

# VEHICLE MODELS AND ESTIMATION OF CONTACT FORCES AND TIRE ROAD FRICTION

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**Abstract:** In this paper a 16 DoF vehicle model is presented and discussed. Then some partial models are considered and justified for the design of robust estimators using sliding mode approach in order to identify the tire-road friction or input variables. The estimations produced are based on split system equations in as cascaded observers and estimators. The first produces estimations of vehicle states.

## 1 INTRODUCTION

In recent years, the increasing demand for the safety in car vehicles has promoted research and development of the technology of active safety. However more and more new active safety systems are developed and installed on vehicles for real-time monitoring and observers for controlling the dynamic stability (EBS, ABS, ESP).

Car accidents occur for several reasons which may involve the driver or vehicle components or environment. Such situations appears when the vehicle is driven beyond the adherence or stability limits. One of the important factors determining vehicle dynamics including safety is road friction and the tire forces (ground-vehicle interactions). In general partial and approximated models are used. They are not fully justified and their validity is often limited. In this work we try to highlight some of the approximations made and give some details allowing to evaluate what is really neglected.

Robust observers looking forward are based on the physics of interacting systems (the vehicle, the driver and the road). However, tire forces and road friction are difficult to measure directly and to represent precisely by some deterministic model equations. In the literature, their values are often deduced by some experimentally approximated models (Gustaf-

son). The knowledge of tire parameters and variables (forces, velocities, wheel and slip), tire forces is essential to advanced vehicle control systems such as anti-lock braking systems (ABS), traction control systems (TCS) and electronic stability program (Ackermann)(Msirdi04)(Canudas03). Recently, many analytical and experimental studies have been performed on estimation of the frictions and contact forces between tires and road.

We focus our work, as presented in this paper, first on modeling and second on on-line estimation of the tires forces (Ackermann)(Msirdi04). We estimate the vehicle state and identify tire forces ((Msirdi03)). The main contribution is the emphasize of the rational behind partial approximated models and the on-line estimation of the tire force needed for control.

Tire forces can be represented by the nonlinear (stochastic) functions of wheel slip. The deterministic tire models encountered are complex and depend on several factors (as load, tire pressure, environmental characteristics, etc.). This makes on line estimation of forces and parameters difficult for vehicle control applications and detection and diagnosis for driving monitoring and surveillance. In (Drakunov)(Canudas03), application of sliding mode control is proposed. Observers based on the sliding mode approach have been also used in (Rabhi04).

In (Ray) an estimation method is based on the least squares algorithm and combined with a Kalman filter to estimate the contact forces. The paper of (Gustafsson) presents an estimation of tire/road frictions by means of a Kalman filter. It gives relevant estimates of the slope of  $\mu$  versus slip ( $\lambda$ ). In (Carlson) estimations of longitudinal stiffness and wheel effective radius are proposed using vehicle sensors and a GPS for low values of the slip.

Robust observers with unknown inputs have been shown to be efficient for estimation of road profile (Imine) and for estimation of the contact forces (Msirdi04)(Rabhi04). Tracking and braking control reduce wheel slip. This can be done also by means of its regulation while using sliding mode approach for observation and control.(Msirdi04)(Rabhi04). This enhances the road safety leading better vehicle adherence and maneuvers ability. The vehicle controllability in its environment along the road admissible trajectories remain an important open research problem.

The proposed estimation procedure has to be robust enough to avoid model complexity. It can then be used to detect some critical driving situations in order to improve the security. This approach can be used also in several vehicle control systems such as Anti-lock Brake Systems (ABS), traction control system (TCS), diagnosis systems, etc... The main characteristics of the vehicle longitudinal dynamics were taken into account in the developed model used to design robust observer and estimations. The estimations are produced using only the angular wheel position as measurement by the specially designed robust observer based on the super-twisting second-order sliding mode. The proposed estimation method is verified through simulation of one- wheel model (with a "Magic formula" as tire model). In a second step of validation we present some application results (on a Peugeot 406) showing an excellent reconstruction of the velocities, tire forces and estimation of wheel radius.

## 2 VEHICLE MODELING

### 2.1 Complete 16 DoF Model

In literature, many studies deal with vehicle modeling (Kien)(Ramirez)(Mendoza). This kind of systems are complex and nonlinear composed with many coupled subsystems: wheels, motor and system of braking, suspensions, steering, more and more inboard and imbedded electronics. Let us represent the vehicle (like eg a Peugeot 406) by the scheme of figure

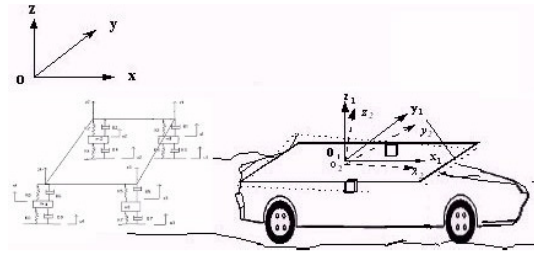


Figure 1: Vehicle dynamics and reference frames.

1 and define the following notations.

The vehicle body receives as excitations external forces and moments following the three axes: - Longitudinal, - Lateral, - Vertical. These come from interaction of the wheels and road, from perturbations (wind for example), gravity and vehicle drive line. Let us consider the basic reference fixed frame  $R$ . We can consider the vehicle as made of 5 sub-systems: chassis with 6 DoF and then 4 wheels with their suspensions. Each of the rear wheels has 2 DoF. The front ones are driven wheels with 3 DoF each. Then we have 16 DoF. Let the generalized variables be in the vector  $q \in R^{16}$ , defined as

$$q^T = [x, y, z, \theta_x, \theta_y, \theta_z, z_1, z_2, z_3, z_4, \delta_3, \delta_4, \phi_1, \phi_2, \phi_3, \phi_4]$$

where  $x, y, z$  represent displacements in longitudinal, lateral and vertical direction. Angles of roll, pitch and yaw are  $\theta_x, \theta_y, \theta_z$  respectively. The suspensions elongations are noted  $z_i$  ( $i = 1..4$ ).  $\delta_i$  stands for the steering angles (for wheels numbered as  $i = 3, 4$ ), finally  $\phi_i$  are angles wheels rotations ( $i = 1..4$ ). Vectors  $\dot{q}, \ddot{q} \in R^{16}$  are respectively velocities and corresponding accelerations.  $M(q)$  is the inertia matrix and  $C(q, \dot{q})\dot{q}$  are coriolis and centrifugal forces. The gravity term is  $G$ . Suspensions forces are  $V(q, \dot{q}) = K_v \dot{q} + K_p q$  with respectively damping and stiffness matrices  $K_v, K_p$ . We can define as dynamic equations of the vehicle by applying the principles fundamental of the dynamics (see (Beurier)):

$$\Gamma + J^T F = M\ddot{q} + C(q, \dot{q})\dot{q} + Kq + G \quad (1)$$

with as parameters only to give an idea

$$M = \begin{bmatrix} \bar{M}_{1,1} & \bar{M}_{1,2} & \bar{M}_{1,3} & 0 & 0 \\ \bar{M}_{2,1} & \bar{M}_{2,2} & \bar{M}_{2,3} & \bar{M}_{2,4} & \bar{M}_{2,5} \\ \bar{M}_{3,1} & \bar{M}_{3,2} & \bar{M}_{3,3} & 0 & 0 \\ 0 & \bar{M}_{4,2} & 0 & \bar{M}_{4,4} & 0 \\ 0 & \bar{M}_{5,2} & 0 & 0 & \bar{M}_{5,5} \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & \bar{C}_{12} & \bar{C}_{13} & 0 & 0 \\ 0 & \bar{C}_{22} & \bar{C}_{23} & \bar{C}_{24} & \bar{C}_{25} \\ 0 & \bar{C}_{32} & \bar{C}_{33} & 0 & 0 \\ 0 & \bar{C}_{42} & 0 & 0 & 0 \\ 0 & \bar{C}_{52} & 0 & 0 & \bar{C}_{55} \end{bmatrix}$$

$$\text{and } J^T = \begin{bmatrix} J_{1,1} & J_{1,2} & J_{1,3} & J_{1,4} \\ J_{2,1} & J_{2,2} & J_{2,3} & J_{2,4} \\ J_{3,1} & J_{3,2} & J_{3,3} & 0 \\ 0 & 0 & 0 & J_{4,4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

This is just to show that we can decompose our system as coupled subsystems. Let us say five coupled subsystems, that we have considered in our previous works. This has been computed using a symbolic computation software considering 16 generalized variables: 6 for position and orientation of body, 4 as suspensions ones, 2 for front wheels steering and 4 as wheels rotations. The matrices  $M$ ,  $C$  and  $K$  are of dimensions  $16 \times 16$ .  $F$  is input forces vector acting on wheels, it has 12 components (3 forces (longitudinal, lateral and normal)  $\times$  4 wheels),  $\Gamma$  represent extra inputs for perturbations.. In the following application this model has been reduced and simplified assuming as nominal behavior a normal driving situation (Msirdi03).

## 2.2 Coupled Sub Models

We can split the previous model, without approximations, in five parts as follows. This leads us to the body's translations dynamics:

$$\begin{bmatrix} F_{LT} \\ F_{TT} \\ F_{NT} \end{bmatrix} = \bar{M}_{11} \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} + \bar{M}_{12} \begin{bmatrix} \ddot{q}_4 \\ \ddot{q}_5 \\ \ddot{q}_6 \end{bmatrix} + \quad (2)$$

$$+ \bar{M}_{13} \begin{bmatrix} \ddot{q}_{11} \\ \ddot{q}_{21} \\ \ddot{q}_{31} \\ \ddot{q}_{41} \end{bmatrix} + \bar{C}_{12} \begin{bmatrix} \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \end{bmatrix} \quad (3)$$

Rotations and orientation motions of the body:

$$\begin{bmatrix} J_{4F} \\ J_{5F} \\ J_{6F} \end{bmatrix} = \bar{M}_{12}^T \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} + \bar{M}_{22} \begin{bmatrix} \ddot{q}_4 \\ \ddot{q}_5 \\ \ddot{q}_6 \end{bmatrix} +$$

$$+ \bar{M}_{23} \begin{bmatrix} \ddot{q}_{11} \\ \ddot{q}_{21} \\ \ddot{q}_{31} \\ \ddot{q}_{41} \end{bmatrix} + \bar{M}_{24} \begin{bmatrix} \ddot{q}_{12} \\ \ddot{q}_{22} \\ \ddot{q}_{32} \end{bmatrix} + \bar{M}_{25} \begin{bmatrix} \ddot{q}_{42} \\ \ddot{q}_{33} \\ \ddot{q}_{43} \end{bmatrix} +$$

$$+ \bar{C}_{22} \begin{bmatrix} \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \end{bmatrix} + \bar{C}_{23} \begin{bmatrix} \dot{q}_{21} \\ \dot{q}_{31} \\ \dot{q}_{41} \end{bmatrix} +$$

$$+ \bar{C}_{24} \begin{bmatrix} \dot{q}_{12} \\ \dot{q}_{22} \\ \dot{q}_{32} \end{bmatrix} + \bar{C}_{25} \begin{bmatrix} \dot{q}_{42} \\ \dot{q}_{33} \\ \dot{q}_{43} \end{bmatrix}$$

Suspensions dynamics:

$$\begin{bmatrix} J_{7F} \\ J_{8F} \\ J_{9F} \\ J_{10F} \end{bmatrix} = \bar{M}_{1,3} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} + \bar{M}_{2,3} \begin{bmatrix} \ddot{q}_4 \\ \ddot{q}_5 \\ \ddot{q}_6 \end{bmatrix} +$$

$$+ \bar{M}_{3,3} \begin{bmatrix} \ddot{q}_{11} \\ \ddot{q}_{21} \\ \ddot{q}_{31} \\ \ddot{q}_{41} \end{bmatrix} + \bar{C}_{32} \begin{bmatrix} \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \end{bmatrix} + \bar{C}_{33} \begin{bmatrix} \dot{q}_{11} \\ \dot{q}_{21} \\ \dot{q}_{31} \\ \dot{q}_{41} \end{bmatrix} +$$

$$+ \begin{bmatrix} R_{11} & & & \\ & R_{21} & & \\ & & R_{31} & \\ & & & R_{41} \end{bmatrix} \begin{bmatrix} q_{11} \\ q_{21} \\ q_{31} \\ q_{41} \end{bmatrix} + \begin{bmatrix} G_7 \\ G_8 \\ G_9 \\ G_{10} \end{bmatrix} \quad (4)$$

The previous 16 DoF model is then equivalent to:

$$\begin{bmatrix} F_T \\ F_2 \\ F_3 \\ 0 \\ U_5 \end{bmatrix} = M \begin{bmatrix} \ddot{\xi}_1 \\ \ddot{\xi}_2 \\ \ddot{\xi}_3 \\ \ddot{\xi}_4 \\ \ddot{\xi}_5 \end{bmatrix} + C \begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \dot{\xi}_3 \\ \dot{\xi}_4 \\ \dot{\xi}_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ R_{33} \cdot \xi_3 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \\ G_3 \\ 0 \\ 0 \end{bmatrix}$$

In the last expression, we can remark that splitting the model can be realized and this model is helpful, when using reduced models, to identify what is neglected regard to our proposed nominal model with 16 DoF. The dynamic equations can be reduced, in case where we assume that motion is normal driving in a normal stait road, to translations and rotations of the body, and wheels plus suspension motions. For translations we find often in literature:

$$\begin{aligned} m\dot{v}_x &= \sum F_x \\ m\dot{v}_y &= \sum F_y \\ m\dot{v}_z &= \sum F_z \end{aligned} \quad (5)$$

where  $m$  is the total mass of the vehicle and  $v = [v_x, v_y, v_z]^T$  describe the vehicle velocities along  $x$ ,  $y$ ,  $z$ . In the left hand side of this approximate model are the forces  $\sum F_x$ ,  $\sum F_y$  and  $\sum F_z$  applied in directions of  $x$ ,  $y$  and  $z$  and the balance of the moments ( $\sum M_x, \sum M_y, \sum M_z$ ), give rotations following the three directions  $x$ ,  $y$  and  $z$ , is given by:

$$J \begin{bmatrix} \ddot{\theta} \\ \ddot{\phi} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} \sum M_x \\ \sum M_y \\ \sum M_z \end{bmatrix} \quad (6)$$

The wheel angular motions can be written:

$$\begin{aligned} \dot{\omega}_{fl} &= \frac{1}{I_\omega} (C_{fl} - R_\omega F_{xf1}) \\ \dot{\omega}_{fr} &= \frac{1}{I_\omega} (C_{fr} - R_\omega F_{xf2}) \\ \dot{\omega}_{rl} &= \frac{1}{I_\omega} (C_{rl} - R_\omega F_{xr1}) \\ \dot{\omega}_{rr} &= \frac{1}{I_\omega} (C_{rr} - R_\omega F_{xr2}) \end{aligned} \quad (7)$$

with  $\omega_f$  and  $\omega_r$  are the rotation velocities of the front and rear wheel,  $C_{mi}$  is the motor couple applied at wheel  $i$  and  $T_i$  is the braking couple applied at wheel  $i$ . Let  $r_1$  be the distance between the center of gravity and the front axis and  $r_2$  the distance between the center of gravity and the rear axis.

### 2.3 Partial Models

The complete model is difficult to use in control applications. It involves several variables which are not available for measurement or not observable. The most part of applications deal with simplified and partial models. Let us consider, for our robust observer, the simplified motion dynamics of a quarter-vehicle model, capturing only nominal behavior (Msirdi04) (Msirdi03). This model retains the main characteristics useful for the longitudinal dynamic. For a global application, this method can be easily extended to the complete vehicle and involve the four coupled wheels. The amount of neglected parts in the modeling can be considered to evaluate robustness of proposed estimators.

Applying Newton's law to one isolated wheel gives:

$$\begin{aligned} m\dot{v}_x &= F_x \\ J_r\dot{\omega} &= T - rF_x \end{aligned}$$

where  $m$  is the vehicle mass and  $J_r$ ,  $r$  are the inertia and effective radius of the tire respectively.  $v_s$  is the linear velocity of the vehicle,  $\omega$  is the angular velocity of the considered wheel.  $T$  is the accelerating (or braking) torque, and  $F_x$  is the tire/road friction force. The tractive (respectively braking) force, produced at the tire/road interface when a driving (braking) torque is applied to pneumatic tire, has opposite direction of relative motion between the tire and road surface. This relative motion exhibits the tire slip properties. The wheel-slip is due to deflection in the contact patch. The longitudinal wheel slip is generally called the slip ratio and is described by a kinematic relation as (Carlson).

$$\lambda = \frac{|v_r - v_x|}{\max(v_r, v_x)} \tag{8}$$

where  $v_r$  is the wheel velocity. Representing the adhesion coefficient as a function of the wheel slip yields the adhesion characteristic  $\mu(\lambda)$ , which depends on the road surfaces as shown in the following figure 2.

The figure 2 shows the relations between coefficient of road adhesion  $\mu$  and longitudinal slip  $\lambda$  for

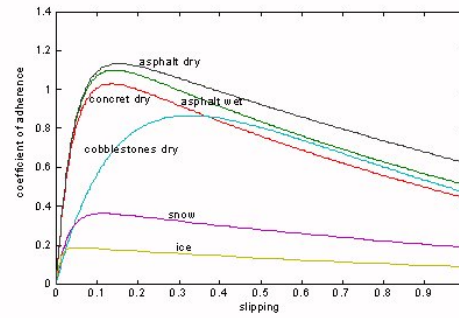


Figure 2: Wheel slip.

different road surface conditions. It can be observed that all curves  $\mu(\lambda)$  start at  $\mu = 0$  for zero slip, which corresponds to the non-braking and non accelerating, free rolling wheel. With a linear increasing slip ratio from 3% to 20%. Beyond this maximum value the slope of the adhesion characteristic is maximum and then slope becomes negative. At a slip ratio of 100% the wheel is completely skidding, which corresponds to the locking of the wheel. The adhesion characteristic plays an essential role for both the design and the validation of ABS. Overall, to improve the performance of an ABS it is desirable to have some real-time information about the adhesion characteristic.

By assuming that the longitudinal forces are proportional to the transversal ones, we can express these forces as follows, where  $F_z$  is the vertical force of the wheel.

$$F_x = \mu F_z \tag{9}$$

The vertical forces that we use in our model are function of the longitudinal acceleration and the height of the center of gravity. The vertical force can be represented as:

$$F_z = \frac{m}{2(l_f + l_r)}(gl_r - h\dot{v}_x) \tag{10}$$

where  $h$  is the height of the center of gravity,  $l_f$  is the distance between the center of gravity and the front axis center of gravity and  $l_r$  is the distance between the center of gravity and the rear axis center of gravity.

## 3 OBSERVER DESIGN

The sliding mode technique is an attractive approach (Davila). The primary characteristic of SMC is that the feedback signal is discontinuous, switching on

one or several manifolds in the state-space. In what follows, we use a second order differentiator in order to derive robust estimates of the tire road friction.

### 3.1 High Order Sliding Mode Observer (HOSM)

In this part we will use a Robust Differentiation Estimator (RDE) to deduce our robust estimations. Consider a smooth dynamics function,  $s(x) \in \mathbb{R}$ . The system containing this variable may be closed by some possibly-dynamical discontinuous feedback where the control task may be to keep the output  $s(x(t)) = 0$ . Then provided that successive total time derivatives  $s, \dot{s}, \ddot{s}, \dots, s^{(r-1)}$  are continuous functions of the closed system state space variables, and the t-sliding point set is non-empty and consist locally of Filippov trajectories.

$$s = \dot{s} = \ddot{s} = \dots = s^{(r-1)} = 0 \quad (11)$$

is non-empty and consists locally of Filippov trajectories. The motion on set (Filippov)(Utkin99) is called r-sliding mode ( $r^{th}$ -order sliding mode) (Orlov)(Levant).

The HOSM dynamics converge toward the origin of surface coordinates in finite time always that the order of the sliding controller is equal or bigger than the sum of a relative degree of the plant and the actuator. To estimate the derivatives  $s_1$  and  $s_2$  without its direct calculations of derivatives, we will use the  $2^{nd}$ -order exact robust differentiator of the form (Levant)

$$\begin{aligned} \dot{z}_0 &= v_0 = z_1 - \lambda_0 |z_0 - s_\omega|^{\frac{2}{3}} \text{sign}(z_0 - s_\omega) \\ \dot{z}_1 &= v_1 = -\lambda_1 \text{sign}(z_1 - v_0)^{\frac{1}{2}} \text{sign}(z_1 - v_0) + z_2 \\ \dot{z}_2 &= -\lambda_2 \text{sign}(z_2 - v_1) \end{aligned}$$

where  $z_0, z_1$  and  $z_2$  are the estimate of  $s_\omega, s_1$  and  $s_2$ , respectively,  $\lambda_i > 0, i = 0, 1, 2$ . Under condition  $\lambda_0 > \lambda_1 > \lambda_2$  the third order sliding mode motion will be established in a finite time. The obtained estimates are  $z_1 = s_1 = \dot{s}_\omega$  and  $z_2 = s_2 = \ddot{s}_\omega$  then they can be used in the estimation of the state variables and also in the control.

### 3.2 Cascaded Observers - Estimators

In this section we use the previous approach to build an estimation scheme allowing to identify the tire road friction. The estimations will be produced in three steps, as cascaded observers and estimator, reconstruction of information and system states step by step. This approach allow us to avoid the observability problems dealing with inappropriate use of the

complete modeling equations. For vehicle systems it is very hard to build up a complete and appropriate model for global observation of all the system states in one step. Thus in our work, we avoid this problem by means of use of simple and cascaded models suitable for robust observers design.

The first step produces estimations of velocities. The second one estimate the tire forces (vertical and longitudinal ones) and the last step reconstruct the friction coefficient.

The robust differentiation observer is used for estimation of the velocities and accelerations of the wheels. The wheels angular positions and the velocity of the vehicles body  $v_x$ , are assumed available for measurements. The previous Robust Estimator is useful for retrieval of the velocities and accelerations.

1<sup>st</sup> Step:

$$\begin{aligned} \dot{\hat{\theta}} &= v_0 = \hat{\omega} - \lambda_0 |\theta - \hat{\theta}|^{\frac{2}{3}} \text{sign}(\theta - \hat{\theta}) \\ \dot{\hat{\omega}} &= v_1 = \hat{\omega} - \lambda_1 \text{sign}(\hat{\omega} - v_0)^{\frac{1}{2}} \text{sign}(\hat{\omega} - v_0) \\ \ddot{\hat{\omega}} &= -\lambda_2 \text{sign}(\hat{\omega} - v_1) \end{aligned}$$

The convergence of these estimates is guaranteed in finite time  $t_0$ .

2<sup>nd</sup> Step: In the second step we can estimate the forces  $F_x$  and  $F_z$ . Then to estimate  $F_x$  we use the following equation,

$$J\dot{\hat{\omega}} = T - R_{ef}\hat{F}_x \quad (12)$$

In the simplest way, assuming the input torques known, we can reconstruct  $F_x$  as follows:

$$\hat{F}_x = \frac{(T - J\dot{\hat{\omega}})}{R_{ef}} \quad (13)$$

$\hat{\omega}$  is produced by the Robust Estimator (RE). Note that any estimator with output error can also be used to enhance robustness versus noise. In our work, in progress actually, the torque  $T$  will be also estimated by means of use of additional equation from engine behavior related to accelerating inputs.

After those estimations, their use in the same time with the system equations allow us to retrieve de vertical forces  $F_z$  as follows. To estimate  $F_z$  we use the following equation

$$\hat{F}_z = \frac{m}{2(l_f + l_r)} (gl_r - h \cdot \dot{\hat{v}}_x) \quad (14)$$

$\hat{v}_x$  is produced by the RE.

3<sup>rd</sup> Step: At this step it only remains to estimate the adherence or friction coefficient. To this end we

assume the vehicle rolling in a normal or steady state situation in order to be able to approximate this coefficient by the following formula

$$\hat{\mu} = \frac{\hat{F}_x}{\hat{F}_z} \tag{15}$$

#### 4 SIMULATION AND EXPERIMENTAL RESULTS

In this section, we give some realistic simulation results in order to test and validate our approach and the proposed observer. In simulation, the state and forces are generated by use of a car simulator called VeDyna (VEDYNA). In this simulator the model involved is more complex than the one of 16 DoF presented in the first part of the paper. Comparing the simplified model to the 16 DoF one, let us evaluate the robustness of estimation. The VeDyna simulated brake torque is shown in figure 3.

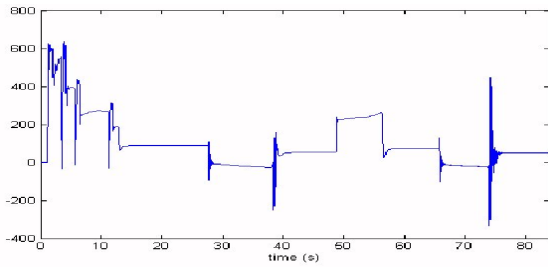


Figure 3: Braking torque.

Figure 4 shows the measured and estimated wheel angular position. This signal is used to estimate velocities and accelerations. Figure 4 shows the esti-

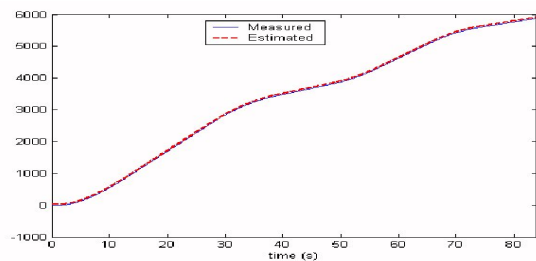


Figure 4: Angular displacements.

imated wheel angles. In the figure 6, we represent the estimation of vehicle velocity. The figure shows the good convergence to the actual vehicle velocity. Figure 7 shows the obtained vehicle acceleration. The

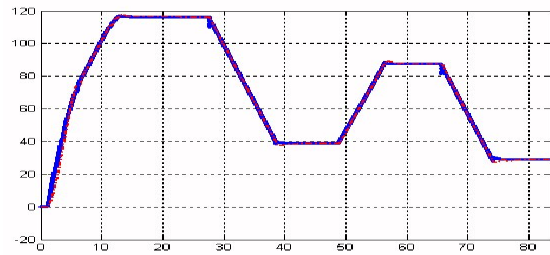


Figure 5: Angular Velocity.

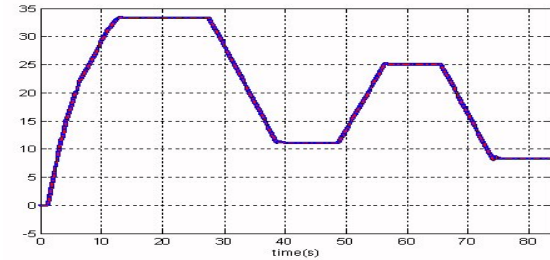


Figure 6: Vehicle velocity.

observer allows a good estimation of angular velocity and acceleration. The last step gives us the estimated

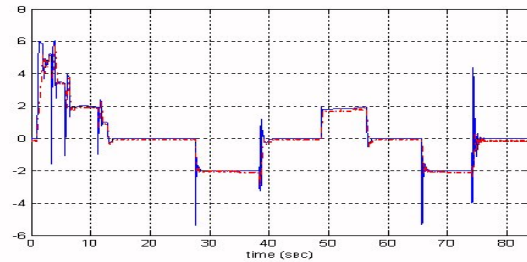


Figure 7: Estimated and measured acceleration.

longitudinal forces  $F_x$  and normal forces  $F_z$  which are presented in figure 8 and 9. Finally road friction coefficient is deduced and presented in figure (10).

#### 5 CONCLUSION

In this work we have tried to highlight all approximations made in general when using simplified models and this paper gives some details allowing to evaluate what is really neglected. In second part of this paper, we have proposed an efficient and robust estimator based on the second order sliding mode differentiator. This is used to build an estimation scheme allowing to identify the tire road frictions and input forces which are non observable when using the complete model

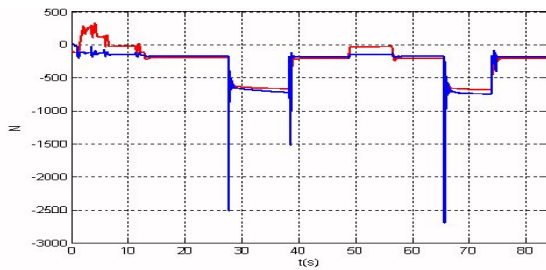


Figure 8: Longitudinal force.

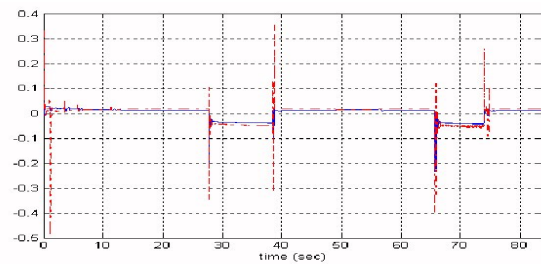
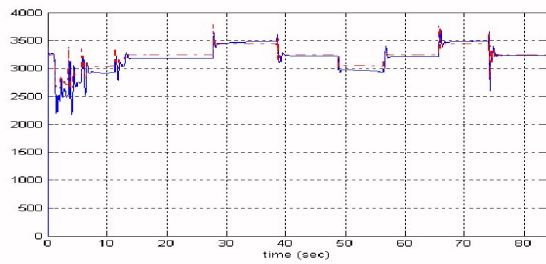


Figure 10: Road friction.


 Figure 9: Normal force  $F_z$ .

and standard sensors. The estimations produced finite time converging measurements of model inputs, in three steps by cascaded observers and estimators. This method shows very good performances in simulations conducted using a more complex model (than the 16 DoF one) involved in VeDyna car simulator. Tire forces (vertical and longitudinal ones) are also estimated correctly. Simulation results are presented to illustrate the ability of this approach to give estimation of both vehicle states and tire forces. The robustness versus uncertainties on model parameters and neglected dynamics has also been emphasized in simulations. Application of this approach with inclusion of torque estimation using a simplified model for the engine behavior, is in progress.

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$$\bar{M}_{3,3} = \begin{bmatrix} M_{7,7} & 0 & 0 & 0 \\ 0 & M_{8,8} & 0 & 0 \\ 0 & 0 & M_{9,9} & 0 \\ 0 & 0 & 0 & M_{10,10} \end{bmatrix};$$

$$\bar{M}_{4,4} = \begin{bmatrix} M_{11,11} & 0 & 0 \\ 0 & M_{12,12} & 0 \\ 0 & 0 & M_{13,13} \end{bmatrix}$$

$$\bar{M}_{5,5} = \begin{bmatrix} M_{14,14} & 0 & 0 \\ 0 & M_{15,15} & 0 \\ 0 & 0 & M_{16,16} \end{bmatrix}$$

## APPENDIX

Definition of the matrices involved in the model.

$$\bar{M}_{11} = \begin{bmatrix} M_{1,1} & 0 & 0 \\ 0 & M_{2,2} & 0 \\ 0 & 0 & M_{3,3} \end{bmatrix}$$

$$\bar{M}_{12} = \bar{M}_{21}^T = \begin{bmatrix} M_{1,4} & M_{1,5} & M_{1,6} \\ M_{2,4} & M_{3,5} & M_{2,6} \\ 0 & M_{3,5} & M_{3,6} \end{bmatrix};$$

$$\bar{M}_{13} = \bar{M}_{31}^T = \begin{bmatrix} M_{1,7} & M_{1,8} & M_{1,9} & M_{1,10} \\ M_{2,7} & M_{2,8} & M_{2,9} & M_{2,10} \\ M_{3,7} & M_{3,8} & M_{3,9} & M_{3,10} \end{bmatrix};$$

$$\bar{M}_{23} = \bar{M}_{32}^T = \begin{bmatrix} M_{4,7} & M_{4,8} & M_{4,9} & M_{4,10} \\ M_{5,7} & M_{5,8} & M_{5,9} & M_{5,10} \\ M_{6,7} & M_{6,8} & M_{6,9} & M_{6,10} \end{bmatrix}$$

$$\bar{M}_{24} = \bar{M}_{42}^T = \begin{bmatrix} M_{4,11} & M_{4,12} & M_{4,13} \\ M_{5,11} & M_{5,12} & M_{5,13} \\ 0 & 0 & 0 \end{bmatrix}$$

$$\bar{M}_{2,5} = \bar{M}_{52}^T = \begin{bmatrix} M_{4,14} & M_{4,15} & M_{4,16} \\ M_{5,14} & M_{5,15} & M_{5,16} \\ 0 & M_{6,15} & M_{6,16} \end{bmatrix}$$

$$\bar{M}_{2,2} = \begin{bmatrix} M_{4,4} & M_{4,5} & M_{4,6} \\ M_{5,4} & M_{5,5} & M_{5,6} \\ M_{6,4} & M_{6,5} & M_{6,6} \end{bmatrix};$$