

USE A NEURAL NETWORKS TO ESTIMATE AND TRACK THE PN SEQUENCE IN LOWER SNR DS-SS SIGNALS

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Abstract: This paper proposes a modified Sanger's generalized Hebbian algorithm (GHA) neural network (NN) method to estimate and track the pseudo noise (PN) sequence in lower signal to noise ratios (SNR) direct sequence spread spectrum (DS-SS) signals. The proposed method is based on eigen-analysis of DS-SS signals. The received signal is firstly sampled and divided into non-overlapping signal vectors according to a temporal window, which duration is a periods of PN sequence. Then an autocorrelation matrix is computed and accumulated by these signal vectors one by one. The PN sequence can be estimated and tracked by the principal eigenvector of autocorrelation matrix in the end. But the eigen-analysis method becomes inefficiency when the estimated PN sequence becomes longer or the estimated PN sequence becomes time varying. In order to overcome these shortcomings, we use a modified Sanger's GHA NN to realize the PN sequence estimation and tracking from lower SNR input DS-SS signals adaptively and effectively.

1 INTRODUCTION

Since the direct sequence spread spectrum (DS-SS, DS) signals have the distinguished capability of anti-jamming and lower probability interception, the DS signals have used broadly in communication, radar, telemetry and telecommand etc for a long time. Usually, the spread spectrum receiver has to perform synchronization before it can start the despreading operation. For the case of DS, this entails establishing complete knowledge of the pseudo noise (PN) sequence and the timing. Synchronization is performed in two stages. The first stage of coarse synchronization is known as PN acquisition and the final stage of maintaining the fine synchronization is called PN tracking. While PN tracking forms an important part of DS synchronization, PN acquisition is a more challenging problem.

Conventional acquisition techniques (Simmon et al., 1994) rely on the knowledge of the internal algebraic structure of the PN spreading sequence to establish synchronization. While they demonstrate good acquisition performance in low noise environments, they tend to break down in environments with high levels of noise and interference because of a high false alarm rate. Furthermore, reliable algebraic techniques for synchronization have yet to be developed for nonlinear codes, or codes with unknown code structure, chip constellations, and residual delay. Additionally, the PN sequences of DS signal have the distinguished function of keeping secrecy. If you have no knowledge of the PN sequence, you could not demodulate the transmitted message symbols generally.

A method of autocorrelation and cyclic autocorrelation was proposed to de-spread the DS

signal (French et al., 1986), which can extract a differentially-encoded estimate of the underlying message sequence from a modulation-on-symbol DS signal (where the spreading PN sequence repeats once per message symbol) on the basis of the periodic structure of these signals. This method attempts to overcome some of these disadvantages by making no assumptions about the internal algebraic structure of the PN spreading sequence. They can operate in the presence of arbitrary delay and for arbitrary codes or chip constellations. Because some spectral correlation computations are required, it is difficult to carry out in real-time. Furthermore, it does only de-spread the DS signal without the PN sequence, but it doesn't utilize or analyze any signal structure information. So far, most of DS packet radio and military systems often require frequent, fast and robust synchronization. Blind estimation and tracking of the PN spreading sequence without the a priori knowledge of its structure and timing is useful in achieving these objectives.

The signal subspace analysis and relational techniques, introduced in (Zhang et al., 2005) (Simic et al., 2005) (Zhan et al., 2005), is precisely such a technique. It is based on the signal subspace analysis of DS signal, and estimates the PN spreading sequence blindly by exploiting cyclostationarity property and eigenstructure of the DS signal. The technique provides perfect estimates of the PN spreading sequence under the assumptions of infinite time-averaging in the presence of arbitrary levels of temporally-white background noise. But the methods proposed in (Zhang et al., 2005) (Simic et al., 2005) (Zhan et al., 2005) belong to a batch method, when the number of samples in a period of observation window becomes too large or the estimated PN sequence becomes time-varying, the computation of matrix decomposition may not be feasible in practice.

This paper proposes an unsupervised adaptive approach of Sanger's generalized Hebbian algorithm (GHA) neural networks (NN) to PN sequence blind estimation and adaptive tracking. It needs the first and second principal component vectors associated with the largest and second largest eigenvalue respectively; and it can deal with too long sampling signal vectors and time-varying cases.

2 SIGNAL MODEL

The base band DS signal $x(t)$ corrupted by the white Gaussian noise $n(t)$ with the zero mean and σ_n^2 variance can be expressed as (French et al., 1986) (Zhang et al., 2005) (Simic et al., 2005) (Zhan et al., 2005)

$$x(t) = s(t - T_x) + n(t) \quad (1)$$

Where $s(t) = d(t)p(t)$ is the DS signal, $p(t) = \sum_{j=-\infty}^{\infty} p_j q(t - jT_c)$, $p_j \in \{\pm 1\}$ is the periodic PN sequence, $d(t) = \sum_{k=-\infty}^{\infty} m_k q(t - kT_0)$, $m_k \in \{\pm 1\}$ is the symbol bits, uniformly distributed with $E[m_k m_l] = \delta(k - l)$, $\delta(\cdot)$ is the Dirac function, $q(t)$ denotes a pulse chip. Where $T_0 = NT_c$, N is the length of PN sequence, T_0 is the period of PN sequence, T_c is the chip duration, T_x is the random time delay and uniformly distributed on the $[0, T_0]$.

According to the above, the PN sequence and synchronization are required to de-spread the received DS signals. But in some cases, we only have the received DS signals. We must estimate the signal parameters firstly (We assume that T_0 and T_c had known in this paper), and then estimate the PN sequence and synchronization.

3 SUBSPACE ANALYSIS BASED ON K-L TRANSFORMATION

The received DS signal is sampled and divided into non-overlapping temporal windows, the duration of which is T_0 . Then one of the received signal vector is

$$\mathbf{X}(k) = \mathbf{s}(k) + \mathbf{n}(k), \quad k = 1, 2, 3, \dots \quad (2)$$

Where $\mathbf{s}(k)$ is the k -th vector of useful signal, $\mathbf{n}(k)$ is the additive white Gaussian noise vector. The dimension of vector $\mathbf{X}(k)$ is $N = T_0 / T_c$. If the random time-delay is T_x , $0 \leq T_x < T_0$, $\mathbf{s}(k)$ may contain two consecutive symbol bits, each modulated by a period of PN sequence, i.e.

$$\mathbf{s}(k) = m_k \mathbf{p}_1 + m_{k+1} \mathbf{p}_2 \quad (3)$$

Where m_k and m_{k+1} are the two consecutive symbol bits, \mathbf{p}_1 (\mathbf{p}_2) is the right (left) part of the PN sequence waveform.

According to K-L transformation, we normalize \mathbf{p}_i by $\mathbf{u}_i = \mathbf{p}_i / \|\mathbf{p}_i\|$, $i=1,2$

$$\mathbf{u}_i^T \mathbf{u}_j = \delta(i-j), \quad i, j=1,2 \quad (4)$$

Where \mathbf{u}_1 and \mathbf{u}_2 are ortho-normal vectors, $\delta(\cdot)$ is a Dirac function. From \mathbf{u}_1 and \mathbf{u}_2 , we have

$$\mathbf{X}(k) = m_k \|\mathbf{p}_1\| \mathbf{u}_1 + m_{k+1} \|\mathbf{p}_2\| \mathbf{u}_2 + \mathbf{n}(k) \quad (5)$$

The autocorrelation matrix of $\mathbf{X}(k)$: \mathbf{R}_X may be estimated as

$$\hat{\mathbf{R}}_X(M) = \frac{1}{M} \sum_{i=1}^M \mathbf{X}(i) \mathbf{X}^T(i) \quad (6)$$

Assume $\mathbf{s}(k)$, $\mathbf{n}(k)$ are mutually independent, substitute Eq.(5) into Eq.(6) yields

$$\begin{aligned} \mathbf{R}_X &= \hat{\mathbf{R}}_X(\infty) = \bar{\mathbf{U}}_s \mathbf{\Lambda}_s \bar{\mathbf{U}}_s^T + \bar{\mathbf{U}}_n \mathbf{\Lambda}_n \bar{\mathbf{U}}_n^T \\ &= \sigma_n^2 \left\{ \left(\gamma_{SNR} \cdot \frac{T_0 - T_x}{T_c} \right) \cdot \bar{\mathbf{u}}_1 \bar{\mathbf{u}}_1^T + \left(\gamma_{SNR} \cdot \frac{T_x}{T_c} \right) \cdot \bar{\mathbf{u}}_2 \bar{\mathbf{u}}_2^T + \mathbf{I} \right\} \quad (7) \end{aligned}$$

Where \mathbf{I} is an identity matrix of dimension $N \times N$, the expectation of m_k is zero. The variance of m_k is σ_m^2 , the symbol is uncorrelated from each other. The energy of PN sequence is $E_p \approx T_c \|\mathbf{p}\|^2$, the variance of $\mathbf{s}(k)$ is $\sigma_s^2 = \sigma_m^2 E_p / T_0$, $\gamma_{SNR} = \sigma_s^2 / \sigma_n^2$. The row vectors of $\bar{\mathbf{u}}_s$ and $\bar{\mathbf{u}}_n$ are corresponding to the eigenvectors of eigenvalue $\lambda_{R1} = [1 + \gamma_{SNR} \cdot (T_0 - T_x) / T_c] \sigma_n^2$, $\lambda_{R2} = (1 + \gamma_{SNR} \cdot T_x / T_c) \sigma_n^2$ and σ_n^2 , and exist $\lambda_{R1} \geq \lambda_{R2} > \sigma_n^2$. It is clear that the eigenvalues of \mathbf{R}_X are dependent on T_x . It is shown in (Anderson, 1963) that the estimated principal eigenvectors have the following behavior:

$$\|\bar{\mathbf{u}}_i - \mathbf{u}_i\| = O(\sqrt{\log \log M / M}), \quad i=1,2,\dots,K.$$

Therefore, $M \rightarrow \infty$, there always exists $\bar{\mathbf{u}}_i = \mathbf{u}_i$, $i=1,2,\dots,K$.

When $T_x \neq 0$, the biggest eigenvalue is λ_{R1} , the sign of its corresponding eigenvector $\mathbf{p}_1 = \text{sign}(\bar{\mathbf{u}}_1)$. The second biggest eigenvalue is λ_{R2} and the sign of its corresponding eigenvector $\mathbf{p}_2 = \text{sign}(\bar{\mathbf{u}}_2)$. We can recover a period PN sequence from $\mathbf{p} = \mathbf{p}_2 + \mathbf{p}_1 = \text{sign}(\bar{\mathbf{u}}_2) + \text{sign}(\bar{\mathbf{u}}_1)$. When $T_x = 0$, λ_{R1} and $\mathbf{p}_1 = \text{sign}(\bar{\mathbf{u}}_1)$ which denote a period of PN sequence.

Because the accumulation of \mathbf{R}_X estimation by Eq. (6) is a de-noise process, we can estimate the PN sequence by decomposition of $\hat{\mathbf{R}}_X$ even when γ_{SNR} is lower. However, the memory size and computational speed will become problems when N becomes bigger. Additionally, it is difficult to use this batch method to realize the PN tracking of DS signals. Since we would like to track slowly varying parameters, we must form a moving average estimate of the correlation matrix based on the J most recent observations

$$\hat{\mathbf{R}}_X(i, J) = \frac{1}{J} \sum_{j=i-J+1}^i \mathbf{X}(j) \mathbf{X}^T(j) \quad (8)$$

It is well known (Anderson, 1963) that the maximum likelihood estimate of the eigenvalues and associated eigenvectors of \mathbf{R}_X is just the eigenvalue decomposition of $\hat{\mathbf{R}}_X(i, J)$. But there are a lot of difficulties in this tracking process by eigenvalue decomposition for it's a batch method. In the following context, we will propose to use the PCA NN to solve these problems.

4 IMPLEMENTATION OF A MODIFIED SANGER'S GHA NEURAL NETWORKS

According to the result of subspace analysis of DS signals based on K-L transformation, we'll have to extract the first and second principal eigenvectors before realizing the whole PN sequence estimation. A two-layer PCA NN is used to estimate the PN sequence in DS signal blindly as in Fig.1. The number of input neurons is given by $N = T_0 / T_c$.

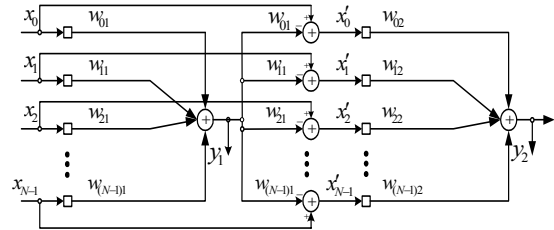


Figure 1: Neural Networks.

Assume $T_x \neq 0$, one of the received signal vectors is

$$\mathbf{X}(t) = \mathbf{X}(k) = [x(t), x(t-T_c), \dots, x[t-(N-1)T_c]]^T \quad (9)$$

$$= [x_0(t), x_1(t), \dots, x_{N-1}(t)]^T$$

Where $\{x_i(t) = x(t-iT_c), i = 0, 1, \dots, N-1\}$ are sampled by one point per chip. The synaptic weight vector is

$$\mathbf{w}_j(t) = [w_{0j}(t), w_{1j}(t), \dots, w_{(N-1)j}(t)]^T \quad (10)$$

Where the sign of $\{w_{ij}(t), i = 0, 1, \dots, N-1, j = 1, 2\}$ denotes the 1st and 2nd i -th bit of estimated PN sequence. The output layer of NN has only two neurons, its output is

$$y_j(t) = \sum_{i=0}^{N-1} w_{ij}(t)x_i(t), \quad j = 1, 2 \quad (11)$$

The synaptic weight $w_{ij}(t)$ is adapted in accordance with a general form of Hebbian learning, as shown by

$$\mathbf{w}_j(t+1) = \mathbf{w}_j(t) + \beta_j y_j(t) \left[\mathbf{X}(t) - \sum_{k=1}^j y_k(t) \mathbf{w}_k(t) \right] \quad (12)$$

Where β_j are the positive step-size parameters. In order to achieve good robust convergence performance, we modified β_j in learning rule Eq.(12) of Sanger's GHA as follows

$$\beta_j = 1/d_j(t+1), \quad (13)$$

$$d_j(t+1) = B_j d_j(t) + y_j^2(t), \quad j = 1, 2$$

Where $B_j, j = 1, 2$, are two positive constants (usually less than 1). Where the Sanger's generalized Hebbian algorithm (GHA) of Eq.(12) for layer of j neurons includes the algorithm of original Hebbian for a single neuron as a special case, that is, $j = 1$.

For a heuristic understanding of how the Sanger's GHA actually operates, we use matrix notation to rewrite the version of the algorithm defined in Eq.(12) as follows

$$\mathbf{w}_j(t+1) = \mathbf{w}_j(t) + \beta_j y_j(t) [\mathbf{X}'(t) - y_j(t) \mathbf{w}_j(t)] \quad (14)$$

Where

$$\mathbf{X}'(t) = \mathbf{X}(t) - \sum_{k=1}^{j-1} y_k(t) \mathbf{w}_k(t) \quad (15)$$

The vector $\mathbf{X}'(t)$ represents a modified form of the input vector. Provided that the first neuron has already converged to the first principal component, the second neuron sees an input vector $\mathbf{X}'(t)$ from which the first eigenvector of the correlation matrix \mathbf{R}_X has been removed. The second neuron therefore extracts the first principal component of $\mathbf{X}'(t)$,

which is equivalent to the second principal component of the original input vector $\mathbf{X}(t)$.

The neuron-by-neuron description above is intended merely to simplify the explanation. In practice, all the neurons in this modified generalized Hebbian algorithm tend to converge together. There is a convergence theorem in (Sanger, 1989) (Haykin, 1999) which can guarantee the convergence of the modified Sanger's GHA NN. It guarantees the GHA NN to find the first j eigenvectors of the correlation matrix \mathbf{R}_X . Equally important is the fact that we do not need to compute \mathbf{R}_X . Rather, the first j eigenvectors of \mathbf{R}_X are computed by the algorithm directly from the input signal. The resulting computational savings can be enormous especially if the dimensionality N of the input space is very large, and the required number of the eigenvectors associated with the j largest eigenvalues of the \mathbf{R}_X is a small fraction of N . This provides best advantage to track the time-varying PN spreading sequence of DS signals adaptively.

5 SIMULATIONS

The experiments mainly focus on the NN implementation. We get principal eigenvectors and performance curves.

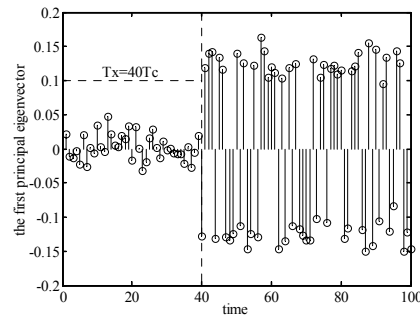


Figure 2: The estimated 1st principal eigenvector.

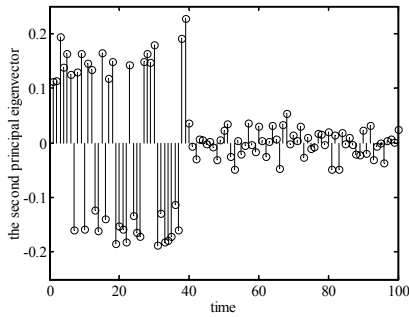


Figure 3: The estimated 2nd principal eigenvector.

Fig.2 and Fig.3 denote the first and second principal eigenvector with $N=100$ bit at $T_x=0.4T_0$. From them, we may estimate the parameter T_x and reconstruct the original PN sequence.

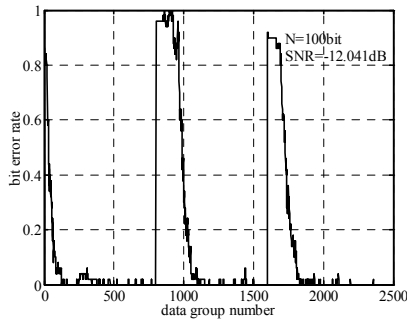


Figure 4: The performance curves of PN tracking.

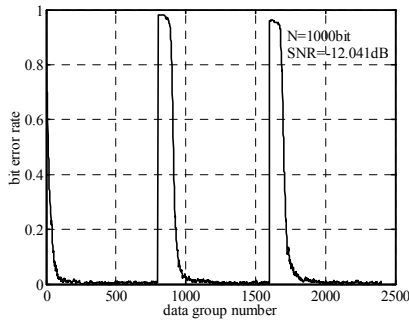


Figure 5: The performance curves of PN tracking.

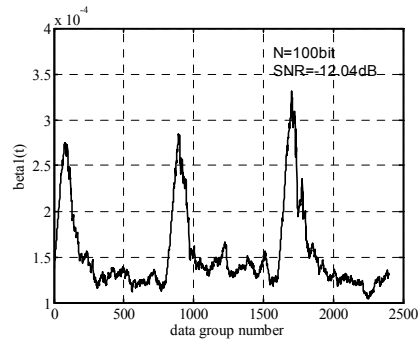


Figure 6: The curve of $\beta_1(t)$.

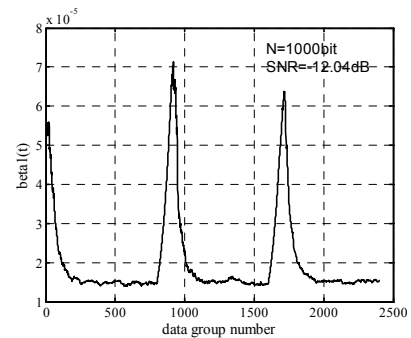


Figure 7: the curve of $\beta_1(t)$.

Fig.4-5 show the tracking performance of the NN under $SNR=-12.04dB$ when the length of PN sequence is $N=100bit$ and $N=1000bit$ respectively. Fig.6-7 show the curves of step-size $\beta_1(t)$ when the case of $N=100bit, SNR=-12.04dB$ and $N=1000bit, SNR=-12.04dB$, respectively. Under the same parameters except the length and content of PN sequence, we study the convergence behavior of the NN in signal scenarios with sudden PN sequence changes. We see in Fig.4-7 that when the PN sequence is longer, the convergence and tracking performance is better.

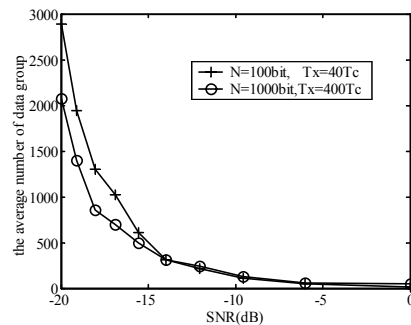


Figure 8: The performance curves of PN estimation.

Fig.8 denotes the performance curves of PN sequence estimation. It shows the time taken for the NN to perfectly estimate the PN sequence for lengths of $N=100\text{bit}$ and $N=1000\text{bit}$ at $T_x/T_0=0.4$. Under the same condition, when the longer the PN sequence is, the better the performance is.

6 CONCLUSIONS

A modified Sanger's GHA NN technique for blind estimation and adaptive tracking of PN sequence of DS signals is developed and demonstrated. The technique, referred to here as the modified Sanger's GHA NN algorithm, exploits the subspace analysis based on K-L transformation of the DS signal to blindly estimate and adaptively track the spreading code and can further despread the underlying message sequence, without knowledge of the content of the PN code or message sequences. The technique is applicable to arbitrary spreading codes and message sequences, and can operate in environments containing arbitrary levels of additive white Gaussian noise in theory.

The technique is demonstrated for the length of PN code $N=100\text{bit}$ and 1000bit DS-SS signal received in -20 dB to 0 dB of additive white Gaussian noise. It is shown that the technique can blindly estimate and adaptively track the PN sequence in the presence of strong additive white Gaussian noise. In (Simic et al., 2005) Simic used the method of eigen-analysis to achieve -5dB of the SNR threshold, moreover, in (Zhan et al., 2005) Zhan use the method of matrix to achieve -12dB SNR threshold, but we can realize threshold of $SNR = -20.0\text{dB}$ easily here, hence the performance of the methods in this paper is more better. The convergence time of the algorithm for PN sequence perfect estimation is also shown to be competitive with conventional despreading techniques (which require knowledge of the spreading code) such as delay-lock loops.

These results show that modified Sanger's GHA NN technique can provide a promising alternative to existing despreading algorithms. The algorithm can be applicable to signals with short code lengths, such as commercial communication signals. The algorithm can be also applicable to signals with longer code lengths, such as military communication signals. It can be further used in management and scout of DS communications.

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