GENERAL FORMULATION OF SYSTEM DESIGN PROCESS

Design Process Formulation as a Controllable Dynamic System

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Keywords: Circuit design, control theory formulation, time minimization.

Abstract: The formulation of the process of analogue circuit design has been done on the basis of the control theory application. This approach produces the set of different design strategies inside the same optimization procedure. Basic equations for this design methodology were elaborated. The problem of the time-optimal design algorithm construction is defined as the problem of a functional minimization of the optimal control theory. By this context the design process is defined as a controllable dynamic system. Numerical results of some electronic circuit design demonstrate the efficiency of the proposed methodology and prove the non-optimality of the traditional design strategy.

1 INTRODUCTION

One of the main problems of a large system design is the excessive computer time that is necessary to achieve the final point of the design process. This problem has a great significance at least for the VLSI electronic circuit design. Any system design methodology includes two main parts: the block of analysis of the mathematical model of the system and optimization procedure that achieves the cost function optimal point during the design process. This is a traditional design approach for the system design and we call it as a Traditional Design Strategy (TDS). There are some powerful methods that reduce the necessary time for the circuit analysis by means of the special sparse matrix techniques (Osterby, Zlatev, 1983), (George, 1984) or by the partitioning of a circuit matrix by branches (Wu, 1976) or by nodes (Sangiovanni-Vincentelli et al, 1977).

Another formulation of the circuit optimization problem was developed in heuristic level some decades ago (Kashirsky and Trokhimenko, 1979). This idea was based on the Kirchhoff laws ignoring for all the circuit or for the circuit part. The special cost function is minimized instead of the circuit equation solving. This idea was developed in practical aspect for the microwave circuit optimization (Rizzoli et al, 1990) and for the synthesis of high-performance analogue circuits (Ochotta et al, 1996) in extremely case, when the total system model was eliminated. The last idea that excludes completely the Kirchhoff laws can be named as the Modified Traditional Design Strategy (MTDS).

More general approach was elaborated in previously work (Zemliak, 2005). This approach can be developed to define the system design problem by means of the optimal control theory.

2 PROBLEM FORMULATION

The design process for any analogue system design can be defined as the problem of the cost function $C(X)$ minimization ($X \in R^N$) with the system of constraints. It is supposed that the minimum of the cost function $C(X)$ achieves all design objects and the system of constraints is the mathematical model of the electronic circuit. It is supposed also that the circuit model can be described as the system of nonlinear equations:

$$g_j(X) = 0 \tag{1}$$

$$j = 1,2,\ldots,M$$

The vector X is separated in two parts: $X = (X', X'')$. The vector $X' \in R^K$ is the vector of independent variables where K is the number of independent variables and the vector $X'' \in R^M$, is the vector of dependent variables, ($N = K + M$).

The optimization process for the cost function $C(X)$ minimization with constrains (1) can be defined in general case by next vector equation:

$$X^{s+1} = X^s + t_s \cdot H^s \tag{2}$$

where s is the iterations number, t_s is the iteration parameter, $t_s \in R^1$, H is the direction of the cost function $C(X)$ decreasing. The system (1) must be solved at each step of the optimization process (2) in this case. The optimization process is realized in R^K . This is a TDS.

The specific character of the design process for the electronic systems consists in fact that it is not necessary to fulfil the conditions (1) for all steps of the optimization process. It is quite enough to fulfil these conditions for the final point only.

The problem (1)-(2) can be redefined. We suppose that all components of the vector X are independent. This is the main idea for the penalty function method application. In this case the vector function H is the function of the cost function $C(X)$ and the additional penalty function $\varphi(X)$: $H^s = f\left(C(X^s), \varphi(X^s)\right)$. The penalty function structure includes all equations of the system (1) and can be defined for example as:

$$\varphi(X^s) = \frac{1}{\varepsilon} \sum_{i=1}^{M} g_i^2(X^s) \tag{3}$$

In this case we define the design problem as the unconstrained optimization (2) in the space R^N without any additional system but for the other type of the cost function $F(X)$. This function can be defined for example as an additive function: $F(X) = C(X) + \varphi(X)$. In this case we reach the minimum of the initial cost function $C(X)$ and comply with the system (1) in the final point of the optimization process. This is a MTDS.

It is possible to generalize the above mentioned idea. We suppose that the penalty function includes a one part of the system (1) only and the other part of this system is defined as constraints. In this case the penalty function includes first Z items only:

$$\varphi(X^s) = \frac{1}{\varepsilon} \sum_{i=1}^{Z} g_i^2(X^s) \tag{4}$$

where $Z \in [0, M]$ and M - Z equations make up one modification of the system (1):

$$g_j(X) = 0 \tag{5}$$

$$j = Z+1, Z+2, \ldots, M$$

This idea can be generalized more in case when the penalty function $\varphi(X)$ includes Z arbitrary equations from the system (1). The total number of different design strategies is equal to 2^M if $Z \in [0, M]$. The optimization procedure is realized in the space R^{K+Z}. The different strategies have different computer times. It is appropriate in this case to define the problem of an optimal design strategy search that has the minimal computer time.

3 CONTROL THEORY APPLY

The problem of optimal design can be defined now as the problem of the optimal control. It is possible to define a design strategy by equations (2), (4) with a variable value of the parameter Z during the all optimization process. It means that we can change the number of independent variables and the number of the terms of the penalty function in each point of the optimization procedure. It is convenient to introduce a vector of the special control functions $U = (u_1, u_2, \ldots, u_M)$ for this aim, where $u_j \in \Omega$; $\Omega = \{0;1\}$. The sense of the control function u_j is next: equation number j is presented in the system (4) and the term $g_j^2(X)$ is removed from the right part of the formula (3) when $u_j = 0$, and on the contrary, the equation number j is removed from the system (4) and is presented in the right part of the formula (3) when $u_j = 1$. The optimization procedure for the design process can be defined in discrete (Eq. (2)) or continuous form. In the last case the design process includes the next principal equations:

$$\frac{dx_i}{dt} = f_i(X, U) \tag{6}$$

$$i = 0,1,\ldots,N$$

$$(1 - u_j) g_j(X) = 0 \tag{7}$$

$$j = 1,2,\ldots,M$$

$$\varphi(X,U) = \frac{1}{\varepsilon}\sum_{j=1}^{M} u_j \cdot g_j^2(X) \qquad (8)$$

The functions of the right hand part of the system (5) depend on the optimization method and can be determined for example for the gradient method as:

$$f_i(X,U) = -\frac{\delta}{\delta x_i} F(X,U) \qquad (9)$$

$$i = 1,2,\dots,K$$

$$f_i(X,U) = -u_{i-K}\frac{\delta}{\delta x_i}F(X,U) + \frac{(1-u_{i-K})}{t_s}\left\{-x_i^s + \eta_i(X)\right\} \qquad (9')$$

$$i = K+1, K+2, \dots, N$$

where $F(X,U) = C(X) + \varphi(X,U)$, x_i^s is equal to $x_i(t-dt)$, the operator $\delta / \delta x_i$ means here $\frac{\delta}{\delta x_i}\varphi(X) = \frac{\partial \varphi(X)}{\partial x_i} + \sum_{p=K+1}^{K+M}\frac{\partial \varphi(X)}{\partial x_p}\frac{\partial x_p}{\partial x_i}$, $\eta_i(X)$ is the implicit function ($x_i = \eta_i(X)$) that is determined by the system (7).

All the control functions u_j depend on the current step of the optimization process. The total number of the different design strategies which are produced inside the same optimization procedure is practically infinite. Among all of these strategies exist one or few optimal strategies that achieve the design objects for the minimum computer time. The function $f_0(X,U)$ is determined as the necessary time for one step of the system (5) integration. The additional variable x_0 is determined as the total computer time T for the system design. In this case we determine the problem of the time-optimal system design as the classical problem of the functional minimization of the control theory. In this context the aim of the design process is to result each function $f_i(X,U)$ to zero for the final time t_{fin}, and to minimize the cost function $C(X)$. The aim of the optimal control is to minimize the total computer time x_0 of the design process. It is necessary to find the optimal behaviour of the control functions u_j during the design process.

The idea of the system design problem formulation as the functional minimization problem of the control theory is not depend of the optimization method and can be embedded into any optimization procedures. In this paper the gradient method and the Davidon-Fletcher-Powell (DFP) method were used.

Now the analogue circuit design process is formulated as a dynamical controllable system. By this formulation we need to find the special conditions to minimize the transition time for this dynamical system.

4 NUMERICAL RESULTS

Some electronic circuits have been designed to demonstrate a new system design approach based on the control theory. The design process has been realized on DC mode. The cost function $C(X)$ has been determined as the sum of the squared differences between beforehand defined values and current values of the nodal voltages for some nodes. Numerical results for the transistor amplifier that is shown in Fig. 1 are discussed below.

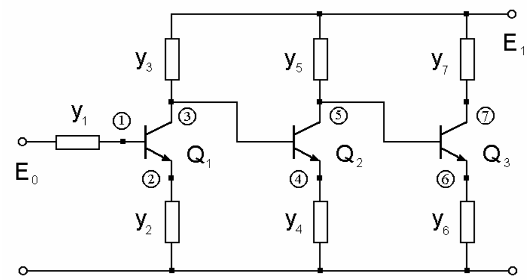

Figure 1: Circuit topology for three-cell transistor amplifier.

The Ebers-Moll static model of the transistor has been used. The analyzed circuit has seven admittance as independent variables $y_1, y_2, y_3, y_4, y_5, y_6, y_7$, ($K=7$) and seven nodal voltages as dependent variables $V_1, V_2, V_3, V_4, V_5, V_6, V_7$, (M=7).

The results of the analysis of the traditional design strategy and some other strategies that have the computer time less than the traditional strategy are given in Table 1. The first line corresponds to the TDS. The last line corresponds to the MTDS. Other nes are the intermediate strategies. The optimal strategies from this table (number 18 and 25 for two optimization procedures respectively) are not optimal in general and the data for the time-optimal strategies are given in Table 2 by means of the control vector variation.

The time gain of the optimal design strategy with respect to the traditional strategy is equal to 285 for the gradient method and 200 for the DFP method. These data show good perspectives for proposed approach. However the potential time gain is realized only in case when we found the algorithm for the optimal control vector construction.

Table 1: Data of some strategies.

N	Control functions	Gradient	method	DFP	method
	vector	Iterations	Total design	Iterations	Total design
	U (u1,u2,u3,u4,u5,u6,u7)	number	time (sec)	number	time (sec)
1	(0 0 0 0 0 0 0)	6379	321.09	854	64.47
2	(0 0 1 0 1 0 1)	922	54.53	764	52.29
3	(0 0 1 0 1 1 0)	1667	80.71	650	46.13
4	(0 0 1 0 1 1 1)	767	35.35	426	22.68
5	(0 0 1 1 1 0 0)	3024	159.67	940	52.71
6	(0 0 1 1 1 0 1)	823	37.73	177	7.71
7	(0 0 1 1 1 1 0)	3068	86.87	450	14.56
8	(0 0 1 1 1 1 1)	553	15.75	170	6.93
9	(0 1 1 0 1 0 1)	465	10.01	101	2.66
10	(0 1 1 0 1 1 0)	1157	31.92	111	3.85
11	(0 1 1 0 1 1 1)	501	8.82	124	2.66
12	(0 1 1 1 1 0 0)	2643	72.66	314	9.24
13	(0 1 1 1 1 0 1)	507	9.24	170	4.62
14	(0 1 1 1 1 1 0)	3070	67.27	423	12.25
15	(1 0 1 0 1 0 1)	1345	28.07	397	16.94
16	(1 0 1 0 1 1 1)	615	10.01	191	4.62
17	(1 0 1 1 1 0 1)	699	10.71	197	4.97
18	(1 0 1 1 1 1 1)	366	4.97	103	1.96
19	(1 1 1 0 1 0 1)	789	10.43	201	4.97
20	(1 1 1 0 1 1 0)	3893	61.53	1158	18.06
21	(1 1 1 0 1 1 1)	749	7.71	148	2.11
22	(1 1 1 1 1 0 0)	4325	90.72	945	19.18
23	(1 1 1 1 1 0 1)	796	8.47	133	2.31
24	(1 1 1 1 1 1 0)	2149	29.26	1104	13.44
25	(1 1 1 1 1 1 1)	2031	5.67	180	0.77

5 CONCLUSIONS

The traditional approach for the analogue circuit design is not time-optimal. The problem of the time-optimum design algorithm can be solved adequately on the basis of the control theory application. The construction of the time-optimal design algorithm is formulated as the problem of a functional minimization of the control theory. This approach can reduce considerably the total computer time for the system design. Analysis of the different electronic systems gives the possibility to conclude that the potential computer time gain of the time-optimal strategy increases when the size and complexity of the system increase. The proposed approach gives the possibility to find the time-optimal algorithm as a solution of the typical problem of the optimal control theory. The optimal structure of the control vector can be finding by the approximate methods of control theory.

ACKNOWLEDGEMENTS

This work was supported by the Mexican National Council of Science and Technology – CONACYT, under project SEP-2004-C01-46510.

REFERENCES

Osterby, O., Zlatev, Z., 1983. *Direct Methods for Sparse Matrices*, Springer-Verlag, N.Y.

George, A., 1984. On Block Elimination for Sparse Linear Systems, *SIAM J. Numer. Anal*. vol. 11, no.3, pp. 585-603.

Wu, F.F., 1976. Solution of Large-Scale Networks by Tearing”, *IEEE Trans. Circuits Syst.*, vol. CAS-23, no. 12, pp. 706-713.

Sangiovanni-Vincentelli, A., Chen, L.K., Chua, L.O., 1977. An Efficient Cluster Algorithm for Tearing Large-Scale Networks, *IEEE Trans. Circuits Syst.*, vol. CAS-24, no. 12, pp. 709-717.

Kashirsky, I.S., Trokhimenko, Y.K., 1979. *The Generalized Optimization of Electronic Circuits*, Tekhnika, Kiev.

Rizzoli, V., Costanzo, A., Cecchetti, C., 1990. Numerical Optimization of Broadband Nonlinear Microwave Circuits, *IEEE MTT-S Int. Symp*., vol. 1, pp. 335-338.

Ochotta, E.S., Rutenbar, R.A., Carley, L.R., 1996. Synthesis of High-Performance Analog Circuits in ASTRX/OBLX, *IEEE Trans. on CAD*, vol.15, no. 3, pp. 273-294.

Zemliak, A., 2005. Generalization of Analog System Design Methodology, *In Proc. 5th WSEAS Int. Conf. on Instrumentation, Measurement, Control, Circuits and Syst*., Cancun, Mexico, pp.114.119.

Table 2: Data of the optimal design strategies.

N	Method	Optimal control functions vector U (u1,u2,u3,u4,u5,u6,u7)	Iterations number	Switching points	Total design time (sec)	Computer time gain
1	Gradient method	(1111111); (1111101)	363	350	1.127	285
2	DFP method	(1111111); (1110111)	69	66	0.322	200