# **DISCRETE DYNAMIC SLIDING SURFACE CONTROL FOR ROBUST SPEED CONTROL OF INDUCTION MACHINE DRIVE**

Abdel Faqir<sup>(1)</sup>, Daniel Pinchon<sup>(2)</sup>, Rafiou Ramanou<sup>(2)</sup> and Sofiane Mahieddine<sup>(2)</sup>

*(1) ECTEI – (Paris, France) faqir@ece.fr (2) LTI - Institut Universitaire de Technologie de l'Aisne (GEII) 13, Avenue François Mitterrand 02880 Cuffies France Tel: +33 (0) 323 764 010, Fax: +33 (0) 323 764 015 daniel.pinchon@u-picardie.fr* 

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Abstract: This paper proposes the discrete dynamic sliding surface control to guarantee the existence of discrete sliding mode and reduce the chattering phenomena for speed control of induction machine drive. In discrete systems, the controller does not control the system during the sampling interval. The great chattering and large control signal are caused by the high switching gain. In this paper, the dynamic sliding surface is introduced to overcome the drawback. By setting the initial value of the dynamic sliding surface, the system can lock to the sliding surface quickly without high switching gain. The control signal can be reduced and the chattering can be eliminated. Furthermore, the induction machine speed control system is used to show this controller's robustness to against the parameter variation and external load. The speed of the induction machine is regulated using the indirect field oriented control (IFOC). Thus, after the application of the IFOC technique by determining the decoupled model of the machine, a discrete sliding surface controller has been applied. Simulation study is used to show the performances of the proposed method and then validated by an experimental prototype.

## **1 INTRODUCTION**

Field oriented control, published for the first time by Blaschke in his pioneering work in 1972, consists in adjusting the flux by a component of the current and the torque by the other component. For this purpose, it is necessary to choose a d-q reference frame rotating synchronously with the rotor flux space vector, in order to achieve decoupling control between the flux and the produced torque. This technique allows to obtain a dynamical model similar to the DC machine.

This technique presents a major drawback. Indeed the behavior of the machine and its command is strongly affected by the variation of the rotor resistance due to the temperature or by the variations of the rotor inductance due to the saturation.

To eliminate this drawback, we propose in this paper, an indirect field oriented method using two sliding mode controllers. Once the decoupled model

of the machine is obtained, a discrete sliding surface control is chosen with an appropriate switching. Simulations have been carried out to verify the effectiveness and the performances of the proposed method.

## **2 SYSTEM DESCRIPTION AND MACHINE MODELLING**

The system is an induction machine fed by a PWM voltage source inverter. The sliding mode controllers are applied to the inverter via reference voltages (Fig. 1).



Figure 1: Discrete Sliding Surface Control structure of an induction machine.

## **2.1 Induction Machine Model**

For the study of the induction machine, we take the following model:

$$
\frac{dX}{dt} = AX + BU \tag{1}
$$

with

$$
A = \begin{bmatrix} -\left(\frac{1}{T_s\sigma} + \frac{1}{T_r} - \frac{\sigma}{\sigma}\right) & \omega_s & \frac{1-\sigma}{\sigma} \frac{1}{L_m T_r} & \frac{1-\sigma}{\sigma} \frac{1}{L_m} \omega_r \\ -\omega_s & -\left(\frac{1}{T_s\sigma} + \frac{1}{T_r} - \frac{\sigma}{\sigma}\right) & -\frac{1-\sigma}{\sigma} \frac{1}{L_m} \omega_r & \frac{1-\sigma}{\sigma} \frac{1}{L_m T_r} \\ \frac{L_m}{T_r} & 0 & -\frac{1}{T_r} & \omega_s l \\ 0 & \frac{L_m}{T_r} & -\omega_{s l} & -\frac{1}{T_r} \end{bmatrix}
$$

$$
B = \begin{bmatrix} \frac{1}{\sigma L_s} & 0 & \frac{1}{\sigma L_s} \\ 0 & \frac{1}{\sigma L_s} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, X = \begin{bmatrix} i_{ds} \\ i_{qs} \\ \phi_{dr} \\ \phi_{qr} \end{bmatrix} \text{ and } U = \begin{bmatrix} v_{ds} \\ v_{qs} \\ v_{qs} \end{bmatrix}
$$

Where

$$
\sigma = \begin{pmatrix} \frac{l^2}{l_{B}} \\ l - \frac{l^2}{l_s} \end{pmatrix} \qquad \omega_r = p\Omega_r, \qquad \omega_s = \omega_{sl} + \omega_r.
$$

The stator voltages  $\begin{pmatrix} v_{ds}, v_{as} \end{pmatrix}$  are considered as control inputs, while the stator currents  $\left( \frac{i}{ds}, i_{gs} \right)$ , the rotor flux  $(\phi_{dr}, \phi_{qr})$  and the speed  $(\Omega_r)$  are considered as state variables.

From the equations (1), the following electrical equations are deduced:

$$
\begin{cases}\n\frac{di_{ds}}{dt} = \frac{-R_{sm}}{\sigma L_s} i_{ds} + \omega_s i_{qs} + \frac{1}{\sigma L_s} \frac{L_m}{T_r L_r} \phi_{dr} + \frac{1}{\sigma L_s} \frac{L_m}{L_r} \omega_r \phi_{qr} + \frac{1}{\sigma L_s} v_{ds} \quad (2) \\
\frac{di_{qs}}{dt} = -\omega_s i_{ds} - \frac{R_{sm}}{\sigma L_s} i_{qs} - \frac{1}{\sigma L_s} \frac{L_m}{L_r} \omega_r \phi_{dr} + \frac{1}{\sigma L_s} \frac{L_m}{T_r L_r} \phi_{qr} + \frac{1}{\sigma L_s} v_{qs} \quad (3)\n\end{cases}
$$

$$
\begin{cases}\n\frac{d\phi_{dr}}{dt} = \frac{L_m}{T_r} i_{ds} - \frac{1}{T_r} \phi_{dr} + \omega_{sl} \phi_{qr}\n\end{cases}
$$
\n(4)

$$
\frac{d\phi_{qr}}{dt} = \frac{L_m}{T_r} i_{qs} - \omega_{sl} \phi_{dr} - \frac{1}{T_r} \phi_{qr}
$$
\n<sup>(5)</sup>

With 
$$
R_{sm} = R_s + \frac{L_m^2}{L_r^2} R_r
$$

The mechanical model is given by:

$$
J\frac{d\Omega_r}{dt} = T_{em} - T_L - K_f \Omega_r \tag{6}
$$

And the electromagnetic torque can be expressed as:

$$
T_{em} = \frac{pL_m}{L_r} \left( \phi_{dr} i_{qs} - \phi_{qr} i_{ds} \right) \tag{7}
$$

### **2.2 Field Oriented Control**

The field orientation is obtained by imposing:

$$
\begin{cases}\n\phi_{dr} = \phi_r \\
\phi_{qr} = 0\n\end{cases}
$$
\n(8)

From the equations (4) and (8), the  $i_{ds}$  reference can be computed in order to impose the flux  $\phi_r$ :

$$
\phi_r = \frac{L_m}{1 + sT_r} i_{ds} \tag{9}
$$

Furthermore, the position  $\theta_s$  of the rotating frame can be estimated using equations (5) and (8):

$$
\theta_{s} = \int \left( \frac{L_{m} i_{qs}}{T_{r} \phi_{r}} + p \Omega_{r} \right) dt
$$
 (10)

With taking into account the field orientation of the machine, the stator equations on d-q axis become:

$$
\begin{cases}\nV_{ds} = \left[\sigma L_s \frac{di_{ds}}{dt} + R_{sm} i_{ds} - \sigma L_s \omega_s i_{qs} - \frac{L_m}{T_r L_r} \phi_r\right] \\
V_{qs} = \left[\sigma L_s \frac{di_{qs}}{dt} + R_{sm} i_{qs} + \sigma L_s \omega_s i_{ds} + \frac{L_m}{L_r} \omega_r \phi_r\right]\n\end{cases} (11)
$$

### **2.3 Decoupling System**

Using the system given by equations (11), we can remark the interaction of both inputs, which makes the control design more difficult.

The first step of our work is to obtain a decoupled system in order to control the electromagnetic torque via stator quadrature current  $i_{qs}$  such as a DC machine.

A decoupled model can be obtained by using two intermediate variables:

$$
v_{ds1} = v_{ds} + emf_d \tag{12}
$$

$$
v_{qs1} = v_{qs} + emf_q \tag{13}
$$

where 
$$
emf_d = \omega_s \sigma L_s i_{qs} + \frac{L_m}{L_r^2} R_r \phi_r
$$
 (14)

and 
$$
emf_q = -\omega_s \sigma L_s i_{ds} - \omega_s \frac{L_m}{L_r} \phi_r + \frac{L_m^2}{L_r T_r} i_{qs}
$$
 (15)  
\n
$$
\begin{bmatrix} v_{ds1} \\ v_{qs1} \end{bmatrix} = M^{-1} \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix}
$$
\n
$$
M = \begin{bmatrix} L & 0 \\ 0 & L \end{bmatrix} \text{ with } L = \frac{L_r T_r}{R_s L_r T_r + L_m^2 + \sigma L_s L_r T_r s}
$$
 (16)

The stator voltages  $(v_{ds}, v_{ds})$  are reconstituted from  $(v_{ds1}, v_{gs1})$  (Fig. 2):



Figure 2: Decoupling control.

## **3 DISCRETE SLIDING SURFACE CONTROL**

#### **3.1 General Concept**

Since Dr. Utkin proposed the variable structure system (VSS), it had been widely discussed and applied in many control systems. Due to the change of the switching gains in control function, the controlled system can vary its own controller according to the external condition. Hence, VSS is robust to against to the system's parameter variation and external disturbance. VSS owns one sliding surface predetermined according to the desired dynamic character. Once the sliding mode locking on the sliding surface, the system response will be directed by this surface. The existence condition of classical sliding mode in continuous system is

$$
S\dot{S} < 0 \tag{17}
$$

Change the differential equation to difference equation. When applying the condition to discrete systems, the existence condition becomes to

$$
S(k)[S(k+1) - S(k)] < 0
$$
 (18)

However, the system is controlled by the controller only in each sampling time. The controller can not modify the response during the sampling interval. It may happen that the condition (18) is not only satisfied but also the sliding motion is divergent. It is shown in fig 3.



Figure 3: Discrete sliding mode.

The condition (18) only makes the sliding motion toward to the sliding surface. However, it can not guarantee the sliding mode convergent to this surface. The condition (18) is only the necessary condition not the sufficient condition in discrete systems. To make up the drawback, we introduces one additional restriction, that is

$$
|\mathbf{S}(\mathbf{k} + \mathbf{1})| < |\mathbf{S}(\mathbf{k})| \tag{19}
$$

Combining equation (18) and (19) can make sure the sliding motion convergent. However, the sliding surface is changed to sliding region shown in fig 4.



Figure 4: Discrete sliding mode with sliding region.

The choice of switching gain becomes three states. This change causes some difficulty in implementation of hardware. To maintain the binary choice, one restriction of different viewpoint, that is

$$
\left|S(k+1)-S(k)\right|<\frac{\xi}{2}\tag{20}
$$

where  $\xi$  is a small positive constant. The varying of each step of sliding motion is restricted. Then, condition (18) makes the sliding mode toward to the surface. The condition (20) makes the sliding motion oscillated on this surface within a small range ξ shown in fig 5.



Figure 5: New discrete sliding mode.

This new sliding mode is  $|S(k)| < \xi$  different to the classical sliding mode  $S(k) = 0$ . Hence, this new sliding mode is called "non-ideal sliding mode." This paper will prove that if the controller makes the system's solution of classical sliding mode asymptotically stable, then the same controller makes the solution of non-ideal sliding mode asymptotically stable, too. Hence, the discussion of non-ideal sliding mode can be done like classical sliding mode. Just like the classical sliding mode, the system's dynamic character is directed by the surface only when the sliding motion is within the range of  $\xi$ . To reduce the time of out of control, the high switching gain is usually chosen to speed up the reaching time. However, in discrete systems the controller only modifies the control signal at each sampling time. High switching gains can speed up the reaching time, but the chattering often be enlarged. To eliminate the chattering and decrease the reaching time, this paper introduces the dynamic sliding surface control (DSSC) rule shown in fig 6.



Figure 6: Discrete sliding mode with dynamic sliding surface.

## **5 SIMULATION RESULTS**



Figure 7: Scheme of speed sliding mode regulation.

The proposed scheme has been simulated using parameters given in the appendix.

The first simulation realized on the AC machine consists in step variation of the reference. Indeed, (Fig. 8) and (Fig. 9) represent the speed and the rotor flux when reference step is first imposed and then an inversion is imposed.

In (Fig. 9) it is clearly shown for rotor flux responses that the decoupling is realized since the direct component of the rotor flux converges to the reference  $(\phi_r)_{ref}$ , and its quadrature component to zero despite the reference variations. Furthermore we can remark that the proposed control scheme presents good tracking capacities since there is no overshoot and no static error (Fig. 8).







Figure 9: Rotor flux components responses.

(Fig. 10) represents the dynamic response of the speed for different values of the load torque. First when the speed reaches its reference value (1500 tr/min), a step of load torque is applied at  $(t=0.7 \text{ s})$ . The electromagnetic torque rises to the new value of the load torque (Fig.11), and the speed is not disturbed. Then when the load torque is decreased to zero  $(t=1.2 \text{ s})$  or to a negative value  $(t=1.5 \text{ s})$  the speed stays on the reference value.

It is clearly shown from the results that the control scheme presents good regulation capacities. Indeed, the external disturbances such as load torque variations are rejected by control system.



Figure 10: Speed response to load torque variations.



Figure 11: Electromagnetic torque responses to load torque variations.

(Fig. 12) and (Fig. 13) illustrate the dynamic response in the phase plane respectively with a normal sliding surface control and with a dynamic discrete sliding surface control.

We can note the apparition of the chattering phenomenon (Fig. 12) with the normal sliding surface control due to the discontinuous characteristic of this function.



Figure 12: Response in the phase plane with the normal sliding surface control.



Figure 13: Response in the phase plane with dynamic discrete sliding surface control.

(Fig. 14) depicts the drive response for different values of the rotor constant time.

It is important to note that the changing parameters are introduced only in the model of the machine. The controller is not involved by these variations.

It is well-known for classic controller that the indirect field oriented control is very sensitive to the rotor constant time variations.

The results shown on (Fig. 14) confirm the robustness quality inherent to the proposed

controller. Indeed, there is no overshot whatever the rotor constant time



Figure 14: Robustness test of the sliding mode control.

## **6 EXPERIMENTAL RESULTS**

Figs. 15 and 16 show the experimental evolution of the position and the experimental phase plane trajectory when the sliding condition is just validated:  $|c_{\perp}| = |s1p|$ . It can be seen that the step reference of 400 steps is reached for  $t = 0.08$  s without overshoot or steady-state error and that there are almost three commutations to reach the reference.



Figure 15: Position and reference (400 steps).



Figure 16: Position and reference for two different inertias.



Figure 17: Phase plane trajectory.

Figs. 17 and 18 show the evolution of the position and the phase plane trajectories for two different inertias  $(\bar{J} = J\text{min} = 0.0023 \text{ kg.m}^2 \text{ and } J = J\text{max} =$  $0.013485$  kg.m<sup>2</sup>).

By comparing the two responses, it can be noted that the reference is always reached without any overshoot or steady-state error whatever the inertia of the drive.



Figure 18: Phase plane trajectory for two different inertias.

From these results, it can be seen that the robustness of the proposed approach between external disturbances and plant parameter variations is experimentally validated.

## **7 CONCLUSION**

In this paper, we have shown that by using a sliding mode control applied to an IFOC, a high-precision positioning of an IM shaft can be achieved whatever the mechanical configuration of the load is. Indeed, the position reference is obtainedwithout any overshoot or static errorwhatever the inertia or the load torque are. Furthermore, it has been shown that the chattering problem around the switching surface can be alleviated using the VSC approach with LFSG. Therefore, the proposed solution can be considered very suitable for induction drive used in robotics or in numerical control of machine tools.

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### **LIST OF PRINCIPAL SYMBOLS**

- *p* : Number of pole pairs.
- $R_{\rm c}$ ,  $R_{\rm r}$  : Stator and rotor resistance.
- $L_s$ ,  $L_r$ : Stator and rotor inductance.
- $T_r$ : Rotor time constant.
- *Lm* : Magnetizing inductance.

 $i_{ds}$ ,  $i_{gs}$  : Stator currents in d-q rotating reference frame.

- $v_{ds}$ ,  $v_{as}$ : Stator voltages d-q rotating reference frame.
- $\phi_{dr}$ ,  $\phi_{ar}$ : Rotor fluxes d-q rotating reference frame.
- <sup>ω</sup>*<sup>r</sup>* : Rotor speed.
- $T_e$ : Electromagnetic torque.
- $\theta_{\rm s}$ : Angular position.
- *s* : Laplace operator (*d* / *dt*).
- $\sigma$ : Coefficient of dispersion.
- *J* : Total rotor inertia constant.