IDENTIFICATION OF MODELS OF EXTERNAL LOADS

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Abstract: In the given work the problem of construction (synthesis) of mathematical model of unknown or little-known external load (EL) on open dynamic system is considered. Such synthesis is carried out by special processing of the experimentally measured response of dynamic system (method of identification). This problem is considered in two statements: the synthesis of EL for single model and the synthesis of EL for models class for the purposes of mathematical modelling. These problems are ill-posed by their nature and so the method of Tikhonov's regularization is used for its solution. For increase of exactness of problem solution of synthesis for models class the method of choice of special mathematical models (MM) is used. The calculation of model of external load for rolling mills is executed.

1 INTRODUCTION

At mathematical modeling of real motion of open dynamic systems is important the correct choice of mathematical model of external load on system. The most accessible information about EL is contained in reactions of object on these loads which can be measured experimentally simply enough from the technical point of view. The determination problem of size and character of change of EL based on results of experimental measuring of object responses has been called the problem of identification of EL (Gelfandbein and Kolosov,1972), (Ikeda, Migamoto and Sawaragi, 1976), (Menshikov, 1983). Such approach has some advantages: the construction of model of EL is being carried out on basis of objective information (experimental measuring); the results of mathematical modeling later on are good even in case of the great inaccuracy of MM.

At construction of mathematical model of concrete dynamic system the different authors use the various simplifying assumptions. The whole set of possible equivalent mathematical models of real object (dynamic system) are being obtain in result (Menshikov, 1985). So it will be useful to build the common model of EL which is the best in some sense for class of possible mathematical models of real object (Menshikov, 1985). The statement of such problem can have application in mathematical modeling, systems control, in detection of faults and so on.

2 STATEMENT OF PROBLEM

We shall suppose for simplicity that with the aid of known internal interactions (for example, measured experimentally) some subsystem of initial dynamic system can be received at which is known one variable status and all external loads except the external load which is being investigated. If at a subsystem two variable statuses are known, then this subsystem is being replaced with more simple subsystem, at which one known variable status executes a role of known external load (Menshikov, 1983,1985,1994).

Let us suppose that the motion of the received open dynamic subsystem is being described by system of the linear ordinary differential equations with constant coefficients

$$\dot{\overline{X}} = B\,\overline{X} + C\,\overline{Z}\ ,$$

$$\overline{Y} = D\,\overline{X} + F\,\overline{Z}\ ,$$

where $\overline{X} = (x_1, x_2, ..., x_n)^*$, $\overline{Z} = (z_1, z_2, ..., z_{n_1})^*$, $\overline{Y} = (y_1, y_2, ..., y_{n_2})^*$ ((.)* is the mark of transposition); $z_1 = z$ is the researched external load; B, C, D, F are matrixes with constant coefficients appropriated dimensions, moreover D is diagonal matrix containing only one not zero element, F is diagonal matrix containing only first zero element, $\overline{X}$ is the vector - function of status variables, $\overline{Y}$ is the vector - function of observed variables.

A problem of determination of scalar model z(t) of EL in many cases can be reduced to the solution of the linear integral equation Volltera of the first kind (Menshikov, 1983,1985,1994)

$$\int_0^t K(t-\tau)\, z(\tau)\, d\tau = u_\delta(t),$$

$$\text{or } A_p\, z = u_\delta,\ z \in Z,\ u_\delta \in U\ ; \qquad (1)$$

where Z,U are B- functional spaces, $A_p : Z \to U$.

The function u_δ is obtained from experiment with a known error δ:

$$\| u_T - u_\delta \|_U \leq \delta ,$$

where u_T is an exact response of object on real external load.

We denote by $Q_{\delta,p}$ the set of functions which satisfy the equation (1) with the exactness of experimental measurements with a fixed operator A_p:

$$Q_{\delta,p} = \{ z : z \in Z, \| A_p\, z - u_\delta \|_U \leq \delta \} .$$

The set of $Q_{\delta,p}$ is unbounded set in norm of space U as A_p is a compact operator (Tikhonov, Arsenin , 1979). Any function from $Q_{\delta,p}$ is the good mathematical model of external load. However not all of them are convenient for further use in mathematical modeling. Let the value some continuous non-negative functional $\Omega[z]$, defined on Z_1 (Z_1 is everywhere dense set in Z) characterizes a degree of use convenience of functions from the set $Q_{\delta,p}$.

Let the function $z_p \in Q_{\delta,p}$ satisfies the condition:

$$\Omega[z_p] = \inf_{z \in Q_{\delta,p} \cap Z_1} \Omega[z] . \qquad (2)$$

Function z_p we shall name as *the solution of synthesis problem of EL model*.

Furthermore there are no reasons to believe that the function z_p will be close to real external load. It is only convenient model of external load to use for mathematical modeling later.

Let the operator A_p depends on vector-parameters of mathematical model $p = (p_1, p_2, \ldots, p_m)^*$, $p \in R^m$. It is supposed that the parameters of mathematical model are determined inexact with some error $p_i^0 \leq p_i \leq \hat{p}_i$, $i = 1,2,3,\ldots,m$. Therefore, the vector-parameters p can accept values in some closed domain $p \in D \subset R^m$. The operator A_p in (1) will correspond to everyone of vector-parameter $p \in D$ and they form some class of operators $K_A = \{A_p\}$.

Let's designate through h size of the maximal deviation of the operators A_p from K_A .

Let's consider the extreme problem (2) of model synthesis of EL $\tilde{z} \in Q_{\delta,h}$ for class of models K_A [2,3,4]. The set of the possible solutions for all A_p has the following form in this case:

$$Q_{\delta,h} = \{ z : z \in Z, A_p \in K_A, \| A_p\, z - u_\delta \|_U \leq \delta + \| z \|_Z \} . \quad (3)$$

Any function from $Q_{\delta,h}$ brings about the response of mathematical model, which coincides with the response of real object with an error, which takes into account an error of experimental measurements and error of a possible deviation of parameters of a vector $p \in D$. A problem of finding of mathematical model $\tilde{z} \in Q_{\delta,h}$ of external load with is convenient for use later was called by analogy to the previous problem by *a problem of models synthesis for a class of models* (Menshikov, 1985).

The set of the solutions of inverse problem of synthesis with fixed operator A_p from K_A contains elements with unlimited norm (incorrect problem), therefore size $\delta + \| z \|_Z$ can be indefinitely large. Formally such situation is unacceptable, as it means, that the error of mathematical modeling is equal to infinity, if as models to use any function from $Q_{\delta,h}$. Hence not all functions from $Q_{\delta,h}$ will be "good" models of EL.

Further we shall believe, that the size $\| u_\delta \|_U$ exceeds an error of experimental measurements δ, i.e. $\delta < \| u_\delta \|_U$. Otherwise the zero element of space Z belongs to set $Q_{\delta,h}$ with any operator $A_p \in K_A$, for which $A_p 0 = 0$. This case does not represent practical interest, as the response u_δ can be received with trivial model of EL.

Let's consider the union of sets of the possible solutions $Q_{\delta,p}$:

$$\hat{Q} = \bigcup_{p \in D} Q_{\delta,p} , \ (\cup - \text{mark of union}).$$

As *the solution of a problem of synthesis for the class of models* z_{midl} we shall accept the *element* from $\hat{Q}$ (instead of set $Q_{\delta,h}$) $z_{midl} \in \hat{Q}$ which satisfies the condition:

$$\Omega[z_{midl}] = \inf_{z \in \hat{Q} \cap Z_1} \Omega[z] .$$

For increase of exactness of problem solution of synthesis for class of models the method of choice of special MM is used (Menshikov, 1997). For the realization of such approach it is necessary to choose within the vectors $p \in D$ some vector $p_0 \in D$ such that

$$\Omega[A_{p_0}^{-1} x] \le \Omega[A_p^{-1} x]$$

for all possible $x \in X$ and all $p \in D$. The operator A_{p_0} with parameter $p_0 \in D$ will be called the special minimal operator.

3 THE UNIFIED MATHEMATICAL MODEL OF EXTERNAL LOAD

Let's consider the problem of construction (synthesis) of EL model $z^{un} \in Z_1$ which provides the best results of mathematical modeling uniformly for all operators $A_p \in K_A$ (Menshikov and Nakonechny, 2005):

$$\| A_p z^{un} - u_\delta \|_U^2 \le \inf_{z_p} \sup_{A_b \in K_A} \| A_b z_p - u_\delta \|_U^2 \quad \text{for all } A_p \in K_A. \tag{4}$$

Let us name function z^{un} as *the unified mathematical model of external load for class K_A*.

Theorem. The function z^{un} exist and steady to small variations of initial data if $\Omega[z]$ is stabilizing functional.

4 IDENTIFICATION OF EXTERNAL RESISTANCE ON ROLLING MILLS

One of the important characteristics of rolling process is the moment of technological resistance (MTR) arising at the result of plastic deformation of metal in the center of deformation. Size and character of change of this moment define loadings on the main mechanical line of the rolling mill. However complexity of processes in the center of deformation do not allow to construct authentic mathematical model of MTR by usual methods. In most cases at research of dynamics of the main mechanical lines of rolling mills MTR is being created on basis of hypothesis and it is being imitated as piecewise smooth linear function of time or corner of turn of the working barrels (Menshikov, 1983,1985,1994). The results of mathematical modeling of dynamics of the main mechanical lines of rolling mills with such model MTR are different among themselves (Menshikov,1994).

In work the problem of construction of models of technological resistance on the rolling mill is considered on the basis of experimental measurements of the responses of the main mechanical system of the rolling mill under real EL (Menshikov,1983,1985,1994). Such approach allows to carry out in a consequence mathematical modeling of dynamics of the main mechanical lines of rolling mills with a high degree of reliability and on this basis to develop optimum technological modes. The four-mass model with weightless elastic connections is chosen as MM of dynamic system of the main mechanical line of the rolling mill (Menshikov, 1983,1985,1994):

$$\ddot{M}_{12} + \omega_{12}^2 M_{12} - \frac{c_{12}}{\vartheta_2} M_{23} - \frac{c_{12}}{\vartheta_2} M_{24} = \frac{c_{12}}{\vartheta_1} M_{eng}(t)$$

$$\ddot{M}_{23} + \omega_{23}^2 M_{23} - \frac{c_{23}}{\vartheta_2} M_{12} + \frac{c_{23}}{\vartheta_2} M_{24} = \frac{c_{23}}{\vartheta_3} M_{rol}^U(t) \; ; \tag{5}$$

$$\ddot{M}_{24} + \omega_{24}^2 M_{24} - \frac{c_{24}}{\vartheta_2} M_{12} + \frac{c_{24}}{\vartheta_4} M_{23} = \frac{c_{24}}{\vartheta_4} M_{rol}^L(t)$$

where $\omega^2_{ik} = c_{ik} (\vartheta_i + \vartheta_k) \vartheta_i^{-1} \vartheta_k^{-1}$, ϑ_k are the moments of inertia of the concentrated weights, c_{ik} are the rigidity of the appropriate elastic connection, M_{rol}^U, M_{rol}^L are the moments of technological resistance put to the upper and lower worker barrel accordingly, $M_{eng}(t)$ is the moment of the engine.

The problem of synthesis of MM of EI can be formulated so: it is necessary to define such external models of technological resistance on the part of metal which would cause in elastic connections of model of fluctuations identical experimental (in points of measurements) taking into account of an error of measurements and error of MM of the main mechanical line of rolling mill. Such type of problems and the methods of their solutions can find applications at construction of MM of EI in other similar situations.

The information on the real motion of the main mechanical line of rolling mill is received by an experimental way (Menshikov, 1983,1985,1994). Such information is being understood as presence of functions $M_{12}(t)$, $M_{23}(t)$, $M_{24}(t)$. Let's consider a problem of construction of models of EL to the upper working barrel. On the lower working barrel all calculations will be carried out similarly. From system (5) the equation concerning required model M_{rol}^U can be received.

$$\int_0^t \sin\omega(t-\tau)\, M_{rol}(\tau)\, d\tau = u_\delta(t)\ .$$

The size of the maximal deviation of the operators $A_T \in K_A$ was defined by numerical methods and it equal h = 0.12. An error initial data for a case Z = U = C [0,T] is equal $\delta = 0.066$ Мнм.

In figure 1 the diagrams of functions z_{midl}, z^{un} for a typical case of rolling on the smooth working barrel are submitted as solution of last equation (Menshikov, 1983,1985,1994).

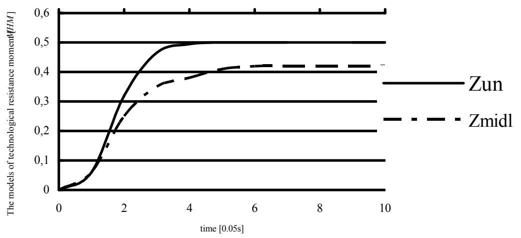

Figure 1: The diagrams of change of models of the moment of technological resistance on the rolling mill.

The results of calculations are showing that the rating from above of accuracy of mathematical modeling with model z^{un} for all $A_T \in K_A$ does not exceed 11 % in the uniform metrics with error of MM parameters of the main mechanical line of rolling mill in average 10 % and errors of experimental measurements 7 % in the uniform metrics.

The calculations of model of EL $\tilde{z}$ for a class of models K_A on set of the possible solutions $Q_{\delta,h}$ was executed for comparison. This function has the maximal deviation from zero as 0.01 Мнм.

In work [4] the comparative analysis of mathematical modeling with various known models of EL was executed. The model of load z^{un} turn out to be correspond to experimental observations in the greater degree [4].

4 CONCLUSIONS

The offered approach to synthesis of mathematical models of external loads on dynamical system can find application in cases when the information about external loads is absent or poor and also for check of hypotheses on the basis of which were constructed the known models of external loads.

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