MODIFIED MODEL REFERENCE ADAPTIVE CONTROL FOR PLANTS WITH UNMODELLED HIGH FREQUENCY DYNAMICS

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Abstract: In this paper we develop a modified MRAC strategy for use on plants with unmodelled high frequency dynamics. The MRAC strategy is made up of two parts, an adaptive control part and a fixed gain control part. The adaptive algorithm uses a combination of low and high pass filters such that the frequency range for the adaptive part of the strategy is limited. This reduces adaptation to unexpected high frequency dynamics and removes low frequency gain wind-up. In this paper we consider two examples of plants with unmodelled high frequency dynamics, both of which exhibit unstable behaviour when controlled using the standard MRAC strategy. By using the modified strategy we demonstrate that robustness is significantly improved.

1 INTRODUCTION

Two of the major challenges in the application of model reference adaptive control (MRAC) strategies are disturbances and plant uncertainty (Aström and Wittenmark, 1995; Sastry and Bodson, 1989; Landau, 1979; Popov, 1973). One effect of disturbances, such as transducer noise, is that control gains can 'wind-up' (Ioannou and Kokotovic, 1984; Virden and Wagg, 2005). An effective way to remove gain wind-up behaviour is to eliminate the inherent zero eigenvalue in the (localised) MRAC system by introducing a complementary low pass filter (Yang et al., 2006). Plants with unmodelled high frequency dynamics are one important case of plant uncertainty, and previous studies have shown how this can cause system instability in many real applications (Rohrs et al., 1985; Nikzad et al., 1996; Crewe, 1998; Neild et al., 2005b).

As an example of using MRAC on plants with higher order unmodelled dynamics, we consider the application of the MRAC to hydraulic shaking tables. Hydraulic shaking tables are widely used in the earthquake engineering community for dynamic testing of structures subjected to extreme loading. Adaptive control is desirable due to the changing dynamics of the test specimen attached to the table when exposed to extreme loading (Stoten and Gómez, 2001). Generally hydraulic actuators may be modelled as first order systems (Neild et al., 2005a), however attaching a large mass, such as the table and payload, to the actuator can lead to significant higher frequency dynamics due to oil column resonance (Nikzad et al., 1996; Crewe, 1998; Neild et al., 2005b).

In this paper we present a modified MRAC algorithm which uses complementary filters at both low and high frequency. We demonstrate that when this new modified MRAC algorithm is applied to systems with unmodelled high frequency dynamics a stable response can be achieved.

2 FORMULATIONS OF MODIFIED MRAC STRATEGY

In this section a brief introduction of ρ/ϕ modified MRAC algorithm is given for a single-input single-output (SISO) system. For more detailed discussions of standard MRAC can be found in (Landau, 1979; Sastry and Bodson, 1989; Aström and Wittenmark, 1995). The system studied in this paper is based on a first-order linear plant approximation given by the transfer function $G(s) = X(s)/U(s) = b/(s+a)$, where $X(s)$ is the plant state ($x(t)$ in the time domain),

$U(s)$ is the control signal and a and b are the plant parameters. The control signal is generated from the state variable and the reference (or demand) signal $r(t)$, using adaptive control gains K and K_r, such that $u(t) = Kx(t) + K_r r(t)$, where K is the feedback adaptive gain and K_r the feed forward adaptive gain. The plant is controlled to follow the output from a reference model $G_m(s) = X_m(s)/R(s) = b_m/(s+a_m)$, where X_m is the state of the reference model and a_m and b_m are the reference model parameters which are specified by the controller designer. The block diagram of MRAC is illustrated by Fig.1.

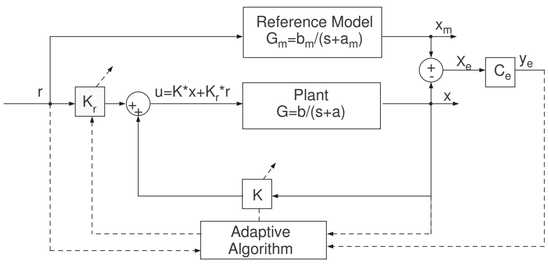

Figure 1: Schematic block diagram of the model reference adaptive control system. K and K_r are the adaptive gains generated using the MRAC algorithm.

The object of the MRAC algorithm is for $x_e \to 0$ as $t \to \infty$, where $x_e = x_m - x$ is the error signal. The dynamics of the system can be rewritten in terms of the error such that

$$\dot{x}_e = (-a+bK)x_e + b(K^E - K)x_m + b(K_r^E - K_r)r, \quad (1)$$

where K^E and K_r^E are Erzberger gains. The Erzberger gains are defined as the linear gains which results in the plant response matching the reference model response (Khalil, 1992);

$$K^E = \frac{a-a_m}{b}, \quad K_r^E = \frac{b_m}{b}. \quad (2)$$

For general model reference adaptive control, the adaptive gains are commonly defined by using Hyperstability rule (Popov, 1973), which is a proportional plus integral formulation

$$\begin{aligned} K(t) &= \alpha \int_0^t C_e x_e x(\tau) d\tau + \beta C_e x_e x(t) + K_0, \\ K_r(t) &= \alpha \int_0^t C_e x_e r(\tau) d\tau + \beta C_e x_e r(t) + K_{r0}, \end{aligned} \quad (3)$$

where α and β are adaptive control weightings representing the adaptive effort and K_0 and K_{r0} are the initial gain values. In the case of a first-order implementation, C_e is a scalar and therefore may be incorporated into the α and β adaptive control weightings.

2.1 Mrac with ρ Modification

The purpose of introducing the ρ modification to the MRAC algorithm is to resolve the problem of gain 'wind-up' observed using standard the MRAC strategy on plants with output disturbances. The modified adaptive gains $K_{m\rho}$ and $K_{rm\rho}$ are given by

$$\begin{aligned} K_{m\rho}(s) &= \tfrac{s}{s+\rho^2} K(s) + \tfrac{\rho^2}{s+\rho^2} K^*(s), \\ K_{rm\rho}(s) &= \tfrac{s}{s+\rho^2} K_r(s) + \tfrac{\rho^2}{s+\rho^2} K_r^*(s), \end{aligned} \quad (4)$$

where ρ is a constant, $K(s)$ and $K_r(s)$ are the standard adaptive control gains in the Laplace domain, and $K^*(s)$ and $K_r^*(s)$ are constant gains. This modification eliminates a zero eigenvalue in the localised error dynamics about the equilibrium point, replacing it with an eigenvalue of $-\rho^2$, hence making all the system eigenvalues asymptotically stable (Yang et al., 2006). The ρ modified MRAC can also be explained in terms of frequency response. A bode plot of Eq.4 is shown in Fig.2(a), we can see how the ρ term works as a low frequency filter on the adaptive gains and stops gain wind-up by pushing gains to fixed values. Experimental tests have demonstrated the effectiveness of ρ modified MRAC on preventing gain wind-up in a small scale motor-driven shaking table (Yang et al., 2006).

2.2 Mrac with ρ/ϕ Modification

In this paper we present an additional modification to MRAC through the use of an additional high frequency complementary filter. A ϕ term is introduced as the complementary filter to reduce adaptation to high frequencies, e.g. due to the unmodelled dynamics. This is illustrated in Fig.2(b).

The ρ/ϕ modified MRAC control gains may be described in the Laplace domain as

$$\begin{aligned} K_m(s) = \; & \tfrac{\phi^2 s}{(s+\rho^2)(s+\phi^2)} K(s) + \tfrac{\rho^2}{s+\rho^2} K^*(s) \\ & + \tfrac{s^2}{(s+\rho^2)(s+\phi^2)} K^*(s), \\ K_{rm}(s) = \; & \tfrac{\phi^2 s}{(s+\rho^2)(s+\phi^2)} K_r(s) + \tfrac{\rho^2}{s+\rho^2} K_r^*(s) \\ & + \tfrac{s^2}{(s+\rho^2)(s+\phi^2)} K_r^*(s), \end{aligned} \quad (5)$$

where ρ and ϕ are constants which need to be selected by the designer, and K^* and K_r^* are steady-state gains, ideally they are set to the values of the Erzberger Gains. K and K_r are the standard MRAC control gains.

By inspecting Eq.5, we note that the modified control gain K_m is made up of an adaptive part and a fixed

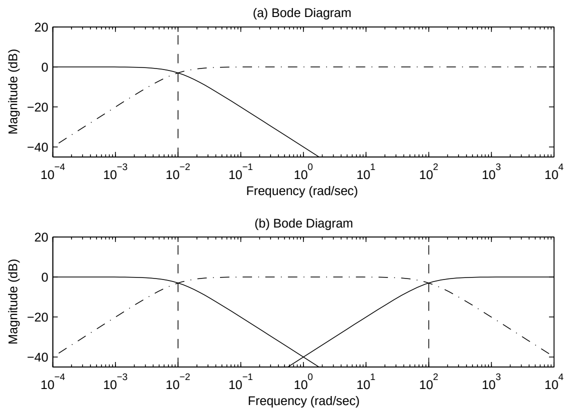

Figure 2: (a) ρ modified MRAC adaptive gain structure. Solid line represents the fixed gain control part, K^* or K_r^*, and dash-dot line represents gains adaptive part, K or K_r. The vertical dash line shows the value of ρ^2 corresponding to the complementary filters break point. (b) ρ and ϕ modified MRAC structure. Solid line represents the fixed gain control part K^* or K_r^*, and dash-dot line represents gains adaptive part K or K_r. Vertical dash line shows the value of ρ^2 and ϕ^2.

gain control part. The first term on the right hand side is the adaptive part, and the second and third terms are fixed gain control terms based on the constant steady-state gain $K^*(s)$. The same situation can be found in the modified gain K_{rm}. Given ρ and ϕ are non-zero real values, the fixed gain part of Eq.5 has all poles on left half plane, hence this part is stable. Now we focus on the stability of the adaptive part of modified control gains. By applying the Laplace transform given zero initial conditions to Eq. 3 we have

$$\frac{K(s)-K_0}{P_1(s)} = \frac{\beta s+\alpha}{s}, \quad \frac{K_r(s)-K_{r0}}{P_2(s)} = \frac{\beta s+\alpha}{s}, \quad (6)$$

where $P_1(s) = C_e X_e(s) X(s)$ and $P_2(s) = C_e X_e(s) R(s)$. We note that a zero pole exists which makes the transfer function marginally stable. Substituting $K(s)$ and $K_r(s)$ in Eq.5 by Eq.6, the adaptive part of Eq.5 becomes

$$\begin{aligned} \frac{K_m(s)-K_0}{P_1(s)} &= \frac{\phi^2(\beta s+\alpha)}{(s+\rho^2)(s+\phi^2)}, \\ \frac{K_{rm}(s)-K_{r0}}{P_2(s)} &= \frac{\phi^2(\beta s+\alpha)}{(s+\rho^2)(s+\phi^2)}. \end{aligned} \quad (7)$$

Comparing Eq.7 with standard MRAC control gains of Eq.6, we noticed that the zero pole in the standard MRAC control gain is replaced by two negative poles, given ρ and ϕ are non-zero real values, and this makes the control gains asymptotically stable.

Now we consider the overall transfer function path from the input signal r to error signal e. Given plant transfer function G and reference model transfer function G_m, the transfer function from reference signal r to plant output x can be written as (Aström and Wittenmark, 1995)

$$G_c(s) = \frac{X(s)}{R(s)} = \frac{K_r G}{1-KG}. \quad (8)$$

So the error signal x_e becomes $X_e(s) = [G_m(s) - G_c(s)]R(s)$, hence the transfer function from reference signal r to error signal x_e can be written as $X_e(s)/R(s) = G_m - G_c$, substituting $G_c(s)$ by Eq.8 and rearranging it we have

$$\frac{X_e(s)}{R(s)} = \frac{G_m - KG_mG - K_rG}{1-KG}. \quad (9)$$

Since the transfer function of plant is $G(s) = b/(s+a)$ and the transfer function of reference model is $G_m(s) = b_m/(s+a_m)$, Eq.9 can be calculated as

$$\frac{X_e(s)}{R(s)} = \frac{b(K_r^E - K_r)s + b_m b(K^E - K) + a_m b(K_r^E - K_r)}{(s+a_m)(s+a-bK)}. \quad (10)$$

Eq.10 represents the error response of the overall system. We notice there are two poles $-a_m$ and $bK - a$ in this transfer function. To make the overall system stable, we need to ensure both poles are on left half plane. Since a_m is defined as positive, the $-a_m$ pole is on left-half plane. To make $bK - a < 0$, the condition of $K < a/b$ need to be satisfied. We notice if $K = K^E = (a - a_m)/b$ this condition will always be satisfied.

As a further insight into Eq.5 and Eq.10, if $\rho = \infty$ and $\phi = 0$, Eq.5 will become $K_m = K^*$ and $K_{rm} = K_r^*$, which means the system will be completely controlled by fixed gains. Hence to increase ρ from 0 and decrease ϕ from infinite means to add weights on fixed gain control. In Eq.10 if $K = K^* = K_r^E$ and $K_r = K_r^* = K_r^E$ the error signal will become zero, which means the system has ideal response. We therefore set K^* and K^* to our best estimate of the Erzberger gains K^E and K_r^E.

3 MODIFIED MRAC APPLIED TO ROHRS EXAMPLE

Knowledge of the Erzberger gains, to set K^* and K^*, is important to the accuracy to the ρ/ϕ modified MRAC algorithm. In many practical situations the Erzberger Gains can not be estimated precisely and in some cases can only be crudely approximated. One such case is a plant with unmodelled high frequency dynamics, for example 'Rohrs model' (Rohrs et al., 1985). In this section we show how the modified MRAC algorithm copes with Rohrs example. The plant transfer function is given as

$$G(s) = \frac{2}{(s+1)} \frac{229}{(s^2+30s+229)}, \quad (11)$$

which is a nominally first order plant $2/(s+1)$ multiply by a second order unmodelled dynamics $229/(s^2+30s+229)$ which has almost critical damping, $\zeta = 1.02$. The plant thus has two poles $s = -15 \pm 2i$ neglected in the model used to design the adaptive controller. The reference model is given as

$$G_m(s) = \frac{3}{s+3}. \quad (12)$$

The initial conditions for both control gains K and K_r are zero. As in Rohrs example the input signal is set as

$$r(t) = 0.3 + 1.85\sin(16.1t), \quad (13)$$

much higher frequency than the nominal first order plant break frequency (1 rad/sec). Nominal Erzberger gains can be calculated according to Eq.2 as $K^E = -1$ $K_r^E = 1.5$. The α/β ratio is chosen as 1, which is the same as nominal plant break frequency.

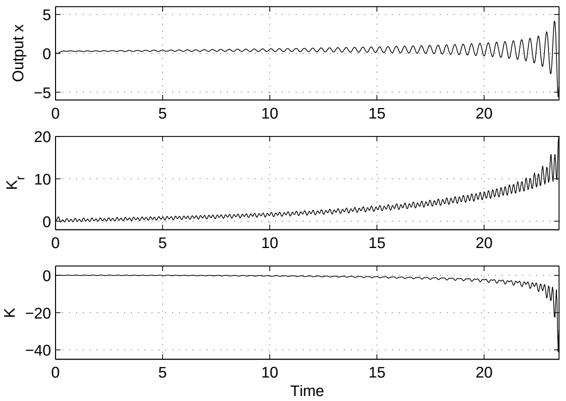

Figure 3: Standard MRAC with unmodelled dynamics (Rohrs model), input signal r(t)=0.3+1.85sin(16.1t), $\alpha = \beta = 1$. The system is unstable.

If the standard MRAC strategy is applied to the nominal first order plant, $G = 2/(s+1)$, the response is stable and the gains tend to the Erzberger values.

However if the higher frequency dynamics, as described by Eq.11 are included in the plant gain wind-up occurs which results in system instability. Fig.3 shows the system response with $\alpha = \beta = 1$, which results in system instability within 25 seconds.

Fig.4 shows the plant response, for the case where with high frequency dynamic are included, using ρ modified MRAC, $\rho^2 = 0.4$ and $\alpha = \beta = 1$. The modified strategy results in a stable response with no gain wind-up.

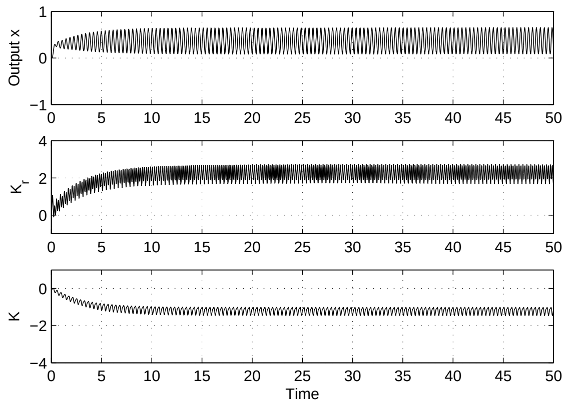

Figure 4: ρ modified MRAC with Rohrs model, input signal r(t)=0.3+1.85sin(16.1t), $\alpha = \beta = 1$, $\rho^2 = 0.4$. The system is stable.

4 MODIFIED MRAC APPLIED TO SHAKING TABLES

In this section, to demonstrate the difference in behaviour due to the ρ and the ϕ modifications, we consider the application of the MRAC strategy to control hydraulic shaking tables. Under general operating conditions, a large hydraulic shaking table used for earthquake tests will have a low frequency demand which is affected by high frequency dynamics due to oil column resonance. Typically system identification of hydraulic shaking tables over the low frequency operating range, around 0-10 Hz, results in a first order approximation to the system dynamics with the break frequency occurring within the operating range. However oil column resonance causes an unmodelled high frequency resonance with low damping, in the order of 10% of critical damping, (Nikzad et al., 1996; Crewe, 1998; Neild et al., 2005b).

To simulate this type of application, we will make the following changes to Rohrs example considered in the last section. Firstly, we change the demand signal frequency to 1 rad/sec, such that it coincides with the nominal plant break frequency:

$$r(t) = 0.3 + 1.85\sin(1t). \quad (14)$$

Secondly, we add white Gaussian noise to the plant output, resulting in an approximate signal to noise ratio of 20, to mimic transducer noise. Thirdly, we change the damping ratio of the higher frequency unmodelled dynamics to $\zeta = 0.1$ to represent the oil column resonance to give the overall plant transfer function:

$$G(s) = \frac{2}{(s+1)}\frac{229}{(s^2+3s+229)}, \quad (15)$$

where the nominal first order plant is still $2/(s+1)$, but the unmodelled dynamics becomes $229/(s^2 + 3s + 229)$. A Bode plot of the plant is given in Fig.5(a). The reference model and other conditions remain unaltered.

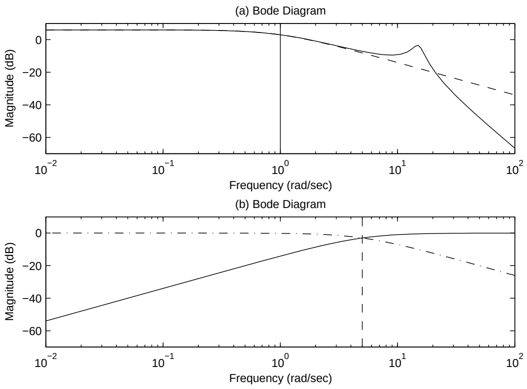

Figure 5: (a) Plant dynamics Bode plot: solid line shows the plant with unmodelled high frequency dynamics, the dash line is the nominal first order plant and the vertical line represents input signal operating frequency 1 rad/sec. (b) ϕ modified MRAC: dash-dot line represents adaptive part of the control gains K or K_r, solid line represents the fix part K^* or K_r^* and the vertical dash line represents the ϕ complementary filter break frequency; $\phi^2 = 5$ rad/sec.

As with Rohrs example, the standard MRAC strategy exhibits gain windup resulting in system instability when applied to the plant with higher frequency dynamics.

Fig.6 shows the control performance using ρ modified MRAC (with $\rho^2 = 0.5$ and $\alpha = \beta = 0.5$). We can see that, in contrast to Rohrs example, this system is still unstable despite the ρ modification. This is because the ρ modification is designed to removing windup rather than the gain oscillations that occur when the unmodelled higher order dynamics has low damping.

Fig.7 is the system response using the ϕ modified MRAC algorithm (with $\phi^2 = 5$ and $\alpha = \beta = 0.5$). The value of ϕ has been selected to reduce gain adaptation at the oil column resonance frequency of 11 rad/sec. The system is stable, with the error and both gains settle around 150 seconds. Comparing Fig.7 with Fig.6, we observe that ϕ plays a different role from ρ in the modified control algorithm. The ϕ modification results in filtering out the unmodelled high frequency dynamics directly to avoid the system adapting to these undesirable dynamics. In this example setting $\phi^2 = 5$ can minimise the gain adaptation to the oil column resonance, as illustrated by Fig.5(b) which shows the resulting complementary filters applied to the adaptive, K, and linear, K^*, gains.

Finally, Fig.8 shows the control result using the combined ρ/ϕ modified MRAC algorithm (with $\alpha = \beta = 0.5$, $\rho^2 = 0.5$ and $\phi^2 = 5$). The system has a stable response, with the error and gains settle within around 10 seconds – faster than when ϕ modified MRAC was used. The reason is that by increasing ρ the fixed gain contribution to the controller, which requires no time to settle, becomes more dominant.

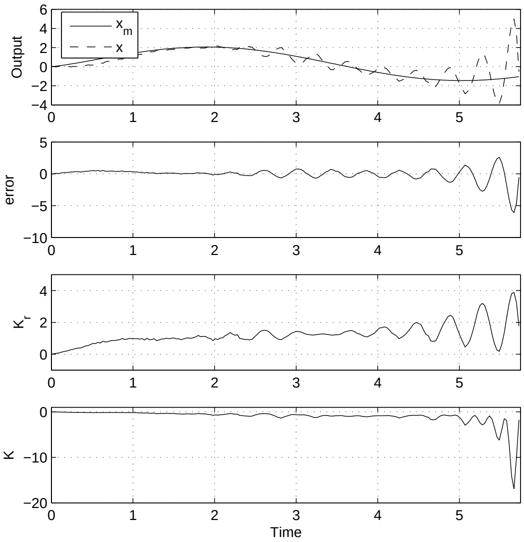

Figure 6: Plant with unmodelled high frequency dynamics, damping ratio 0.1, controlled by ρ modified MRAC. Input signal r(t)=0.3+1.85sin(1t), $\alpha = \beta = 0.5$, $\rho^2 = 0.5$. System is unstable.

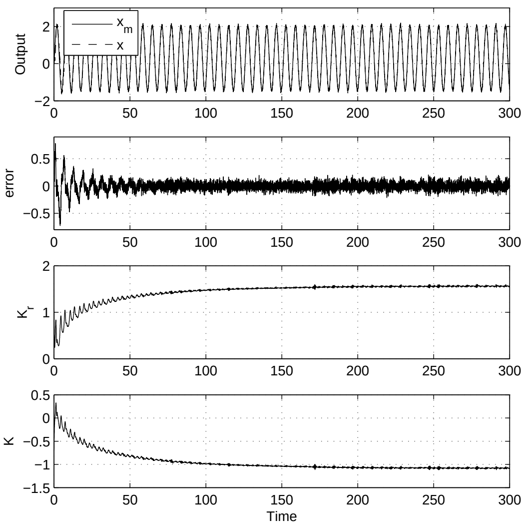

Figure 7: Plant with unmodelled high frequency dynamics, damping ratio 0.1, controlled by ϕ modified MRAC. Input signal r(t)=0.3+1.85sin(1t), $\alpha = \beta = 0.5$, $\phi^2 = 5$. System is stable. Error and gains settle within around 150 seconds.

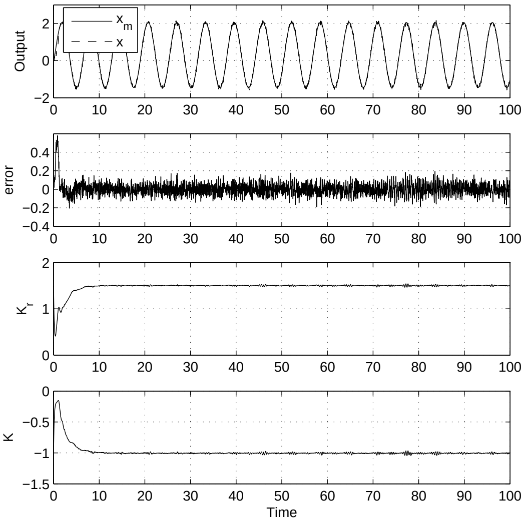

Figure 8: Plant with unmodelled high frequency dynamics, damping ratio 0.1, controlled by ρ/ϕ modified MRAC. Input signal r(t)=0.3+1.85sin(1t), $\alpha = \beta = 0.5$, $\rho^2 = 0.5$, $\phi^2 = 5$. System is stable. Error and gains settle within around 10 seconds.

5 CONCLUSION

In this paper we have introduced a ρ/ϕ modified MRAC strategy and tested it on plants with unmodelled high frequency dynamics. The modified MRAC strategy is made up of two parts, an adaptive control part and a fix gain control part. In the frequency domain, the ρ and ϕ modifications are first-order complementary filters which replace the adaptive gain with a fixed gain at low and high frequency respectively. Two types of unmodelled high frequency dynamics are considered. Firstly using Rohrs model, in which the unmodelled dynamics are almost critical damped, it was observed that ρ modified MRAC eliminated the gain wind-up. Secondly when the plant has lightly damped unmodelled dynamics case, similar to the oil column dynamics observed with hydraulic shaking table control, using ϕ modified MRAC prevents the system adapting to unmodelled high frequency dynamics, hence stabilizing the system. Simulation results show that ϕ modification results in filtering off the unmodelled high frequency dynamics directly to avoid the system adapting to these undesirable dynamics whereas the ρ modification eliminates gain wind-up. Hence the ρ/ϕ modified MRAC is a effective way to control systems with unmodelled high frequency dynamics.

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REFERENCES

Aström, K. J. and Wittenmark, B. (1995). *Adaptive control*. Addison-Wesley, second edition.

Crewe, A. (1998). *The Characterisation and Optimisation of Earthquake Shaking Table Performance.* PhD thesis, University of Bristol.

Ioannou, P. and Kokotovic, P. (1984). Instability analysis and improvement of robustness of adaptive control. *Automatica*, 20(5):583–594.

Khalil, H. (1992). *Nonlinear Systems*. Macmillan:New York.

Landau, Y. (1979). *Adaptive control:The model reference approach*. Marcel Dekker:New York.

Neild, S., Drury, D., and Stoten, D. (2005a). An improved substructuring control strategy based on the mcs control algorithm. *Proceedings of the I. Mech. E. Part I, Journal of Systems and Control Engineering*, 219(5):305–317.

Neild, S., Stoten, D., Drury, D., and Wagg, D. (2005b). Control issues relating to real-time substructuring experiments using a shaking table. *Earthquake Engineering and Structural Dynamics*, 34(9):1171–1192.

Nikzad, K., Ghaboussi, J., and Paul, S. (1996). Actuator dyanamics and delay compensation using neurocontrollers. *Journal of Engineering Mechanics*, 122-10:966–975.

Popov, V. (1973). *Hyperstability of control systems*. Springer.

Rohrs, C., Valavani, L., Athans, M., and Stein, G. (1985). Robustness of continuous-time adaptive control algorithms in the presence of unmodeled dynamics. *IEEE Trans. Automat. Contr*, AC-30:881–889.

Sastry, S. and Bodson, M. (1989). *Adaptive control : Stability, convergence and robustness*. Prentice-Hall : New Jersey.

Stoten, D. and Gómez, E. (2001). Real-time adaptive control of shaking tables using the minimal control synthesis algorithm. *Phil. Trans. R Soc. Lond. A.*, 359:1697–1723.

Virden, D. and Wagg, D. (2005). System identification of a mechanical system with impacts using model reference adaptive control. *Proc. IMechE. Pat I: J. Syst. Control Eng.*, 219:121–132.

Yang, L., Neild, S., Wagg, D., and Virden, D. (2006). Model reference adaptive control of a nonsmooth dynamical system. *Nonlinear Dynamics*, 46(3):323–335.