ON COMPUTING MULTI-FINGER FORCE-CLOSURE GRASPS OF 2D OBJECTS

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Abstract: In this paper, we develop a new algorithm for computing force-closure grasps of two-dimensional (2D) objects using multifingred hand. Assuming hard-finger point contact with Coulomb friction, we present a new condition for multi-finger to form force-closure grasps. Based on the central axis of contact wrenches, an easily computable algorithm for force-closure grasps has been implemented and its efficiency has been demonstrated by examples.

1 INTRODUCTION

Grasping remains one of the fundamental problems in robotics. Research has been directed towards the design and control of multifingred dexterous robot hand to increase robot dexterity and adaptability (Li J-W., Jin M-H. and Liu H. 2003).

A main property of a multi-finger stable grasp is force-closure. It's the ability to balance any external object wrenches by applying appropriate finger wrenches at the contact points. In other words, a grasp on an object is force-closure if and only if arbitrary force and torque can be exerted on the object through the fingers (Yan-Bin Jia 2004). It's complicated to assure that the applied finger forces remain in the friction cone at all times so as to avoid fingers slippage on the object surface (Murray R., Li Z. and Sastry S. 1994).

Human can use more than three/four fingers of his hand to manipulate objects. During such tasks, there exists a lot of contact points between the hand and grasped object. The question is: how can we evaluate or compute force-closure of such grasps?

In this paper, we are focused on the problem of computing force-closure of multifingered grasps of 2D objects. We develop a new approach for force-closure test independently of fingers's number. This quality is obtained using the mechanical properties of the grasp wrench.

2 RELATED WORK

Force-closure test is an essential problem in grasping. However, The notion of force-closure does not directly yield a method for force-closure test (Sudsang A. and Phoka T. 2005). Some necessary and sufficient conditions for force-closure were formulated in order to derive force-closure tests. A commonly used necessary and suffecient force-closure condition given by (Salisbury J.K. and Roth B. 1982) allowed a force-closure test to be performed by checking whether the origin is strictly inside the convex hull of the primitive contact wrenches. This test also provided an underlying idea to recent work in grasping (D. Ding, Y-H Liu, and S. Wang 2001). Nguyen (Nguyen, V.D. 1988) formally demonstrated for 2-fingered grasps that non-marginal equilibrium grasps achieve force-closure. Recently, (Li J-W., Jin M-H. and Liu H. 2003) proposed a necessary and suffecient condition for 3-fingered force-closure grasps based on (Ponce J. and Faverjon B. 1995) and developed an algorithm for three-finger force-closure test. Their method begins by the processing of friction cones using an operation called disposition H, then, they attack the problem of determining the intersection of the three double-side friction cones.

The rest of the paper is organized as follow, in section 3, we present the background of grasp wrenches central axes and the relationship between these axes and grasp force-closure. In section 4, we propose a new multi-finger force-closure condition. Hence, a novel algorithm is presented, which its implementation needs little geometric computations. In section 5, we present some multi-finger grasps examples. Finally, we conclude with future works.

3 CENTRAL AXIS OF GRASP WRENCHES

Based on Coulomb friction model, a contact force is constrained to lie in a friction cone centered about the internal surface normal at contact point.

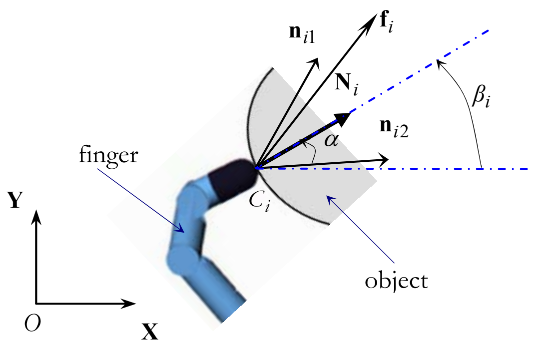


Figure 1: Contact between the finger and an object showing friction cone.

As shown in figure 1, a friction cone at C_i is bounded by vectors $\mathbf{n}_{i1}$ and $\mathbf{n}_{i2}$, and any force $\mathbf{f}_i$ is a nonnegative combination of these two vectors. In 2D case, contact forces are

$$\mathbf{f}_i = a_{i1}\,\mathbf{n}_{i1} + a_{i2}\,\mathbf{n}_{i2} \quad i = 1,\ldots,m \tag{1}$$

With $a_{i1} \geq 0, a_{i2} \geq 0$ to avoid fingertips slippage. m is the number of contact points.

If $\mathbf{N}_i$ is the surface normal at the contact point C_i and α is the friction angle that depends on materials in contact (finger and object) then,

$$\beta_i = Atan2(\mathbf{N}_i\,\mathbf{Y}, N_i\,\mathbf{X}) \tag{2}$$

and

$$\mathbf{n}_{i1} = \begin{pmatrix} Cos(\beta_i+\alpha) \\ Sin(\beta_i+\alpha) \end{pmatrix} ; \ \mathbf{n}_{i2} = \begin{pmatrix} Cos(\beta_i-\alpha) \\ Sin(\beta_i-\alpha) \end{pmatrix} \tag{3}$$

The contact wrench produced by $\mathbf{f}_i$ reduced at the point O is defined by

$$\mathbf{w}_i = \begin{bmatrix} \mathbf{f}_i \\ \mathbf{OC}_i \times \mathbf{f}_i \end{bmatrix} = \begin{bmatrix} a_{i1}\,\mathbf{n}_{i1} + a_{i2}\,\mathbf{n}_{i2} \\ \mathbf{OC}_i \times (a_{i1}\,\mathbf{n}_{i1} + a_{i2}\,\mathbf{n}_{i2}) \end{bmatrix} \tag{4}$$

The external wrench applied by the robotic hand on the grasped object is given by

$$\mathbf{W}_{c/o} = \sum_{i=1}^{m} \mathbf{w}_i = \begin{bmatrix} \mathbf{F}_c \\ \boldsymbol{\tau}_{c/o} \end{bmatrix} \tag{5}$$

With

$$\begin{aligned} \mathbf{F}_c &= \sum_{i=1}^{m} \mathbf{f}_i = \sum_{i=1}^{m} (a_{i1}\,\mathbf{n}_{i1} + a_{i2}\,\mathbf{n}_{i2}) \\ \boldsymbol{\tau}_{c/o} &= \sum_{i=1}^{m} (\mathbf{OC}_i \times (a_{i1}\,\mathbf{n}_{i1} + a_{i2}\,\mathbf{n}_{i2})) \end{aligned} \tag{6}$$

In two-dimensional grasps case, we have

$$\mathbf{F}_c = (F_{cx}, F_{cy}, 0)^T \text{ and } \boldsymbol{\tau}_{c/o} = (0, 0, \tau_{z/o})^T \tag{7}$$

With

$$\begin{cases} F_{cx} = \mathbf{X} \cdot \sum_{i=1}^{m} (a_{i1}\,\mathbf{n}_{i1} + a_{i2}\,\mathbf{n}_{i2}) \\ F_{cy} = \mathbf{Y} \cdot \sum_{i=1}^{m} (a_{i1}\,\mathbf{n}_{i1} + a_{i2}\,\mathbf{n}_{i2}) \\ \tau_{z/o} = \mathbf{Z} \cdot \sum_{i=1}^{m} (\mathbf{OC}_i \wedge (a_{i1}\,\mathbf{n}_{i1} + a_{i2}\,\mathbf{n}_{i2})) \end{cases} \tag{8}$$

Poinsot's theorem: "*Every collection of wrenches applied to a rigid body is equivalent to a force applied along a fixed axis (central axis) and a torque around the same axis*" (Murray R., Li Z., Sastry S. 1994). Using this theorem, points of the central axis Δ_C of contact wrench are given by

$$\Delta_C = \begin{cases} \dfrac{\mathbf{F}_c \times \boldsymbol{\tau}_{c/o}}{\mathbf{F}_c^{\,2}} + \lambda \mathbf{F}_c & if\ \mathbf{F}_c \neq \mathbf{0} \\ \lambda \boldsymbol{\tau}_{c/o} & if\ \mathbf{F}_c = \mathbf{0} \end{cases} : (\lambda \in \Re) \tag{9}$$

The axis Δ_C is a directed line through a point. For $\mathbf{F}_c \neq \mathbf{0}$, the central axis is a line in the

$\mathbf{F}_c$ direction going through the point Q_0 such as $\mathbf{OQ}_0 = (\mathbf{F}_c \times \boldsymbol{\tau}_{c/o}) / \mathbf{F}_c^{\,2}$.

For $\mathbf{F}_c = \mathbf{0}$, the axis is a line in the $\boldsymbol{\tau}_{c/o}$ direction going through the origin (Murray R., Li Z., Sastry S. 1994).

In two-dimentional case with non null forces ($\mathbf{F}_c \neq \mathbf{0}$), the torque around the central axis is zero. The force $\mathbf{F}_c$ is an invariant vector and always parallel to Δ_C. Figure 2 shows the central axis in 2D grasps when $\mathbf{F}_c \neq \mathbf{0}$. it is Characterized by the following equation

$$y = \left(\frac{F_{cy}}{F_{cx}}\right) \cdot x + \left(\frac{\tau_{z/o}}{F_{cx}}\right) \tag{10}$$

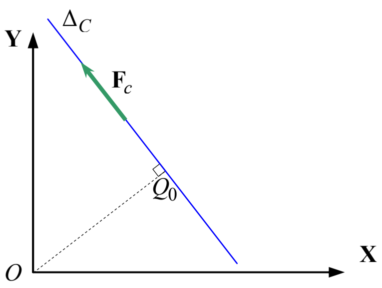
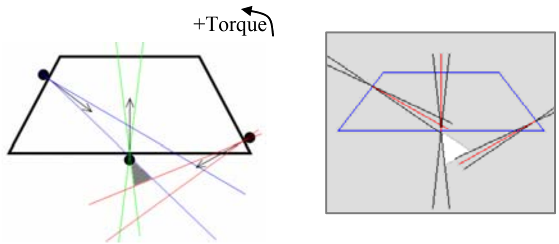


Figure 2: Central axis parameters.

In figure 3-a, we present a first example of three-finger 2D grasp. By varying forces $\mathbf{f}_i$ randomly (in orientation and amplitude) inside friction cones (the friction angle), figure 3-b illustrates all possible central axes of grasp wrenches.

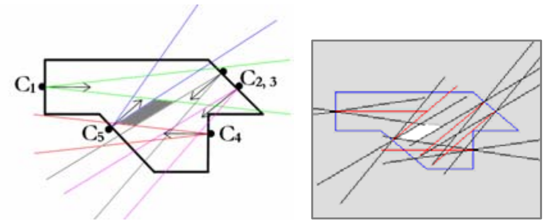
There is no central axis passing through the gray region. In this region, positive torque can't be exerted on the object through the finger contacts. This grasp can't achieve force-closure. Exactly, the grasp can not achieve torque-closure because the object turn around the gray region in figure 3-a.



(a) (b)

Figure 3: a) no force-closure 2D grasp, b) all central axes of grasp wrenches ($\alpha = 5°$).

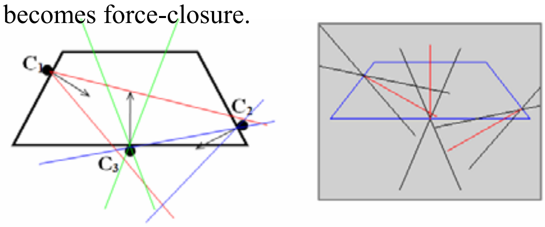
A second example is shown in figure 4; we present a non-force-closure grasp using five contact points. This grasp is instable and the object turn around $\mathbf{Z}$ axis in the gray region (figure 4-a).



(a) (b)

Figure 4: a) non-force-closure five-fingers grasp; b) central axes of grasp wrenches ($\alpha = 10°$).

When a grasp is force closure, the central axes of grasp wrenches can wholly sweep the plan $(\mathbf{X}, \mathbf{Y})$. In the third example, shown in figure 5, we use the same finger's configuration as figure 3 but we change the friction angle $\alpha = 20°$. The grasp becomes force-closure.



(a) (b)

Figure 5: a) three fingers force-closure 2D grasp, b) central axes of grasp wrenches ($\alpha = 20°$).

According to these three examples, we can conclude that the distribution of central axes can confirm if a grasp is force-closure or not (for any contact points number).

4 FORCE-CLOSURE AND EQUILIBRIUM CONDITION

In 2D grasps and based on Poinsot's theorem, we can give the following definition.

Definition 1: Any external wrench applied by the robotic hand on a 2D object is equivalent to a force along a central axis of this wrench. When the force is equal to zero, the external wrench is equivalent to a torque about the grasp plan normal.

Definition 2: A grasp on an object is force-closure if and only if any arbitrary force and torque can be exerted on the object through the finger contacts (Yan-Bin Jia 2004). There are other force-closure definitions, but this one is more useful for our deduction.

Definition 3: A grasp is said to achieve equilibrium when it exists forces (not all of them being zero) in the friction cones at the fingertips such that the sum of the corresonding wrenches is zero (Sudsang A., Phoka T. 2005).

4.1 Equilibrium Condition

During objects grasp operations there exist two kinds of external wrenches applied on the manipulated object, task wrench (applied by the environment) and contact wrench (applied by the robotic hand fingers). Based on *definitions 1* and *definitions 3*, we derive a new proposed necessary and sufficient equilibrium condition.

Proposition 1: A multifingers grasp is said to achieve equilibrium if and only if the central lines of contact wrench and task wrench have the same support and opposite direction.

Proof:

i) *Sufficient Condition*:

the external contact wrench given by equation (5) and task wrench is given by

$$\mathbf{W}_{t/o} = \begin{bmatrix} \mathbf{F}_t \\ \boldsymbol{\tau}_{t/o} \end{bmatrix} \quad (11)$$

The object is in equilibrium if:

$$\mathbf{W}_{c/o} + \mathbf{W}_{t/o} = \{\mathbf{0}\} \Rightarrow \begin{cases} \mathbf{F}_c = -\mathbf{F}_t \\ \boldsymbol{\tau}_{c/o} = -\boldsymbol{\tau}_{t/o} \end{cases} \quad (12)$$

From Relation (9), the central axis of contact wrench is defined by

$$\mathbf{OP}_c = \frac{\mathbf{F}_c \times \boldsymbol{\tau}_{c/o}}{\mathbf{F}_c^{\,2}} + \lambda_c \mathbf{F}_c \quad (\lambda_c \in \Re) \quad (13)$$

Substuting (12) in (13) lead to

$$\mathbf{OP}_c = \frac{\mathbf{F}_t \times \boldsymbol{\tau}_{t/o}}{\mathbf{F}_t^{\,2}} + \lambda_t \mathbf{F}_t \quad (\lambda_t = -\lambda_c \in \Re) \quad (14)$$

Relation (14) defines the central axis of task wrench given by

$$\mathbf{OP}_t = \frac{\mathbf{F}_t \times \boldsymbol{\tau}_{t/o}}{\mathbf{F}_t^{\,2}} + \lambda_t \mathbf{F}_t \quad (\lambda_t \in \Re) \quad (15)$$

In the case $\mathbf{F}_t = \mathbf{0}$, the points P_c are given by

$$\mathbf{OP}_c = \lambda_c \boldsymbol{\tau}_{c/o} = \lambda_t \boldsymbol{\tau}_{t/o} \; : \; (\lambda_t = -\lambda_c \in \Re) \quad (16)$$

In both cases, Relations (14) and (16), the two wrenches (contact and task) should have the same central line with opposite directions. ■

ii) *Necessary Condition:*

Now, if we consider two wrenches reduced at the same point O and they have the same central line with opposite directions. We have two cases:

a) if $\mathbf{F}_t = \mathbf{0}$ then the central axis of the task wrench is defined by the unit vector $\mathbf{U}_t$ where:

$$\mathbf{U}_t = \frac{\boldsymbol{\tau}_{t/o}}{\|\boldsymbol{\tau}_{t/o}\|}$$

If the two wrenches have the same central line with opposite direction then the contact central axis is defined by the following unit vector:

$$\mathbf{U}_c = -\mathbf{U}_t = \frac{\boldsymbol{\tau}_{c/o}}{\|\boldsymbol{\tau}_{c/o}\|}$$

We conclude that:

$$Sgn(\boldsymbol{\tau}_{c/o} \cdot \mathbf{U}_c)\, Sgn(\boldsymbol{\tau}_{t/o} \cdot \mathbf{U}_c) < 0 \quad (17)$$

Sgn is the sign function that computes the sign of the leading coefficient of expression.

b) if $\mathbf{F}_t \neq \mathbf{0}$, having the same central axis with opposite direction implies

$$Sgn(\mathbf{F}_c \cdot \mathbf{U}_c)\, Sgn(\mathbf{F}_t \cdot \mathbf{U}_c) < 0 \quad (18)$$

Where $\mathbf{U}_c$ and $\mathbf{U}_t$ define the unit vectors of the two central axes. We have:

$$\mathbf{U}_c = \frac{\mathbf{F}_c}{\|\mathbf{F}_c\|} \quad ; \quad \mathbf{U}_t = \frac{\mathbf{F}_t}{\|\mathbf{F}_t\|}$$

Using hypothesis that there is one central line and form relation (9), we have

$$\frac{\mathbf{F}_c \times \boldsymbol{\tau}_{c/o}}{\mathbf{F}_c^{\,2}} = \frac{\mathbf{F}_t \times \boldsymbol{\tau}_{t/o}}{\mathbf{F}_t^{\,2}} \quad (19)$$

Then, replacing $\mathbf{F}_c = \|\mathbf{F}_c\| \mathbf{U}_c$; $\mathbf{F}_t = -\|\mathbf{F}_t\| \mathbf{U}_c$ in Relation (19) we obtain

$$\left(\frac{\boldsymbol{\tau}_{c/o}}{\|\mathbf{F}_c\|} + \frac{\boldsymbol{\tau}_{t/o}}{\|\mathbf{F}_t\|} \right) = \lambda \cdot \mathbf{U}_c \; : \quad (\lambda \in \Re) \quad (20)$$

In 2D case, the equation (20) can be only satisfied when $\lambda = 0$. therefore, the two torques have opposite signes:

$$Sgn(\boldsymbol{\tau}_{c/o} \cdot \mathbf{Z})\, Sgn(\boldsymbol{\tau}_{t/o} \cdot \mathbf{Z}) < 0 \quad (21)$$

Relations (17, 18 and 21) imply that the contact wrench can generate grasp wrenches that opposite the external task wrench. Robotic hand can control its fingers force to produce the appropriate force/torque magnitude that achieving equilibrium.■

4.2 Force-Closure Condition

In particular, force-closure implies equilibrium, but there are wrench systems that achieve equilibrium but not force closure (Li Jia-Wei. and Cia He-Gao, 2003).

Using force-closure condition in *definition 2*, we can derive this definition

Definition 4: A grasp is force closed, if and only if it is in equilibrium for any arbitrary wrench (Bicchi A., Kumar V. 2000, Nguyen, V.D. 1988). Thus, force closure implies, fingers contact wrenches can balance any external task wrenches.

According to *proposition 1* and *definition 4*, we propose a new force-closure necessary and sufficient condition.

Proposition 2: A multifingred grasp of 2D objects is said to achieve force-closure if and only if the central axis of the fingers contact wrenches can sweep the grasp plan at any direction.

4.3 Force-Closure Test Algorithm

According to the *proposition 2*, we present a new algorithm for computing 2D multi-fingers grasps of arbitrary object.

Based on the central axis equation defined in relation (10), this central line can sweep the plan in all directions if

$$\forall (k_1, k_2) \in \Re^2, \exists \Delta_c \text{ Satisfy } y = k_1 \cdot x + k_2$$

Where

$$k_1 = \left(\frac{F_{cy}}{F_{cx}}\right) \quad ; \quad k_1 = \left(\frac{\tau_{z/o}}{F_{cx}}\right)$$

In other word, for any axis on the $(\mathbf{X}, \mathbf{Y})$ plan or along the vertical $\mathbf{Z}$, this axis must be one of the grasp wrench central axes.

This condition implies that the quantities k_1 and k_2 must take all real number, therefore

$$\begin{cases} F_{cx} \in [-\infty, +\infty] \\ F_{cy} \in [-\infty, +\infty] \\ \tau_{c/o} \in [-\infty, +\infty] \quad \forall O \end{cases} \tag{21}$$

The third sub-condition is function of the reduced point of the torque, to cover the entire grasp plan; we test this condition at all the vertices of the intersection of the m double-side friction cones (named B_k). In general case of m contact points, the number of intersection points is given by

$$N_{B_k} = 4 \cdot \sum_{k=1}^{m-1} (m-k) \tag{22}$$

Hence, a multifingred 2D grasp is said to achieve force-closure if each of these inequalities are true.

$$\left| \mathbf{X} \cdot \sum_{i=1}^{m} (\mathbf{n}_{i1} + \mathbf{n}_{i2}) \right| < \sum_{i=1}^{m} \left(|\mathbf{X} \cdot \mathbf{n}_{i1}| + |\mathbf{X} \cdot \mathbf{n}_{i2}| \right) \tag{23-1}$$

$$\left| \mathbf{Y} \cdot \sum_{i=1}^{m} (\mathbf{n}_{i1} + \mathbf{n}_{i2}) \right| < \sum_{i=1}^{m} \left(|\mathbf{Y} \cdot \mathbf{n}_{i1}| + |\mathbf{Y} \cdot \mathbf{n}_{i2}| \right) \tag{23-2}$$

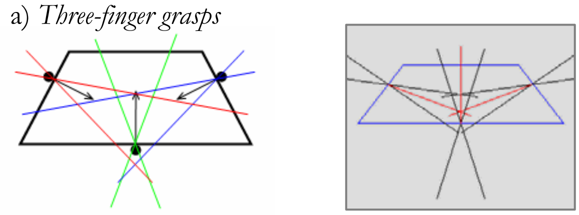
$$\left| \sum_{i=1}^{m} \mathbf{Z} \cdot \left(\mathbf{B}_k \mathbf{C}_i \wedge (\mathbf{n}_{i1} + \mathbf{n}_{i2}) \right) \right| < \sum_{i=1}^{m} \left(|\mathbf{Z} \cdot (\mathbf{B}_k \mathbf{C}_i \wedge \mathbf{n}_{i1})| + |\mathbf{Z} \cdot (\mathbf{B}_k \mathbf{C}_i \wedge \mathbf{n}_{i2})| \right) \tag{23-3}$$

From mechanical viewpoint, inequality (23-1) implies that fingers can generate forces along $\mathbf{X}$ and $-\mathbf{X}$, (23-2) means that fingers can exert force on the object along $\mathbf{Y}$ and $-\mathbf{Y}$. If the last inequality (23-3) is true $\left(for\ k = 1 \ldots N_{B_k}\right)$ then the finger can exert torque on object about the vertical axis $\mathbf{Z}$ in both directions.

5 EXAMPLES

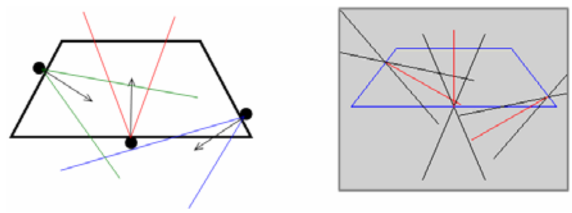
We present bellow some grasp examples using three, four and five fingers. In both cases (force-closure and no force-closure), we show the distribution of grasp wrench central axes.

a) *Three-finger grasps*



(a) (b)

Figure 6: a) a three-finger force-closure 2D grasp, b) central axes of grasp wrenches ($\alpha = 15°$).



(a) (b)

Figure 7: a) a three-finger force-closure 2D grasp, b) central axes of grasp wrenches ($\alpha = 20°$).

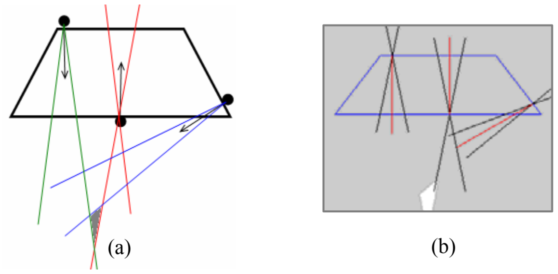


Figure 8: a) no force-closure 2D grasp, b) central axes of grasp wrenches ($\alpha = 10°$). Grasp wrenches can't generate a negative torque in grey zone.

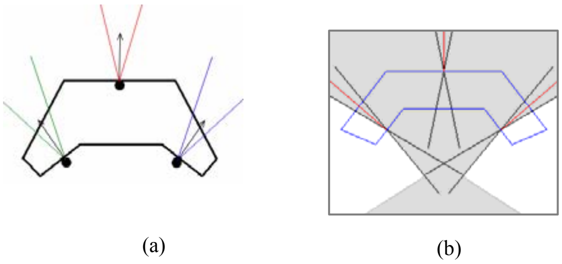


Figure 9: a) no force-closure 2D grasp, b) central axes of grasp wrenches ($\alpha = 10°$).Grasp wrenches can't exert a force along $(-\mathbf{Y})$ axis and can't generate torques in two-direction in unreachable zones in (b).

b) *Four-finger grasps*

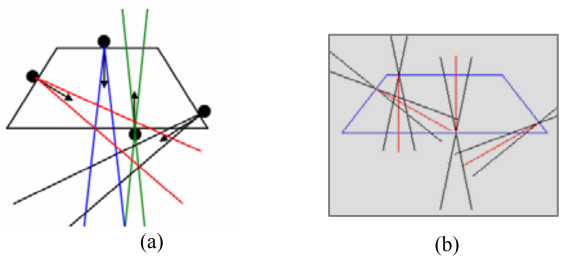


Figure 10: a) four-finger force-closure 2D grasp, b) central axes of grasp wrenches ($\alpha = 10°$).

c) *Five-finger grasps*

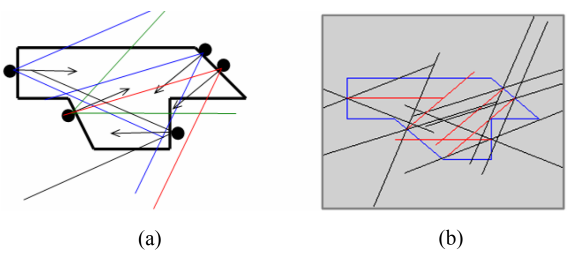


Figure 11: a) five-finger force-closure 2D grasp, b) central axes of grasp wrenches ($\alpha = 25°$).

6 CONCLUSION AND FUTURE WORK

We have presented a new equilibrium and force-closure conditions for multifingred 2D grasps. A novel algorithm for computing 2D multi-finger force-closure grasps of arbitrary objects was developed, which is very simple and needs little geometric computations. Therefore, it can be implemented in real-time multifingred grasp programming. Our future work will be concentrated on the extending of this algorithm to the 3D grasps and the quality measurement of grasps.

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