DIRECTIONAL CHANGE AND WINDUP PHENOMENON

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Abstract: The paper addresses two inherently connected problems, namely: windup phenomenon and directional change in controls problem for multivariable systems. By comparing two ways of performing anti-windup compensation and two different saturation modes a new definition of windup phenomenon for multivariable systems has been obtained, changing definitions present in the literature. It has been shown that avoiding directional change does not have necessarily to mean that windup phenomenon has been avoided too.

1 INTRODUCTION

Consideration of control limits is crucial for achieving high control performance (Peng et al., 1998). There are two ways in which one can consider possible constraints during synthesis of controllers, e.g. imposing constraints during the design procedure, what leads to difficulties with obtaining explicit forms of control laws. The other way is to assume the system is linear and, subsequently, having designed the controller for unconstrained system – impose constraints, what requires then additional changes in control system due to presence of constraints.

A situation when because of, e.g., constraints (or, in general, nonlinearities) internal controller states do not correspond to the actual signals present in the control systems is referred as windup phenomenon (Walgama and Sternby, 1993; Horla, 2004). It is obvious that due to control signal constraints not taken into account during a controller design stage, one can expect inferior performance because of infeasibility of computed control signals.

There are many methods of compensating the windup phenomenon (Peng et al., 1998; Walgama and Sternby, 1993), but a few work well enough in the case of multivariable systems. In such a case, apart from the windup phenomenon itself, one can also observe directional change in the control vector due to, say, different implementation of constraints, what could affect direction of the original, i.e. computed, control vector.

The paper aims to compare two strands in controller design subject to constraints, as mentioned before, and two ways of anti-windup compensation with respect to directional change in controls.

As a result, a new definition of windup phenomenon will be obtained with respect to directional change in controls, which in the case of multivariable systems cannot be omitted.

2 ANTI-WINDUP COMPENSATION

There are two general schemes in anti-windup compensation (AWC) connected with controller design. If the controller has been designed for the case of a linear plant, i.e. with no constraints, introducing them would require certain (most often) heuristic modifications in the control law that usually feed back the difference in between computed $\underline{v}_t$ and constrained control vector $\underline{u}_t$. This is referred in the literature as a posteriori AWC (Horla, 2006a; Horla, 2006b).

The second AWC is incorporated implicitly into the controller, i.e. when controller generates feasible control vector only (belonging to the domain $\mathscr{D}$ of all control vectors for which a certain control performance index J_t is of finite value), what is addressed as a priori AWC.

3 A POSTERIORI AWC

One of the most popular AWCs (Peng et al., 1998) are those based on the RST equation, which in the case of multivariable systems is of the form (Horla, 2004)

$$R(q^{-1})\underline{v}_t = -S(q^{-1})\underline{y}_t + T(q^{-1})\underline{r}_t\,, \qquad (1)$$

where

$$\begin{aligned} R(q^{-1}) &= I + R_1 q^{-1} + \cdots + R_{nR} q^{-nR}\,, \\ S(q^{-1}) &= S_0 + S_1 q^{-1} + \cdots + S_{nS} q^{-nS}\,, \\ T(q^{-1}) &= T_0 + T_1 q^{-1} + \cdots + T_{nT} q^{-nT} \end{aligned}$$

are controller polynomial matrices of appropriate sizes, designed for the unconstrained case, $\underline{y}_t \in R^p$ is the output vector, $\underline{v}_t \in R^m$ is the control vector, $d > 0$ is a dead-time.

When the nonlinearities, such as control limits, are taken into consideration the computed vector $\underline{v}_t$ is different from the constrained, i.e. applied, control vector $\underline{u}_t$. In such a case one can modify the control law according to AWC schemes given below (Horla, 2004; Peng et al., 1998).

- Deadbeat AWC (DB)

  $$\underline{v}_t = (I - R(q^{-1}))\underline{u}_t - S(q^{-1})\underline{y}_t + T(q^{-1})\underline{r}_t\,. \quad (2)$$

  The controller is fed back with the constrained control vector, thus no lack of consistency occurs.
- Generalised AWC (G)

  A matrix $A_o(q^{-1})$ of observer polynomials with $nA_o \le nR$ is added

  $$\begin{aligned} A_o(q^{-1})\underline{v}_t &= (A_o(q^{-1}) - R(q^{-1}))\underline{u}_t - \\ &\quad - S(q^{-1})\underline{y}_t + T(q^{-1})\underline{r}_t\,. \quad (3) \end{aligned}$$
- Conditioning technique AWC (CT)

  The control vector and reference signal are computed as

  $$\begin{aligned} \underline{v}_t &= (I - R(q^{-1}))\underline{u}_t - S(q^{-1})\underline{y}_t + \\ &\quad + (T(q^{-1}) - T_0)\underline{r}_t^r + T_0 \underline{r}_t\,, \quad (4) \\ \underline{r}_t^r &= \underline{r}_t + T_0^{-1}(\underline{u}_t - \underline{v}_t)\,. \quad (5) \end{aligned}$$

  A special case of CT is Modified conditioning technique AWC (MCT) where instead of T_0^{-1} there is an inversion of matrix $(T_0 + \Upsilon)^{-1}$ which is responsible for the rate of modification of the reference vector subject to constraints.

  In the CT case, often outputs that were intended to me unmodified, are modified due to conditioning technique. The latter is a result of directional change, that is given rise by anti-windup compensation. The two issues are therefore connected.
- Generalised conditioning technique AWC (GCT)

  The restoration of the consistency is performed by modifying the filtered reference vector, i.e. computing the so-called feasible filtered reference vector,

  $$\begin{aligned} \underline{v}_t &= (I - Q(q^{-1})R(q^{-1}))\underline{u}_t + \\ &\quad + T_{2,0}\underline{r}_{f,t} + (T_2(q^{-1})L(q^{-1}) - T_{2,0})\underline{r}_{f,t}^r + \\ &\quad - Q(q^{-1})S(q^{-1})\underline{y}_t\,. \quad (6) \\ \underline{r}_{f,t} &= Q(q^{-1})L(q^{-1})^{-1}T_1(q^{-1})\underline{r}_t\,, \quad (7) \\ \underline{r}_{f,t}^r &= \underline{r}_{f,t} + T_{2,0}^{-1}(\underline{u}_t - \underline{v}_t)\,, \quad (8) \end{aligned}$$

  Where $T = T_2 T_1$ with monic T_1, and nonsingular $T_{2,0}$.

4 DEFINITION OF WINDUP PHENOMENON IN MULTIVARIABLE SYSTEMS

Currently, one can meet the following definition of windup phenomenon in multivariable systems with its connections to directional change (Walgama and Sternby, 1993):

Solving the windup phenomenon problem does not mean that constrained control vector is of the same direction as computed control vector.

On the other hand, avoiding directional change in control enables one to avoid windup phenomenon.

In further parts of this paper, it has been shown where the latter definition holds, and in what cases it is invalid.

5 DIRECTIONAL CHANGE PHENOMENON, AN EXAMPLE

Let us suppose that two-input two-output system is not coupled and both loops are driven by separate controllers (with no cross-coupling). The system output $\underline{y}_t$ is to track reference vector comprising two sinusoid waves. It corresponds in the (y_1, y_2) plane to drawing a circular shape.

As it can be seen in the Fig. 1a, the unconstrained system performs best, whereas in the case of cut-off saturation of both elements of control vector (Fig. 1b) the tracking performance is poor. In the application for, e.g., shape-cutting performance of the system from Fig. 1c is superior. Nevertheless, it is to be borne in mind that the system is always perfectly decoupled.

6 PLANT MODEL, CONTROL PROBLEM

The following multivariable CARMA plant model will be of interest

$$A(q^{-1})\underline{y}_t = B(q^{-1})\underline{u}_{t-d}\,, \tag{9}$$

with left co-prime polynomial matrices

$$A(q^{-1}) = I + \begin{bmatrix} -1.4 & -0.1 \\ 0.1 & -1.0 \end{bmatrix} q^{-1} + \begin{bmatrix} 0.49 & 0 \\ 0 & 0.25 \end{bmatrix} q^{-2},$$

$$B(q^{-1}) = I + \mathrm{diag}\{0.5, 0.5\}\, q^{-1}$$

and $d = 1$.

The plant is cross-coupled and comprises fourth-order matrices in the transfer matrix representation (being stable and minimumphase).

6.1 RST Controller (a Posteriori AWC)

It is assumed that the plant is controlled by a multivariable pole-placement controller with characteristic polynomial matrix

$$A_M(q^{-1}) \quad = \quad I + \mathrm{diag}\{-0.5, -0.5\}\, q^{-1}\,.$$

The controller is given in RST structure with polynomial matrices $R(q^{-1})$ and $S(q^{-1})$ resulting from Diophantine equation

$$A(q^{-1})R(q^{-1}) + q^{-d}B(q^{-1})S(q^{-1}) = A_M(q^{-1})A_o(q^{-1})\,, \tag{10}$$

with $A_o(q^{-1}) = I - 0.2Iq^{-1}$, $T(q^{-1}) = KA_o(q^{-1})$, $A_M(1) = B(1)K$.

Having imposed control limits upon the RST controller requires implementing a posteriori AWC techniques in order to restore good performance quality.

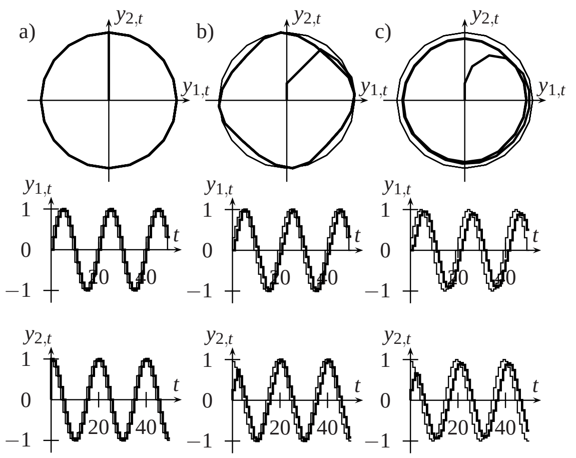

Figure 1: a) unconstrained system, b) cut-off saturation, c) direction-preserving saturation.

6.2 Optimised Controller (a Priori AWC)

In comparison, the RST pole-placement controller performance will be compared with a priori AWC controller, namely multivariable pole-placement controller utilising the theory of predictive control and convex optimisation techniques.

In order to enable such a comparison, the predictive controller has been deprived of all its advantages – the prediction horizon has been chosen as one step, thus the optimal constrained control vector is searched (Horla, 2006a; Horla, 2006b)

$$\underline{u}_t^\star : J_t(\underline{u}_t^\star) = \inf_{\underline{u}_t \in \mathscr{D}(J_t)} \{J_t(\underline{u}_t)\}\,,$$

where its jth component has been symmetrically constrained

$$|u_{j,t}| \quad \leq \quad \alpha_j\,,$$

where $\quad \underline{u}_t = \begin{bmatrix} u_{1,t} & u_{2,t} & \cdots & u_{m,t} \end{bmatrix}^T.$

The performance index has been chosen as a sum of squared tracking errors resulting from a reference model output

$$J_t \quad = \quad \left\| \underline{r}_{M,t+d} - \underline{\hat{y}}_{t+d} - \underline{\hat{\hat{y}}}_{t+d} \right\|_2^2\,, \tag{11}$$

where one-step ($d = 1$) prediction of system output comprises as in (11) forced and free-response output vectors.

The performance index can be rewritten into quadratic form

$$J_t = \left(G\underline{u}_t + \underline{\hat{\hat{y}}}_{t+d} - \underline{r}_{M,t+d}\right)^T \left(G\underline{u}_t + \underline{\hat{\hat{y}}}_{t+d} - \underline{r}_{M,t+d}\right), \tag{12}$$

and the optimisation can be performed with the use of its linear matrix inequality (LMI) form with the last two LMIs responsible for control constraints, as below

$$\begin{array}{ll} \min & \gamma \\ \text{s.t.} & \begin{bmatrix} I & \star \\ \underline{u}_t^T(G^TG)^{1/2} & \left(\begin{array}{c} \gamma - (\underline{r}_{M,t+d} - \underline{\hat{\hat{y}}}_{t+d})^T \times \\ \times(\underline{r}_{M,t+d} - \underline{\hat{\hat{y}}}_{t+d}) + \\ +2(\underline{r}_{M,t+d} - \underline{\hat{\hat{y}}}_{t+d})^T G\underline{u}_t \end{array}\right) \end{bmatrix} \geq 0\,, \\ & \mathrm{diag}\{\alpha_1 - u_{1,t}, \ldots, \alpha_m - u_{m,t}\} \geq 0\,, \\ & \mathrm{diag}\{\alpha_1 + u_{1,t}, \ldots, \alpha_m + u_{m,t}\} \geq 0\,, \end{array} \tag{13}$$

where $\star$ detones a symmetrical entry, and G is an impulse-response matrix.

7 SIMULATION STUDIES

The simulations have been performed for two controllers: for a pole-placement controller with a group of a posteriori AWCs and LMI-based predictive pole-placement controller.

In order to evaluate control performance connected with anti-windup compensation performance the following performance indices have been introduced

$$J = \frac{1}{N}\sum_{i=1}^{2}\sum_{t=1}^{N}|r_{i,t}-y_{i,t}|, \qquad (14)$$

$$\overline{\varphi} = \frac{1}{N}\sum_{t=1}^{N}|\varphi(\underline{v}_t)-\varphi(\underline{u}_t)| \ [^\circ], \qquad (15)$$

where (14) corresponds to mean absolute tracking error on both outputs and (15) is a mean absolute direction change in between computed and constrained control vector.

The control vector has been constrained in all cases to $\alpha_1 = \pm 0.2$ on the first input and $\alpha_2 = \pm 0.3$ for the second output. The reference vector is a square-wave signal of amplitude ± 1 and simulation horizon $N = 150$.

Performance indices have been given in the Tab. 1.

Table 1: Performance indices for a) cut-off saturation, b) direction-preserving saturation.

		–	*DB*	*G*	*CT*	*MCT*	*GCT*
a)	J	1.1539	1.1539	1.1539	1.1539	1.1536	1.1516
	$\overline{\varphi}$	0.9003	0.9004	1.1861	1.1862	1.1093	1.5495
		–	*DB*	*G*	*CT*	*MCT*	*GCT*
b)	J	1.1772	1.1672	1.1677	1.1677	1.1670	1.1655
	$\overline{\varphi}$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

As it can be seen from Tab. 1a, and Figs. 4, 5, cut-off saturation causes directional change, which is visible during reference vector changes. It is necessary to alter the decoupling of the plant, in order to restore high control performance. As it can be seen, the greater the mean absolute direction change, the lesser the performance index is. Thus, it can be said, that directional change supports anti-windup compensation.

On the other hand, as in Tab. 1b, and Figs. 6, 7, direction-preserving saturation does not cause directional change. Preserving a constant direction causes performance indices to increase, dependless of the method of anti-windup compensation. The coupling is clearly visible during tracking when reference vector changes. Constant direction prevents the controller from decoupling the plant – performance is inferior.

In order to stipulate the differences in between saturation methods, two GCT-AWC cases have been chosen and compared with LMI-based approach.

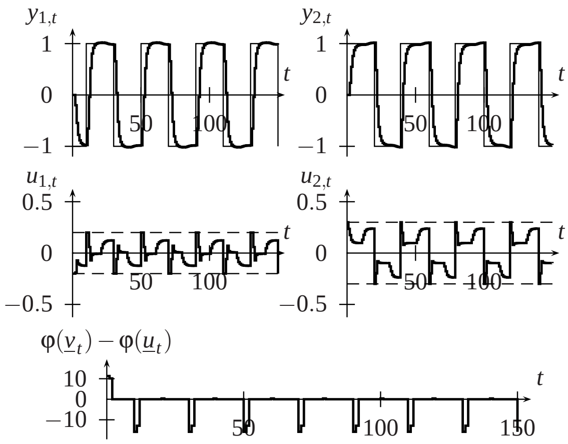

Figure 2: Overall performance for cut-off saturation with GCT-AWC.

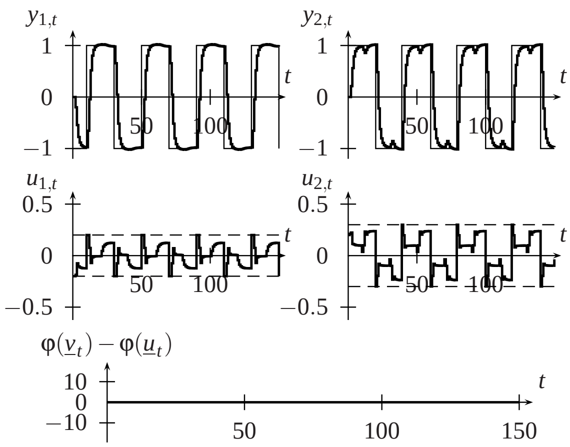

Figure 3: Overall performance for direction-preserving saturation with GCT-AWC.

In the Fig. 8 one can see a priori anti-windup compensator performance, where only feasible control actions are generated. The performance indices in such a case are of the best values, i.e. $J = 1.0450$, $\overline{\varphi} = 64.7063^\circ$.

Having compared Figs. 2–8 it can be said, that in order to achieve the best performance one has to alter the direction of a computed control vector. The greater the directional change is, the better the control performance.

For a priori AWC, visible changes in control direction result from decoupling phase, i.e. whenever control vector encounters constraints it has to be constrained in such a way as to achieve the high control performance (the last plot corresponds to the angle difference in between control vector computed in the unconstrained case using optimisation algorithm, and constrained a priori AWC control vector).

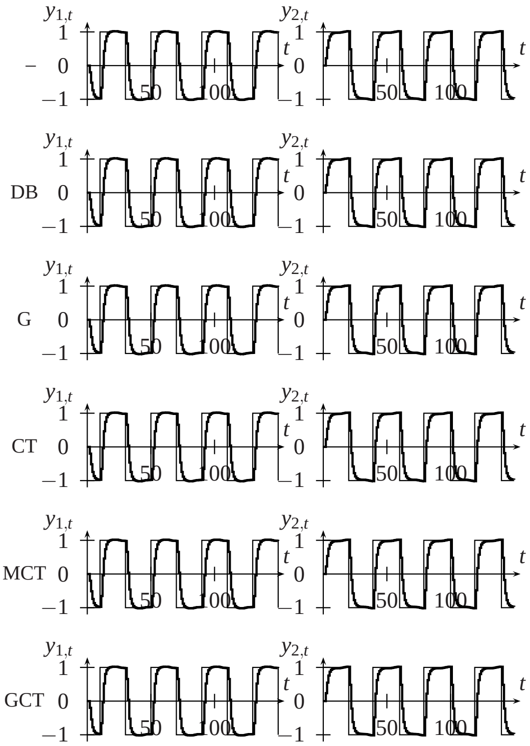

Figure 4: Tracking performance for cut-off saturation.

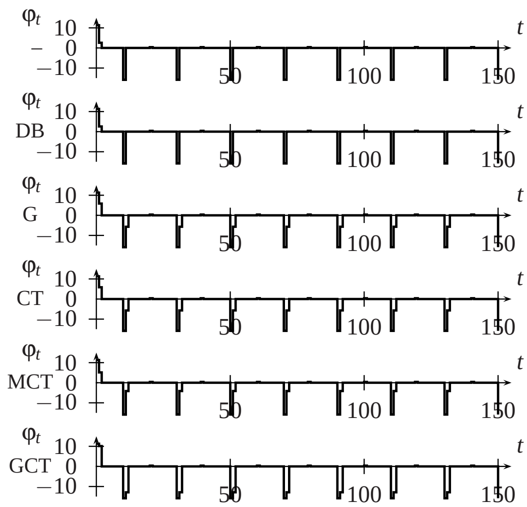

Figure 5: Directional change for cut-off saturation.

On the other hand, having constrained the control vector alters decoupling, thus its direction has to be additionally altered, what is mostly visible in Fig. 8.

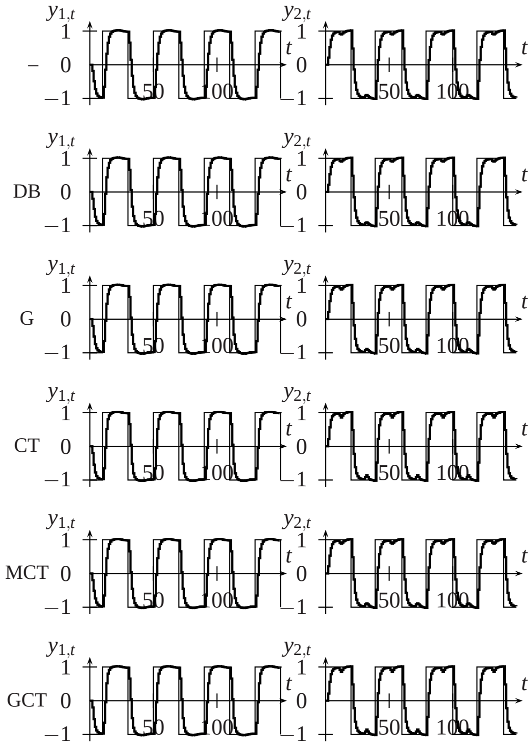

Figure 6: Tracking performance for direction-preserving saturation.

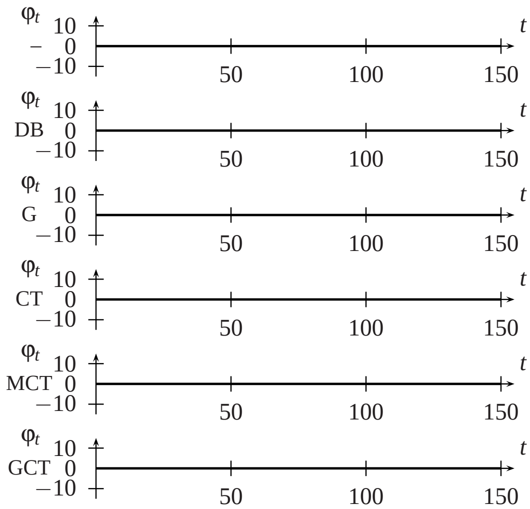

Figure 7: Directional change for direction-preserving saturation.

Finally, in the Fig. 9 it has been shown that both a priori and a posteriori AWCs need to alter direction of controls in order to restore high quality of tracking performance. In addition, a priori AWC has no fixed structure, nor decoupling compensator, thus changes

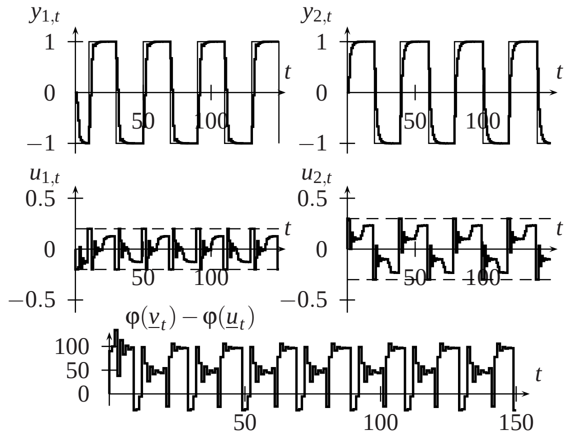

Figure 8: Overall performance for LMI-based control with a priori AWC, $J = 1.0450$, $\overline{\varphi} = 64.7063°$.

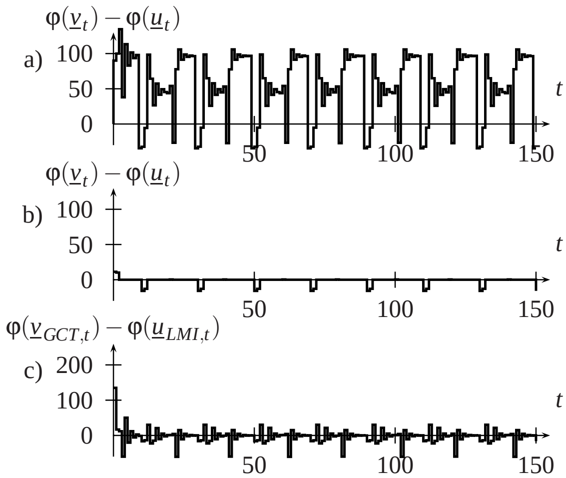

Figure 9: A comparison of direction changes a) a priori AWC, b) a posteriori GCT-AWC with cut-off saturation, c) a priori AWC vs. GCT-AWC with cut-off saturation.

in control direction must be greater than in the case of GCT-AWC with cut-off saturation.

The comparison of angle difference in between unconstrained control vector computed for GCT case and constrained control vector computed by a priori AWC, shows that approximately a priori AWC acts in the direction of unconstrained controller with GCT-AWC, i.e. both of them try to get the system's performance as close as possible to the performance of ideal pole-placement subject to no constraints.

Nevertheless, the simulations have shown that in order to obtain a good performance one has to change direction of control vector. Without the latter one will observe coupling.

8 SUMMARY – NEW DEFINITION OF WINDUP PHENOMENON IN MULTIVARIABLE SYSTEMS

One can formulate a new definition:

Solving the windup phenomenon problem does not ***have to*** *mean that constrained control vector is of the same direction as computed control vector* ***if cross-coupling is present in the control system.***

On the other hand, avoiding directional change in control enables one to avoid windup phenomenon ***if and only in the plant is perfectly decoupled or is not coupled at all.*** (what due to the constraints is hardly ever met)

Such a definition definitely changes the way one should look at windup phenomenon and its connection with directional change problem.

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