

ROBUST ADAPTIVE WAVELET NEURAL NETWORK TO CONTROL A CLASS OF NONLINEAR SYSTEMS

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Abstract: This paper deals with the synthesis of a Wavelet Neural Network adaptive controller for a class of second order systems. Due to its fast convergence, the wavelet neural network is used to approximate the unknown system dynamics. The proposed approximator will be on-line adjusted according to the adaptation laws deduced from the stability analysis. To ensure the robustness of the closed loop system, a modified sliding mode control signal is used. In this work, variable sliding surface is considered to reduce the starting energy without deteriorating the tracking performances. Furthermore, the knowledge of the upper bounds of both the external disturbances and the approximation errors is not needed. The global stability of the closed loop system is guaranteed in the sense of Lyapunov. Finally, a simulation example is presented to illustrate the efficiency of the developed approach.

1 INTRODUCTION

In last decade, active research has been carried out in neural network control (Omidvar, 97) (Noriega, 98) (Lin, 98). The characteristics of fault tolerance, parallelism and learning suggest that they may be good candidates for implementing real-time adaptive control for nonlinear dynamical systems. It has been proven that an artificial neural network can approximate a wide range of nonlinear functions to any desired degree of accuracy under certain conditions (Omidvar, 97). It is generally understood that the selection of the neural network training algorithm plays an important role for most neural network applications. In the conventional gradient-descent-type weight adaptation, the sensitivity of the controlled system is required in the online training process (Lin, 98). However, it is difficult to acquire sensitivity information for unknown or highly nonlinear dynamics. Moreover, the local minimum of the performance index remains variable (Omidvar, 97). In practical control applications, it is desirable to have a systematic method for ensuring the

stability, robustness, and performance properties of the overall system. Recently, several neural network control approaches have been proposed based on Lyapunov stability theory (Fabri, 96) (Farrell, 98) (Seshagiri, 00). One main advantage of these control schemes is that the adaptive laws were derived based on the Lyapunov synthesis method and therefore, the stability of the control system is guaranteed. However, some constraint conditions should be assumed in the control process, e.g., the approximation error, optimal parameter vectors, or higher order terms in a Taylor series expansion of the nonlinear control law are bounded. Also, the prior knowledge of the controlled system may be required, e.g., the external disturbance is bounded or all states of the controlled system are measurable. These requirements are not easy to satisfy in practical control applications. Recently, Wavelet Neural Networks (WNN) have become a very active subject in many scientific and engineering research areas (Zhang, 95) (Kostka, 00) (Lin, 03) (Ho, 05). The WNN have been proposed as a universal tool for functional approximation, which combine the capability of artificial neural networks in learning

and the capability of wavelet decomposition. The WNN allows resolving the conventional problem of poor convergence or even divergence encountered in other kinds of neural networks. It can also increase convergence speed (Delyon, 95) (Hsu, 06).

Sliding mode control is unique in its ability to achieve accurate, robust, decoupled tracking for a class of nonlinear time-varying systems in the presence of disturbances and parameter variations (Utkin, 77) (Salamci, 01). The tracking of the desired trajectory is achieved through two phases: an approach phase, where the system is controlled to attain a predefined sliding surface, and a sliding phase along the sliding surface. However, in order to deal with the presence of modelling imprecision and disturbances, the control law has to be discontinuous across the sliding surface. Since the implementation of the associated control switching is necessarily imperfect, this leads to chattering which involves high control activity and may excite high-frequency dynamics and can, therefore, damage the plant (Slotine, 91). To resolve this problem, many solutions have been proposed in the literature (Slotine, 91)- (Lin, 02) (Berstecher, 01) (Hwang, 01) (Lin, 95) (Wai, 04). In (Slotine, 91), a boundary-layer in the neighbourhood of the sliding surface has been defined to obtain a continuous behaviour of the control signal across this surface. Based on the same idea, a fuzzy system has been used to define this boundary layer and to exploit the human knowledge (Lin, 02). To remove the discontinuity in the control signal, some approaches combining sliding mode control and classical controller using a fuzzy supervisor can be cited (Berstecher, 01) (Hwang, 01) (Lin, 95). These methods resolve the problem related to the chattering phenomenon. However, to design the switching signal assuring the approaching phase, the upper bounds of both the external disturbances and the structural uncertainties must be well known. To overcome these problems, the authors of (Hamzaoui, 04) (Wai, 04) have proposed an approximation of the switching signal by an adaptive fuzzy system to eliminate the chattering phenomenon without requiring any particular knowledge about the upper bounds of both approximation errors and external disturbances. Nevertheless, the global stability of the closed loop system in these approaches is guaranteed only for a good approximation level or for a particular choice of the initial values of the adjustable parameters.

This paper proposes a wavelet Neural Network Adaptive Control (WNNAC) for a class of second-order nonlinear, uncertain and perturbed systems; this controller combines the advantages of WNN

identification and the robustness of sliding mode control. The control law is composed of two parts. The first one represents the WNN identifier that perform the online system dynamic function estimation. This identifier is adjusted according some adaptations laws deduced from the stability analysis. The second part of the control law represents the robust term which ensure the robustness of the closed loop system in the sense of sliding mode technique. This term is synthesised such that knowing of the upper bounds of the external disturbances is not required. Furthermore, the proposed control law uses a variable sliding surface to reduce the starting energy obtained by a classical sliding surface. The stability of the closed loop system is stated using the Lyapunov theory. To illustrate the efficiency of the proposed approach, a numerical simulation example is considered.

The paper is organised as follows: Section 2 illustrates the description of WNN networks. Section 3 is dedicated to the formulation and the investigation of the control problem. In section 4, we present the synthesis of the proposed controller whose design procedure is explained in section 5. To show the efficiency of the proposed approach, a simulation example is presented in section 6.

2 DESCRIPTION OF WAVELET NEURAL NETWORKS (WNNs)

Wavelet neural networks are special case of feed-forward neural networks. The main difference between the artificial neural networks (ANN) and WNN is that, in ANN the nonlinearities are approximated by superposition of sigmoid functions. However, in WNN, nonlinearities are approximated by superposition of wavelet functions (Oussar, 98). Similar to ANN, WNN are also shown to have universal approximation property (Yoo, 05) (Sureshababu, 99).

Similar to the ANN, the WNN consists of an input layer, a hidden layer, and an output layer. The WNN model structure shown in Figure 1 consists of ' n ' input neurons (x_1, x_2, \dots, x_n) in the input layer, equal to the number of input variables. The input neurons are connected to the next layer of neurons, called the hidden layer neurons which make use of wavelets as transformation functions. These neurons are termed as "wavelons". In this work, the Mexican hat (1) is used as a 'mother' wavelet ψ .

$$\psi(x) = (1 - x^2) \times \exp(-0.5x^2) \quad (1)$$

Several daughters of wavelets ψ_j can be constructed by translating and dilating the mother wavelet ψ according to (2).

$$\psi_j(z) = \psi\left(\frac{\sigma_j - t_j}{d_j}\right) \quad (2)$$

where $\sigma_j = \sum_{i=1}^n v_{ij} x_i$, v_{ij} is input scaling vector while t_j , and d_j represent the translation and dilation factors of the wavelet.

For i and j represent the indices of input, hidden, and output layers respectively, the output from the hidden wavelon, H_j is given by (3).

$$H_j(x) = \sum_{i=1}^n \psi\left(\frac{\sigma_j - t_j}{d_j}\right) \quad (3)$$

This output is connected directly to the output layer neurons. The output layer usually consists of a linear output neuron. Mathematically, the final production obtained from figure 1 can be represented by (4).

$$Y_{output} = \underline{C}^T \Psi + \underline{A}^T X + B \quad (4)$$

with $\underline{C}^T = [c_1 \ \dots \ c_j]^T$, $\Psi = [H_1 \ \dots \ H_j]$ and $\underline{A}^T = [a_1 \ \dots \ a_n]$

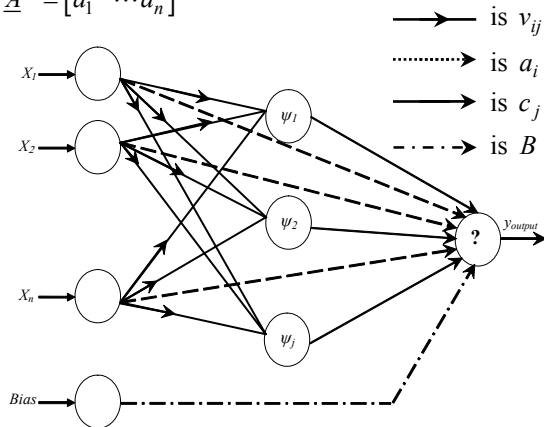


Figure 1: The structure of the used WNN.

The approximator in figure 1 includes three independent adaptable parts; constant, linear and non-linear given by B , \underline{A} and \underline{C} respectively. Thus, the approximator (4) is able to estimate efficiently all possible systems static or dynamics by managing learning rate of each part.

3 PROBLEM STATEMENT

Consider the following 2nd order system:

$$\ddot{y} = f(y, \dot{y}) + g(y, \dot{y})u + d \quad (5)$$

where f and g are two unknown continuous functions. u and y designate the input and output of the system respectively while d is an external disturbance assumed to be unknown but bounded. The input-output system (5) includes a large class of non-linear second-order systems likes Duffing oscillator and mass-spring-damper system (Bartoloni, 97) (Roup, 01) (Chang, 05).

In this work, we assume that the function $g(y, \dot{y})$ can be written as a sum of a known nominal term and an uncertain as follows

$$g(y, \dot{y}) = g_0(y, \dot{y}) + \delta_g(y, \dot{y})$$

In this case, equation (5) can be rewritten as:

$$\ddot{y} = f(y, \dot{y}) + g_0(y, \dot{y})u + \delta_d \quad (6)$$

where $\delta_d = \delta_g u + d$.

The objective of this work is to synthesize a robust controller, based on sliding mode, able to force the output of the system y to follow a bounded reference trajectory y_d under the constraint that all involved signals are bounded.

We denote the tracking error by $e = y_d - y$ and the sliding surface by:

$$s = \dot{e} + \lambda e \quad (7)$$

Since using a large value of the slop λ gives a fast system response. However, a too large value can lead to overshoot and even instability. From the other side, small value of λ results a slow system response. To overcome this problem, the slope can be adapted according to tracking error value $e(t)$ (Liu, 05). In this case, equation (7) becomes

$$s = \dot{e} + \lambda(e)e \quad (8)$$

where $\lambda(e) = \frac{k_\lambda}{|e| + \varepsilon_\lambda}$, k_λ is a given positive constant

and ε_λ is a small positive scalar given by the designer.

Differentially according time of equation (8) gives

$$\begin{aligned} \dot{s}(t) &= \ddot{e}(t) + \lambda \dot{e}(t) + \dot{\lambda} e(t) \\ &= \ddot{y}_d - \ddot{y} + \lambda \dot{e}(t) + \dot{\lambda} e(t) \end{aligned} \quad (9)$$

Using (6), equation (9) becomes

$$\dot{s}(t) = \ddot{y}_d - f - g_0 u - \delta_d + \lambda \dot{e}(t) + \lambda e(t) \quad (10)$$

$$\dot{s}(t) = -f - \delta_d + \lambda e(t) - g_0 u + \ddot{y}_d + \lambda \dot{e}(t) \quad (11)$$

$$\dot{s}(t) = -F(y, \dot{y}, y_d) - g_0 u + \ddot{y}_d + \lambda \dot{e}(t) \quad (12)$$

where $F(y, \dot{y}, y_d) = f + \delta_d - \lambda \dot{e}(t)$.

In the next section, the problem of $F(y, \dot{y}, y_d)$ term estimation will be treated

4 THE WNN ADAPTIVE CONTROLLER SYNTHESIS

It is worthy to say that if the system output and its time derivative converge to their reference signal, the unknown function $F(y, \dot{y}, y_d)$ goes to $F(y_d, \dot{y}_d)$. To approximate $F(y, \dot{y}, y_d)$ we can use a WNN in the form

$$\hat{F}(y_d, \dot{y}_d) = \underline{C}^T \psi + \underline{A}^T Y_d + B$$

with y_d and \dot{y}_d as inputs (Chang, 05).

Consider the pre-assigned constraint regions of \underline{C} , \underline{A} and B defined respectively as:

$$\Omega_C = \{\underline{C} / \|\underline{C}\| \leq M_c, M_c > 0\}$$

$$\Omega_A = \{\underline{A} / \|\underline{A}\| \leq M_A, M_A > 0\}$$

$$\Omega_B = \{B / \|B\| \leq M_B, M_B > 0\}$$

According to the approximation theorem, there exists a finite optimal value of $F(y_d, \dot{y}_d)$ noted by:

$$\hat{F}^*(y_d, \dot{y}_d) = \underline{C}^{*T} \psi + \underline{A}^{*T} Y_d + B^*$$

($\underline{C} \in \Omega_C$, $\underline{A} \in \Omega_A$ and $B \in \Omega_B$) such that:

$$\delta_F = F(y_d, \dot{y}_d) - \hat{F}^*(y_d, \dot{y}_d)$$

hence, equation (12) can be rewritten as:

$$\dot{s}(t) = -\hat{F}^*(y_d, \dot{y}_d) + \delta_F - g_0 u + \ddot{y}_d + \lambda \dot{e}(t) + \delta_d \quad (13)$$

To guarantee the global stability of the closed loop system and the convergence of the tracking error to zero, we propose the following control law:

$$u(t) = g_0^{-1} \left[-\hat{F}^*(y_d, \dot{y}_d) + \ddot{y}_d + \lambda \dot{e} + \frac{s}{\rho^2} \right] \quad (14)$$

Substituting (14) in (13) yields to:

$$\begin{aligned} \dot{s}(t) = & -\hat{F}^*(y_d, \dot{y}_d) + \hat{F}(y_d, \dot{y}_d) - \ddot{y}_d \\ & - \lambda \dot{e} + \ddot{y}_d + \lambda \dot{e} + \delta_F + \delta_d - \frac{s}{\rho^2} \end{aligned} \quad (15)$$

which can be rewritten as

$$\begin{aligned} \dot{s}(t) = & \underline{\tilde{C}}^T \psi + \underline{\tilde{A}}^T y_d + \tilde{B} \\ & + \delta_F + \delta_d - \frac{s}{\rho^2} \end{aligned} \quad (16)$$

To study the stability of the closed loop system and to find the adaptation laws for the adjustable parameters, we consider the following Lyapunov function:

$$v(t) = \frac{1}{2} s^2 + \frac{1}{2\gamma_C} \underline{\tilde{C}}^T \underline{\tilde{C}} + \frac{1}{2\gamma_A} \underline{\tilde{A}}^T \underline{\tilde{A}} + \frac{1}{2\gamma_B} \tilde{B}^2 \quad (17)$$

The time derivative of $v(t)$ is given by:

$$\dot{v}(t) = s\dot{s} + \frac{1}{\gamma_C} \underline{\tilde{C}}^T \dot{\underline{\tilde{C}}} + \frac{1}{\gamma_A} \underline{\tilde{A}}^T \dot{\underline{\tilde{A}}} + \frac{1}{\gamma_B} \tilde{B} \dot{\tilde{B}} \quad (18)$$

Substituting (16) in (18) gives:

$$\begin{aligned} \dot{v}(t) = & s \left(\underline{\tilde{C}}^T \psi + \underline{\tilde{A}}^T y_d + \tilde{B} + \delta_F + \delta_d - \frac{s}{\rho^2} \right) \\ & + \frac{1}{\gamma_C} \underline{\tilde{C}}^T \dot{\underline{\tilde{C}}} + \frac{1}{\gamma_A} \underline{\tilde{A}}^T \dot{\underline{\tilde{A}}} + \frac{1}{\gamma_B} \tilde{B} \dot{\tilde{B}} \\ \dot{v}(t) = & s\delta_F + s\delta_d - \frac{s^2}{\rho^2} + \frac{1}{\gamma_C} \underline{\tilde{C}}^T (\dot{\underline{\tilde{C}}} + \gamma_C s \psi) \\ & + \frac{1}{\gamma_A} \underline{\tilde{A}}^T (\dot{\underline{\tilde{A}}} + \gamma_A s y_d) + \frac{1}{\gamma_B} \tilde{B} (\dot{\tilde{B}} + \gamma_B s) \end{aligned} \quad (19)$$

Choosing the following adaptation law

$$\dot{\underline{\tilde{C}}} = -\gamma_C s \psi \quad (20)$$

$$\dot{\underline{\tilde{A}}} = -\gamma_A s y_d \quad (21)$$

$$\dot{\tilde{B}} = -\gamma_B s \quad (22)$$

leads to:

$$\dot{v}(t) = s\delta_F + s\delta_d - \frac{s^2}{\rho^2} \quad (23)$$

$$\begin{aligned} \dot{v}(t) = & -\frac{s^2}{4\rho^2} + 2\frac{s}{2\rho} \rho \delta_F - \frac{s^2}{4\rho^2} \\ & + 2\frac{s}{2\rho} \delta_d - \frac{s^2}{2\rho^2} \end{aligned} \quad (24)$$

$$\dot{v}(t) = -\left(\frac{s}{\rho} - \rho\delta_F\right)^2 - \left(\frac{s}{\rho} - \rho\delta_d\right)^2 + \rho^2\delta_F^2 + \rho^2\delta_d^2 - \frac{s^2}{2\rho^2} \quad (25)$$

$$\dot{v}(t) \leq -\frac{s^2}{2\rho^2} + \rho^2\delta_F + \rho^2\delta_d^2 \quad (26)$$

$$\text{Let } \Omega_s = \left\{ \frac{s}{\|s\|} \leq \mu, \mu > 0 \right\}.$$

According to the regions $\Omega_s, \Omega_C, \Omega_A$ and Ω_B , there is a sufficient large constant V_{\max} such that $V_{\max} \leq \max_{s \in \Omega_s, C \in \Omega_C, A \in \Omega_A, B \in \Omega_B} v(t)$ (Chang-05).

Afterward, we can define

$$\Omega_e = \left\{ \frac{\dot{e}}{\|\dot{e}\|} \leq \sqrt{2V_{\max}} \right\}$$

and

$$\Omega_e = \left\{ \frac{e}{\|e\|} \leq \sqrt{\frac{2\varepsilon_\lambda^2 V_{\max}}{k_\lambda^2}} \right\}$$

In the case where the sliding surface is outside the region Ω_s , for choosing

$$\frac{1}{\rho^4} > \frac{\delta_F^2 + \delta_d^2}{\mu^2}$$

there exists a positive constant ζ such that:

$$\dot{v}(t) \leq -\zeta s^2 \quad (27)$$

From the definition of the constraint region, we have $v(0) \leq V_{\max}$. From the inequality (23), we obtain $v(t) \leq v(0) \leq V_{\max}$ which implies that $s \in \Omega_s$ for all $t \geq 0$ and therefore $e \in \Omega_e$ and $\dot{e} \in \Omega_e$ (Chang-05). Hence, $\Omega_s \times \Omega_C \times \Omega_A \times \Omega_B$ is an invariant set, we can conclude that all the variables are bounded. Since the Lyapunov function is negative outside the constraint set Ω_s , then sliding surface is Uniformly Ultimately Bounded (UBB) and hence the tracking error is also UBB.

By integrating the above inequality between 0 and T, we obtain:

$$v(T) - v(0) \leq -\int_0^T \frac{s^2}{2\rho^2} dt + \int_0^T \rho^2 \delta_F dt + \int_0^T \rho^2 \delta_d^2 dt$$

$$\int_0^T \frac{s^2}{2\rho^2} dt \leq v(0) - v(T) + \int_0^T \rho^2 \delta_F dt + \int_0^T \rho^2 \delta_d^2 dt \quad (28)$$

Since $v(T) \geq 0$, we have

$$\int_0^T \frac{s^2}{2\rho^2} dt \leq \int_0^T \rho^2 \delta_F dt + \int_0^T \rho^2 \delta_d^2 dt$$

or

$$\int_0^T s^2 dt \leq 4\rho^2 \int_0^T \delta_F dt + 4\rho^2 \int_0^T \delta_d^2 dt \quad (29)$$

Using the Barbalat's lemma (Wang, 94), one can see that the sliding surface converges asymptotically to zero despite the presence of external disturbances. Hence, the sliding surface is attractive, i.e., if the system attains the surface, it remains and converges toward the origin as demonstrated in (Utkin, 99).

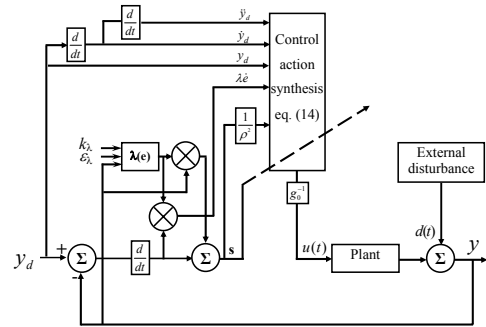


Figure 2: Control scheme of the proposed approach.

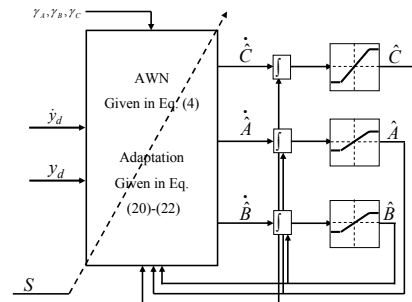


Figure 3: Adaptive Wavelet Network illustration.

5 DESIGN PROCEDURE

The control scheme of the proposed approach and the adaptive WNN block are given respectively by figures 2 and 3. The control law (14) can be synthesised according to the following steps:

Step 1: define the interval range for each input (y_d, \dot{y}_d) and the corresponding wavelet parameters (dilation and translation) such that input range is covered uniformly. To adapt on-line the adjustable parameters \underline{A} , \underline{B} and \underline{C} , we define the learning

rates γ_A , γ_B and γ_C . To increase approximator nonlinearity, we choose γ_C bigger than γ_A and γ_B .

Step 2: specify the parameters k_λ and ε_λ to calculate the sliding surface given in (8). To deduce the control law (14), we choose ρ . This value represents a trade-off between the initial starting energy and the time response in transient state.

Step 3: update the adjustable parameters of the WNN according to equation (20), (21) and (22).

6 SIMULATION EXAMPLE

In order to validate the proposed controller, the Wing-Rock Motion (WRM) system is considered in simulation. Indeed, some combat aircrafts often operate at subsonic speeds and high angles of attack. These aircrafts may become unstable due to oscillation, mainly a rolling motion known as WRM (Lan, 95) (Lin, 02). the dynamics of WRM system can be described in a state variable by:

$$\ddot{y} = b_0 + b_1 y + b_2 \dot{y} + b_3 |y| \dot{y} + b_4 |\dot{y}| \dot{y} + b_5 y^3 + u + d \quad (30)$$

Where y is the roll angle, u is the control action and d is the external disturbance. The parameters related to $b_i (i=1, \dots, 5)$ are the aerodynamic parameters given by:

$$b_0 = 0, \quad b_1 = -.01859521, \quad b_2 = .015162375, \\ b_3 = .06245153, \quad b_4 = .00954708 \text{ and } b_5 = .02145291.$$

According to the design procedure given in section 5, we will define at first the AWNN. Indeed if we consider that $y_d = \cos(t)$ being the desired reference trajectory, then the interval range of the desired output (as well as the actual system output in case of perfect tracking) will belongs always to the closed interval $[-1, 1]$. For the purpose of reliability and to give some relaxation to our controller, the interval $[-1.5, 1.5]$ is considered as a universe of discourse for both of input and hidden layers in the WNN approximator. The number of wavelons (wavelet neurons) components used in the hidden layer are four which is enough for covering the interval $[-1.5, 1.5]$ adequately. For a WNN approximator, the number of waveleons components used in the hidden layer depends on the network input's interval and dynamic complexity for the system to be approximated. Translation parameters selection is considered in a way that guarantees the uniform covering of the $[-1.5, 1.5]$ interval simply using linspace Matlab[®] instruction. Dilation parameters

specify the intersection amount between wavelons activation function (daughter function) which is chosen to be 0.5 such that the horizontal axes $y = 0.5$ contains these intersection points. The adjustable parameters \underline{A} , B and \underline{C} are initialized to zero such that random initialization case is avoided since it doesn't gives neither same initials nor same training speed. Alternatively, these parameters might be chosen through some trials to achieve favourable transient control performance. For constants γ_A , γ_B and γ_C corresponding to the learning rates, it is important to recognize there effects to approximate accurately the nonlinear system and to avoid masking nonlinear property in the structure shown in figure 1. For this, it is better choosing γ_C much bigger than γ_A and γ_B . In our example, the values $\gamma_c = 10$, $\gamma_B = 0.05$ and $\gamma_A = 0.05$ have been considered. According to the second step in the design procedure, the variable sliding mode may be achieved through variation in surface slope (λ). For this we choose $k_\lambda = 20$ and $\varepsilon_\lambda = 0.5$.

Several simulations have been done and figures 4 to 6 show the results obtained for $\rho = 0.5$ where the system is subjected to external disturbance with $d = 0.3 \sin(2t)$. Figures 4 to 6 give the simulation results for two different initial positions. We remark that the system attains the reference trajectory rapidly despite that the initial condition is so far from the reference one. Short response time reflects the good convergence of WNN. Furthermore, figure 6 shows the elimination of the chattering phenomenon and the absence of the abrupt variations appearing in classical sliding mode control. Comparing the obtained results with those in the case of linear sliding surface (λ constant), the proposed approach guarantees the same tracking performances with 40% initial control action reduction.

7 CONCLUSION

In this paper a robust adaptive wavelet neural network to control a class of nonlinear systems was presented. The combination of WNN and sliding mode control allows to develop a robust controller to guarantee the good tracking performances and the closed loop system stability. Considering a variable sliding surface reduces the starting energy without deteriorating the tracking performances.

Furthermore, no knowledge about of the upper bounds of both the external disturbances and the approximation errors is required to synthesis the control law. Simulation results have been presented to show the efficiency of the proposed approach. Current work is dedicated to the generalisation of this method to n^{th} multi-input multi-output systems.

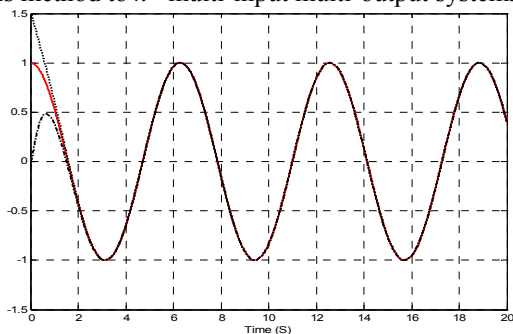


Figure 4: Evolution of the system output and its reference signal (-: y_d , ...: $y(t)$ with $y(0)=1.5$; -.-: $y(t)$ with $y(0)=0$).

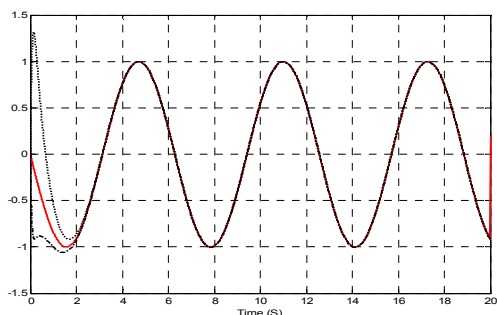


Figure 5: Evolution of the state \dot{y} and its reference signal (-: \dot{y}_d , ...: \dot{y} with $y(0)=1.5$; -.-: \dot{y} with $y(0)=0$).

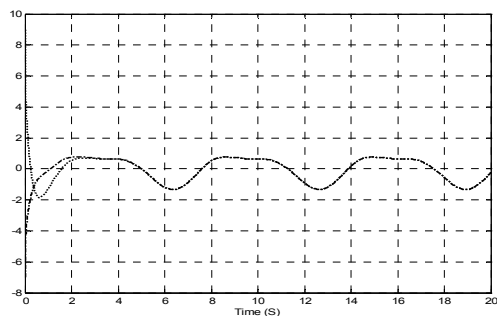


Figure 6: The control signal applied to the system (...: $u(t)$ with $y(0)=1.5$; -.-: $u(t)$ with $y(0)=0$).

REFERENCES

- Antsaklis, P. J., Jun. 1995. vol. 15, Issue 3, pp. 5 - 7. In *Intelligent Learning Control IEEE Control Syst. Mag.* IEEE Press.
- Astrom, K. J., Wittenmark, B., 1995. In *Adaptive Control*. Addison-Wesley, 2nd edition.
- Bartoloni, G., Ferrara, A., Usai, E., 1997. vol. 33 pp. 2203-2212. In *Automatica, Output tracking control of uncertain nonlinear second-order systems*. Elsevier.
- Berstecher, R.G, Palm, R., Unbehauen, H.D., 2001. vol. 48, pp. 18-31. In *An Adaptive Fuzzy Sliding Mode Controller, IEEE Trans. on Indus. Electr.*, IEEE press.
- Broderick, R. L., 2005. vol. 1, pp. 6.C.2-01-10. In *daptive verification for an on-line learning neural-based flight control system, the 24th Digital Avionics Systems Conference*, IEEE Press.
- Chang, Y. C., Yen, H. M., 2005. vol. 35, pp. 1311-1316. In *Adaptive output Feedback control for a class of uncertain nonlinear systems using neural networks, IEEE Trans. Systems, Man, & Cybernetic*. IEEE Press.
- Chang, Y.-C., 2005. part B: cybernetics vol. 35, no. 6 pp.1108- 1119. In *Intelligent Robust Control for Uncertain Nonlinear Time-Varying Systems and Its Application to Robotic Systems, IEEE Trans. on syst. Man & Cyber.*, IEEE Press.
- Delyon, B., Juditsky, A., Benveniste, A., 1995. vol. 6, no. 2, pp.332-348. In *Accuracy analysis for wavelet approximations, IEEE Trans. Neural Networks*. IEEE Press.
- Fabri, S., Kadirkamanathan, V., Sept. 1996. vol. 7, pp. 1151-1167, *Dynamic structure neural networks for stable adaptive control of nonlinear systems, IEEE Trans. Neural Networks* IEEE press.
- Farrell, J., Baker, W., 1993. pp. 237-262. In *Learning control systems*, In Passino, K. M., Antsaklis, P. J. (Eds.), *An Introduction to Intelligent and Autonomous Control*. Kluwer Academic Publishers.
- Farrell, J. A., Sept. 1998. vol. 9, pp.1008-1020 *Stability and approximator convergence in nonparametric nonlinear adaptive control,* *IEEE Trans. Neural Networks*. IEEE press.
- Grman, J., Ravas, R., Syrová, L., 2001. vol. 1, no. 1, pp. 25-28. In *Application of Neural Networks in Multifrequency Eddy-Current Testing, Measurement Science Review*. Slovak Academy of Sciences.
- Hamzaoui, A., Essounbouli, N., Zaytoon, J., 2004. vol. 218 (4), pp. 287-298. In *Fuzzy sliding mode control with fuzzy switching function for nonlinear uncertain MIMO Systems, Journal of Systems and Control Engineering*.
- Ho, D. W. C., Li, J., Niu, Y., 2005. vol. 16, no. 3, pp. 625-635. *Adaptive Neural Control for a Class of Nonlinearly Parametric Time-Delay Systems, IEEE Trans. on Neural Networks*, IEEE Press.
- Hsu, C.F., Lin, C.M., Lee, T.T., 2006. vol. 17, no. 5, pp. 1175-1183. In *Wavelet Adaptive Backstepping Control*

- for a Class of Nonlinear Systems, *IEEE Trans. On Neural Networks*, IEEE Press.
- Huang, S. N., Tan, K. K., Lee, T. H., 2002. vol. 38, no. 2, pp. 227–233. In *Automatica, Adaptive motion control using neural network approximations*, Elsevier.
- Hwang, C.-L., Kuo, C.-Y., 2001. vol. 9 (2), pp. 238-252. In *A stable adaptive fuzzy sliding-mode control for affine nonlinear systems with application to four-bar linkage systems*, *IEEE Trans. On Fuzzy Syst.* IEEE press.
- Kostka, P., Tkacz, E. J., Nawrat, Z., Malota, Z., 2000. pp. 2463-2465. In *An application of wavelet neural networks for heart valve prostheses characteristic*, *Proceedings of the 22nd Annual EMBS International Conference*, IEEE Press.
- Lan, C. E., Chen, Y., Lin, K. J., 1995. vol. 32, pp. 905–910. In *Experimental and analytical investigations of transonic limit-cycle oscillations of a flap-eron*, *Journal Aircraft*, American Institute of Aeronautics and Astronautics.
- Lin, C. M., Hsu, C. F., 2002. vol. 25, pp. 1163–1165, 2002. In *Recurrent neural network adaptive control of wing rock motion*, *Journal of guidance, control, and dynamics*. American Institute of Aeronautics and Astronautics.
- Lin, S.C., Chen, Y.Y., Mar. 1995. vol. 3, pp. 1103-1110. In *A GA-based fuzzy controller with sliding mode*, *Proc. IEEE Int. Conf. on Fuzzy Systems*. IEEE Press.
- Lin, F. J., Hwang, W. J., Wai, R. J., 1998. vol. 145, no. 2, pp. 105–110, *Ultrasonic motor servo drive with on-line trained neural network model-following controller*, *Proc. IEE—Elect. Power Applicat.* . IEEE press.
- Lin, W.-S., Chen, C.-S., 2002. vol. 149, pp. 193-202. In *Robust adaptive sliding mode control using fuzzy modelling for a class of uncertain MIMO nonlinear systems*, *IEE Proceedings: Control Theory and Applications*, IEEE Press.
- Lin, F.J., Wai, R. J., Chen, M. P., 2003. vol. 20, no. 6, pp. 686-698. In *Wavelet Neural Network Control for Linear Ultrasonic Motor Drive via Adaptive Sliding-Mode Technique*, *IEEE Trans. On Ultrasonics, Ferroelectrics & Frequency Control*, IEEE Press.
- Liu, Z. L., Nov. 2005. vol. 152, no. 6, pp. 615- 620 *Reinforcement adaptive fuzzy control of wing rock phenomena*, *IEE proc. Control theory appl.* IEE press.
- Narendra, K. S., Parthasarathy, K., Mar. 1990. vol. 1, no. 1, pp. 4–27, 1990. In *Identification and control of dynamic systems using neural networks*, *IEEE Trans. Neural Net.* IEEE press.
- Noriega, J. R., Wang, H, Jan. 1998. vol. 9, pp. 27–34. *A direct adaptive neural-network control for unknown nonlinear systems and its application*, *IEEE Trans. Neural Network*, IEEE Press.
- Omidvar, O., Elliott, D. L., 1997. *Neural Systems for Control*. New York: Academic.
- Oussar, Y., Rivals, I., Personnaz, L., Dreyfus, G., 1998. vol. 20, pp. 173-188, 1998. In *Training wavelet networks for nonlinear dynamic input-output modeling*, *Neurocomputing*, Elsevier
- Psillakis, H. E., Alexandridis, A. T., Mar. 2006. vol. 153, No. 2. In *Adaptive neural motion control of n-link robot manipulators subject to unknown disturbances and stochastic perturbations*. *IEE Proc. Control Theory Appl.* IEE press.
- Roup, A. V., Bernstein, D. S., 2001. vol. 46, pp. 1821-1825. In *Adaptive stabilization of a class of non-linear systems with parametric uncertainties*, *IEEE Trans. Automatic Control*, IEEE Press.
- Salamci, M. U., Ozgoren, M. K., 2001. vol. 23, no. 4, pp. 719-727. *Sliding mode control with optimal sliding surfaces for missile autopilot design*, *Journal of Guidance, Control and Dynamics*, , 2001.
- Sastry, S., Bodson, M., 1989. In *Adaptive Control: Stability, Convergence, and Robustness*. Prentice-Hall.
- Seshagiri, S., Khalil, H. K., Jan. 2000. vol. 11, pp. 69–79, *Output feedback control of nonlinear systems using RBF neural network*, *IEEE Trans. Neural Networks* IEEE press.
- Slotine, J.-J. E., Li, W., 1991. In *Applied Nonlinear Control*. Prentice-Hall.
- Sureshbabu, N., Farrell, J. A., Feb. 1999. vol. 44, no. 2, pp.412-417. In *Wavelet-Based System Identification for Nonlinear Control*, *IEEE Transactions on automatic control*, IEEE Press.
- Utkin, V.I., 1977. vol. 22, pp. 212-222. *Variable structure systems with sliding modes*, *IEEE Trans. Automatic Control*, IEEE Press.
- Utkin, V. I., 1999. vol. 22, pp. 212-222. In *Variable structure systems with sliding modes*, *IEEE Trans. Automatic Control*, IEEE Press.
- Wai, R.J., Lin, C. M., Hsu, C.F., 2004. vol. 143, pp. 295-310. In *Adaptive fuzzy sliding mode control for electrical servo drive*, *Fuzzy Sets & Systems*, Elsevier.
- Wang, L.-X., 1994. In *Adaptive fuzzy systems and control*, Prentice-Hall.
- Wong, L.K., Leung, F.H.F., Tam, P.K.S., 1998. vol. 1, pp. 296-301. In *Combination of sliding mode controller and PI controller Using fuzzy logic controller*, *IEEE Int. Conf. on Fuzzy Systems*. IEEE Press.
- Yoo, S. J., Park, J. B., Choi, Y. H. Mar. 2005. vol. 3, no. 1, pp. 43–55. In *Stable Predictive Control of Chaotic Systems Using Self-Recurrent Wavelet Neural Network*, *International Journal of Control, Automation, and Systems*, IEEE Press.
- Zhang, Q., Walter, G. G., Miao, Y., Lee, W. N. W., Jun. 1995. vol. 43, pp. 1485-1496. In *Wavelet neural networks for function learning*, *IEEE Trans. Signal Proc.*, IEEE Press.