 0$ *as* $t \to \infty$*) provided that* k_1 *satisfies exactly the same conditions as in Theorem 1.*

Proof. Identical to that of Theorem 1.

Another alternative proposal to the original observer is the following

$$\begin{aligned} \dot{\hat{x}} &= p + k_0\tilde{x}, \\ \dot{p} &= k_1 sgn(\hat{x}) + k_2\tilde{x}. \end{aligned} \qquad (4)$$

Theorem 3. *The velocity observer (4) also ensures global asymptotic regulation of* $\dot{\tilde{x}}(t)$ *(i.e.,* $\dot{\tilde{x}}(t) \to 0$ *as* $t \to \infty$*) on the condition that* k_1 *satisfies the same restriction as in Theorem 1.*

Proof. The same case as for the proof of Theorem 2 above.

These innovative observers are very similar to the one in Theorem 1, but they present significant differences. The observer in Theorem 2 introduces an inversion of the sign in the estimation dynamic that produces a filtering effect. The observer in Theorem 3 brings forward an estimation error $\hat{x}$ inside the signum function, with the intent of reducing the high-frequency content the error signal $\tilde{x}$ present in (2).

3 NUMERICAL EXPERIMENTS

Consider a linear motion of unit mass

$$\ddot{x} = u - f, \qquad (5)$$

where f is the friction force. Assuming for a moment that $f = 0$ and $k_1 = 10$, the requirement imposed by all theorems is satisfied. We complete the velocity observers by setting $k_0 = k_2 = 10$. We also construct the following PID controller, which makes the closed-loop system globally asymptotically stable (if $f = 0$)

$$u = -k_p(x - x_d) - k_i \int (x - x_d)dt - k_d\dot{x}, \qquad (6)$$

with $k_v = 6$, $k_p = 3$, $k_i = 4$, and the position reference
set at $x_d = 1$ [m], see details in (Canudas de Wit et al.,
1995). Because friction force can produce limit cy-
cles within the system when the control law has an in-
tegrating action (Canudas de Wit et al., 1995), we in-
corporate a friction force in (5) by invoking the LuGre
model and its standard parameters given in (Canudas
de Wit et al., 1995); *i.e.*, f is obtained as a nonlinear
dynamic. Simulation results are depicted in Figure 1
where the position and velocity of the system are pic-
tured. These plots are a recreation of the experiment
presented in (Canudas de Wit et al., 1995). Obviously,
the PID controller (6) incorporates velocity measure-
ment, which for this simulation was assumed to be
available.

At this point, we repeated the previous simulation
using the true velocity in the control law and imple-
menting the three observers solely to estimate the ve-
locity, obtaining the results shown in Figure 2. Since
in the experimental application the velocity would
not be available for use, the simulation was again re-
peated, this time invoking the observers from Theo-
rems 1, 2, and 3 in the control law (*i.e.*, replacing $\dot{x}$ by
its corresponding $\hat{\dot{x}}$ in (6)). The results are portrayed
in Figures 3, 4, and 5, respectively. Note that cha-
ttering appears with the observer from Theorem 1 and
not with the others. It is important to stress that cha-
ttering is undesirable in a physical system because the
high-frequency switching can damage the system, as
well as activate unmodelled dynamics (G. Bartolini,
1998) and (Hung, 1993).

Let us return to the previously considered case
where the PID controller (6) is employed using the
exact velocity, which is only observed. If we slightly
modify the observer parameter values to $k_1 = 5$
and $k_0 = k_2 = 1$, the velocity observation obtained
with the observer in (2) becomes highly oscillatory
whereas the two observers proposed still provide
good results, as shown in Figure 6. If furthermore
we now feed the corresponding velocity estimation
into the PID controller (6), the first observer (stated
in Theorem 1) is much more sensible to variations in
its parameters. This causes instability of the closed-
control-loop (simulation pictures were then omitted
for the first case); nevertheless, the other two new ob-
servers yield acceptable good simulation results, see
Figures 7 and 8.

4 APPLICATION TO AN INDUSTRIAL EMULATOR

To have a more realistic comparison among the ob-
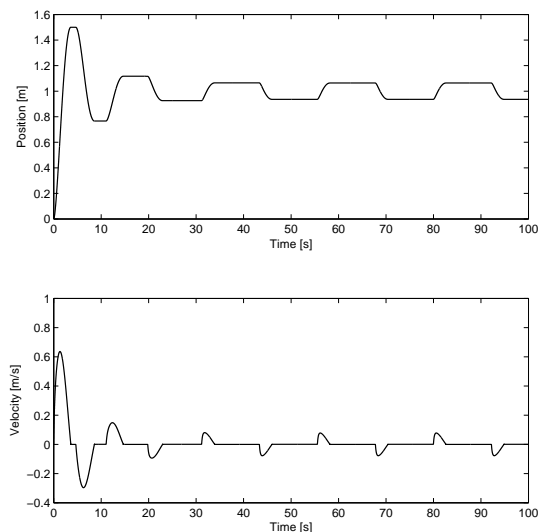servers stated in Section 2, we proceed to implement

Figure 1: PID position control of a second-order system
that incorporates Lugre friction model - see (Canudas de
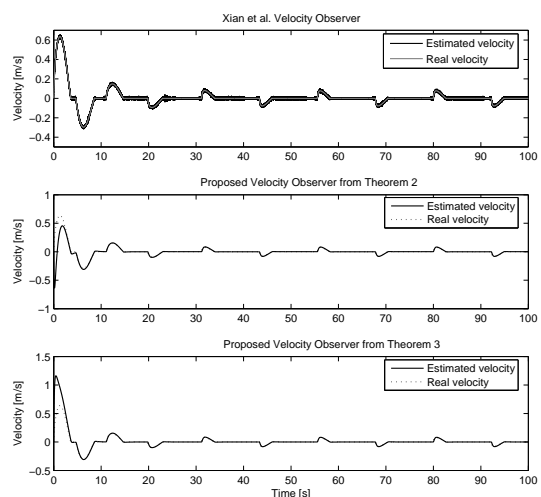Wit et al., 1995).

Figure 2: Comparison of the three estimators (for $k_{0} = k_1 = k_2 = 10$): 1) Top with Theorem 1, 2) Middle with
Theorem 2, and 3) Bottom with Theorem 3.

them on an experimentation testbed.

4.1 Experimental Setup

The experiments were performed on an ECP Model
220 industrial emulator which includes a PC-based
control platform and a DC brushless servo system
(ECP, 1995). The mechatronic system includes two
motors, one as servo actuator and the other as dis-
turbance input (not used here), a power amplifier,
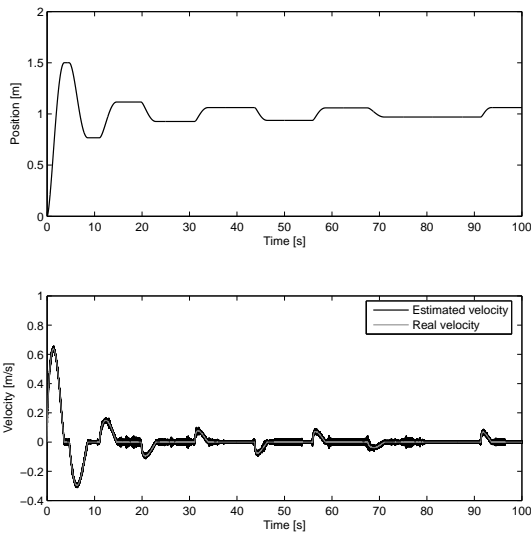and two encoders which provide accurate position

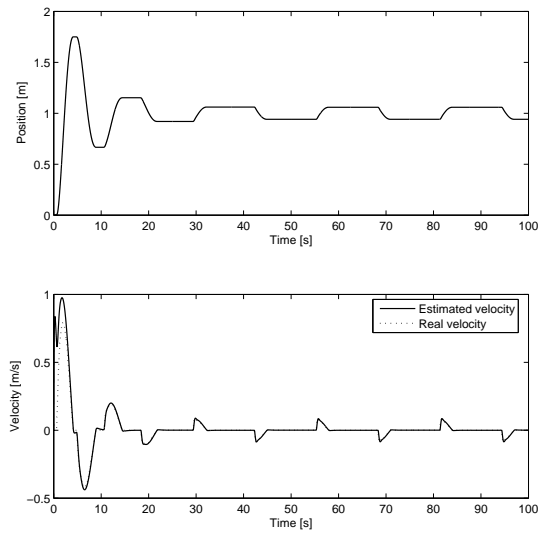Figure 3: PID control incorporating velocity estimation from Theorem 1.

Figure 4: PID control employing velocity estimation given in Theorem 2.

Figure 5: PID control utilizing the observer stated in Theorem 3.

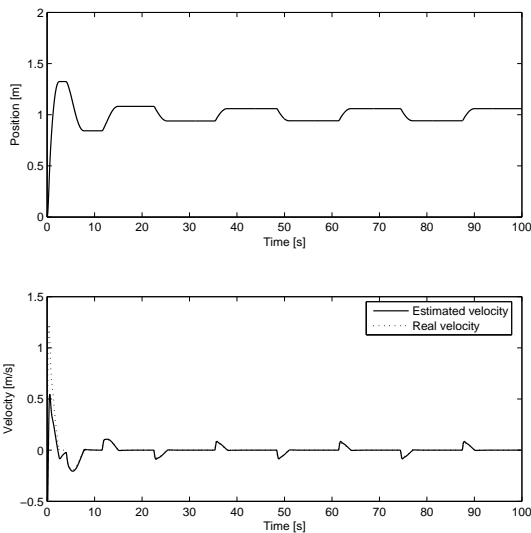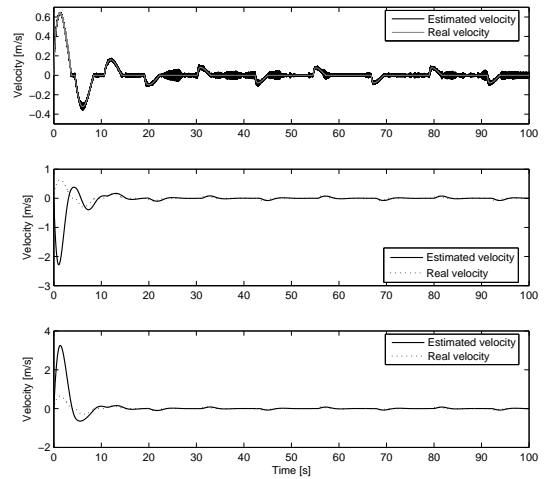

Figure 6: Comparison of the three estimators (for $k_0 = k_2 = 1$ and $k_1 = 5$): 1) Top with Theorem 1, 2) Middle with Theorem 2, and 3) Bottom with Theorem 3.

measurements; i.e., 4000 lines per revolution with $4\times$ hardware interpolation giving 16000 counts per revolution to each encoder; 1 count (equivalent to 0.000392 radians or 0.0225 degrees) is the lowest angular position measurable (ECP, 1995). The system was set up to incorporate inertia and friction brake. The drive and load disks were connected via a $4:1$ speed reduction (see Figure 9). In order to demonstrate that the system is subject to the notorious effects of friction, we calculated according to the procedure described by (R. Kelly and Campa,

2000) the following friction coefficients for the system: $F_v = 0.05772[Nmsec/rad]$ (viscous friction coefficient) and $F_c = 0.43032[Nm]$ (Coulomb friction).

A Pentium 4, 2.80 GHz CPU, 512 MB RAM, computer running under Windows XP is programmed to implement the controller together with the interface medium ECP USR Executive 5.1, a C-like programming language (ECP, 1995). The system contains a data-acquisition board for digital to analog conversion and a counter board to read the position encoder outputs into the servo system. The minimum servo-loop closure sampling time T_s is 0.884 ms.
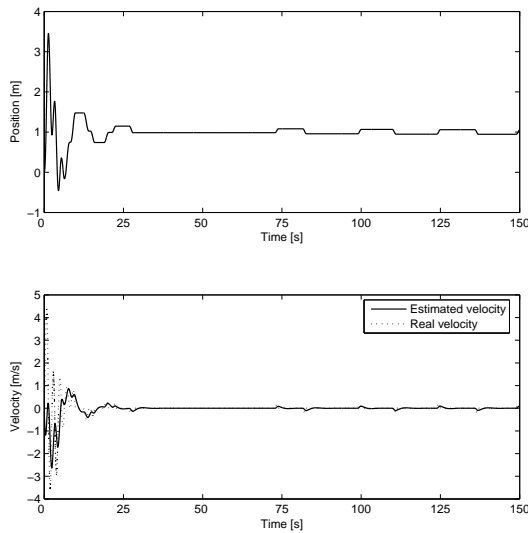
Figure 7: PID control containing the velocity as given by Theorem 2.
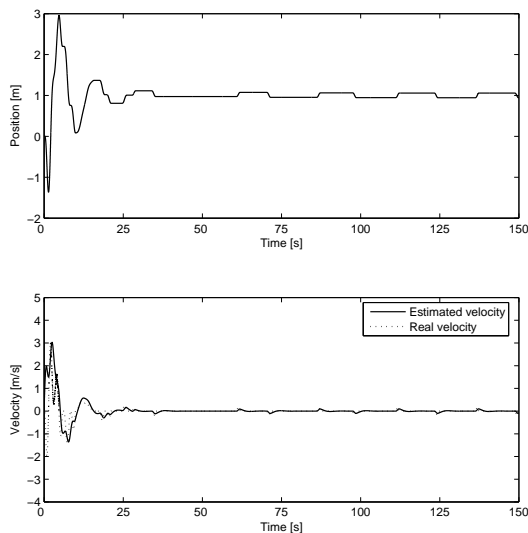

Figure 8: PID control making use of the observer in Theorem 3.

The output voltage signal generated by the system is in the range of $\pm 5V$ and is delivered to the motor drive via the DAC, the measurement feedback is a position signal (in counts or radians), measured at the shaft of each of the two disks by the optical rotary incremental position encoders, then it is read by the microcomputer by means of the counter board and delivered into the PC. A software interface has been built to easily transfer the raw data collected from the plant (by means of the ECP USR Executive program) to the Matlab workspace environment, in order to
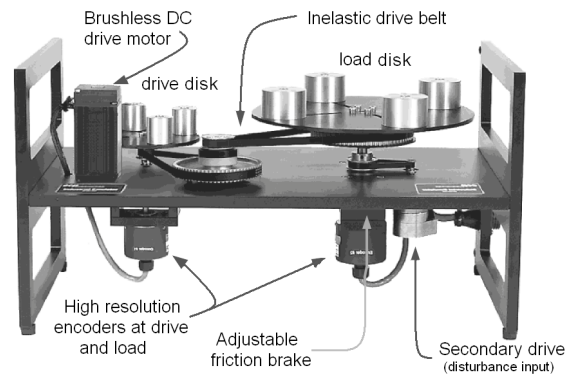

Figure 9: Mechanical system with friction.

display the results. The load disk is weighted with 4 masses of 0.50 kg each (at a radius of r = 10.0 cm) while the drive disk remains unweighted (see Figure 9). It is worth mentioning that the mechanical system has encoders which give accurate position measurements, nevertheless no direct velocity sensing is available (ECP, 1995). In this scenario, we implemented the aforementioned velocity observers, obtaining the results pictured in Figures 10 – 12.

4.2 Experimental Results

The implemented control law is as follows

$$u = -k_p(x - x_d) - k_i \int (x - x_d)dt - k_d \hat{\dot{x}},$$

with $k_d = 0.0011$, $k_p = 0.135$, and $k_i = 0.4$. The desired reference position was set to $x_d = 100 \, [counts] = 0.0392 \, [rad] = 2.25 \, [deg]$.

As it can be noted in Figure 10, when the observer stated in Theorem 1 is employed, this produces an oscillation of relatively high-frequency and amplitude into the system. The observer (4), after the transient, eliminates both the amplitude and the frequency of this oscillation (Figure 11). The estimator from Theorem 3 further reduces the amplitude and duration of the transient, even more than observer (4), eliminating chattering as seen in Figure 12. Moreover, note that the position limit cycles in Figures 11–12, caused by friction in PID control of servo drives (Canudas de Wit et al., 1995), have the same rectangular-like waveform pattern as in Figure 1, and can be distinctly identified.

5 CONCLUSIONS

Two new velocity-observation designs are presented, and experimentally validated, for use in mechanical
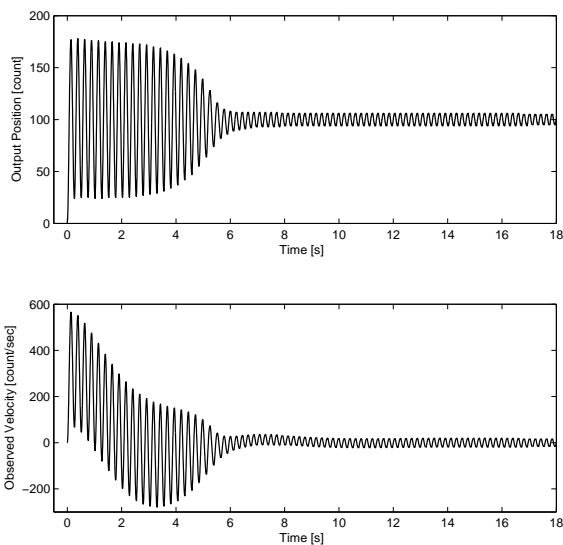
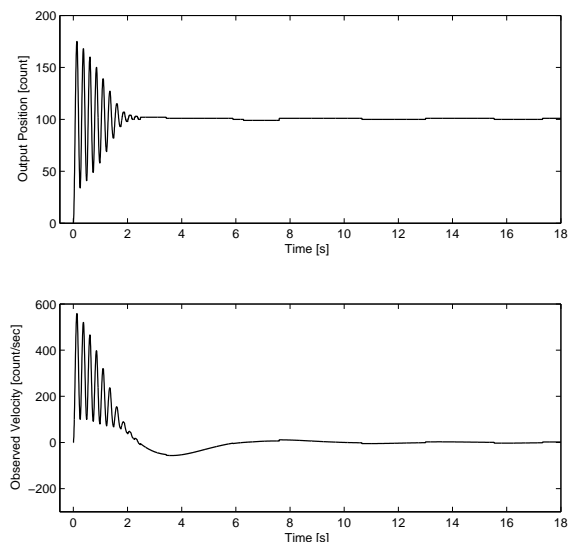Figure 10: Experimental results using the observer from Theorem 1.

Figure 11: Experimental results employing the observer from Theorem 2.

Figure 12: Experimental results utilizing the observer from Theorem 3.

ACKNOWLEDGEMENTS

REFERENCES

Armstrong-Hétlouvry, B., Dupont, P., and Canudas de Wit, C. (1994). A survey of models, analysis tools and compensation methods for the control of machines with friction. *Automatica*, 30(7):1083–1138.

Arteaga, M. A. and Kelly, R. (2004). Robot control without velocity measurements: new theory and experimental results. *IEEE Trans. Robot. Automat*, 20(2):297–308.

Berghuis, H. and Nijmeijer, H. (1993). Global regulation of robots using only position measurements. In *Systems and Control Letters*. vol. 21, no. 4, pp. 289–293.

Canudas de Wit, C. and Fixot, N. (1991). Robot control via robust estimated state feedback. In *IEEE Trans. on Aut. Control*. vol. 36, no. 12, pp. 1497–1501.

Canudas de Wit, C. and Fixot, N. (1992). Adaptive control of robot manipulators via velocity estimated feedback. In *IEEE Trans. on Aut. Control*. vol. 37, no. 8, pp. 1234–1237.

Canudas de Wit, C., Olsson, H., Åström, K. J., and Lischinsky, P. (1995). A new model for control of systems with friction. In *IEEE Trans. on Aut. Control*. vol. 40, no. 3, pp. 419–425.

Choi, J., Misawa, E., and Young, G. (1999). A study on sliding mode state estimation. *J. dyn. syst. meas. control*, 121:255–260.

systems with friction where only-position measurements are available. As it can be appreciated from numerical and experimental results, the proposed observer schemes are more efficient than their precursor in that chattering is eliminated from the velocity-observed signal. It is worth emphasizing that the presented observers (3) and (4) are especially interesting for industrial purposes, for they assure that the velocity-acquisition hardware can, without difficulty, be replaced by an analogous inexpensive software performing the same function.
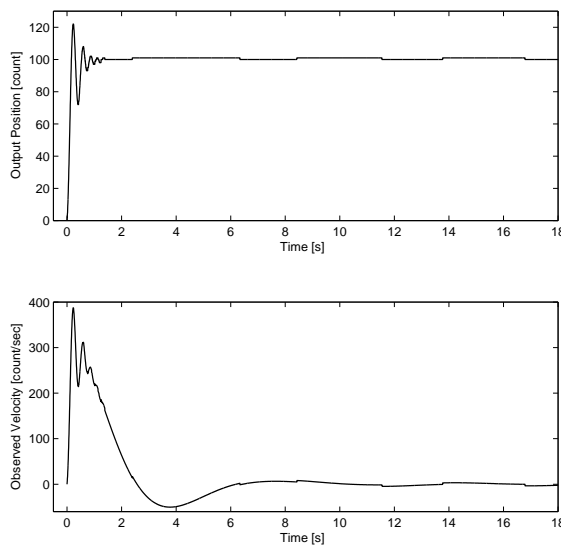
ECP (1995). Manual for model 220 industrial emulator/servo trainer. In *Educational Control Products*. California 91367, USA.

G. Bartolini, A. Ferrara, E. U. (1998). Chattering avoidance by second-order sliding mode control. In *IEEE Trans. on Automatic Control*. vol. 43, no. 2, pp. 241–246.

Hung, J. C. (1993). Chattering handling for variable structure control systems. In *Proc. of the IECON '93*. vol. 3, pp. 1968–1972, Maui, Hawaii, USA.

R. Kelly, J. L. and Campa, R. (2000). A measurement procedure for viscous and coulomb friction. In *IEEE Trans. Instrum. Meas.* vol. 49, no. 4, pp. 857–861.

Xian, C., de Queiroz, M. S., Dawson, D. M., and McIntyre, M. L. (2004). A discontinuous output controller and velocity observer for nonlinear mechanical systems. In *Automatica*. 40, pp. 695–700.

Xiong, Y. and Saif, M. (2001). Sliding mode observer for nonlinear uncertain systems. *IEEE Trans. on Aut. Control*, 46(12):2012–2017.