USING NOISE TO IMPROVE MEASUREMENT AND INFORMATION PROCESSING

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Abstract: This paper proposes a synthetic presentation on the phenomenon of stochastic resonance or improvement through the action of noise. Several situations and mechanisms are reported, demonstrating a constructive role of noise, in the context of measurement and sensors, data and information processing, with examples on digital images.

1 INTRODUCTION

In the context of measurement and information processing, it is progressively realized that noise is not always a nuisance, but that it can sometimes play a beneficial role (Wiesenfeld and Moss, 1995; Andò and Graziani, 2001; Chapeau-Blondeau and Rousseau, 2002; Chapeau-Blondeau and Rousseau, 2004). Such a constructive role of noise can be observed in diverse situations, through different cooperative mechanisms, assessed by various measures of performance to quantify the improvement. The term "stochastic resonance" is used as a common name to unify these situations where a measure of performance is improved to culminate (resonate) at a maximum when the level of noise is raised. In the recent years, many forms of stochastic resonance have been introduced and analyzed. Experimental observations of stochastic resonance have been obtained in many areas, for instance with electronic circuits, optical devices, neuronal processes, nanotechnologies.

In the present paper, we propose an overview of some basic mechanisms of stochastic resonance, including some recent ones, that we organize in a synthetic perspective. We specially focus on possible constructive role of noise for measurement and sensors, data and information processing. For illustration, we provide examples on digital images, a class of signals not so often considered for applying stochastic resonance, which gives us the opportunity of also showing new examples.

2 NOISE-SHAPED SENSORS

In this section, we show a constructive action of the noise that can be used to shape the input–output characteristic of devices and we give an example of application illustrated with saturating imaging sensors.

We consider a device with static nonlinear input–output characteristic $g(\cdot)$. This device is in charge of the transmission or the processing of an information carrying signal s so as to produce the output signal y with

$$y = g(s) , \qquad (1)$$

where input signal s and output signal y may be function of time or space. Let us assume that the input–output characteristic $g(\cdot)$ of our device is not optimally adapted to transmit or process the input signal s. Therefrom, one may look for another device with a more suitable input–output characteristic shape. As an alternative, we are going to show that it is also sometimes possible to modify the input–output characteristic $g(\cdot)$ of such a device without having to change any physical parameter of the device itself.

We introduce a noise η in the input–output relation of Eq. (1) which becomes $y = g(s+\eta)$. This noise η can be a native noise due to the physics of the device or a noise purposely injected to the input of the device. Then, since the device is no longer deterministic, an effective or average input–output characteristic can be defined as $g_{\text{eff}}(\cdot)$ given by the expectation

$$g_{\text{eff}}(s) = \mathrm{E}[y] = \int_{-\infty}^{+\infty} g(u) f_\eta(u-s) du , \qquad (2)$$

with $f_\eta(u)$ the probability density function of the noise η. In presence of the noise η, the shape of the device input–output characteristic, $g_{\text{eff}}(\cdot)$ in Eq. (2), is now controlled by $g(\cdot)$ and by the noise probability density function $f_\eta(u)$. Therefore, a modification in the response of a memoryless device can be obtained thanks to the presence of a noise η which makes it possible to shape the input–output characteristic of the device without changing the device itself.

In practice, the modified input–output characteristic $g_{\text{eff}}(\cdot)$ of the device in Eq. (2) is not directly available. Yet, it is possible to have a device presenting an approximation of the response of Eq. (2) by averaging N acquisitions y_i with $i \in \{1, \dots N\}$ to produce

$$y = \frac{1}{N}\sum_{i=1}^{N} y_i = \frac{1}{N}\sum_{i=1}^{N} g(s+\eta_i)\ , \qquad (3)$$

where the N noises η_i are white, mutually independent and identically distributed with probability density function $f_\eta(u)$. Practical implementation of the process of Eq. (3) can be obtained, as proposed in (Stocks, 2000) for 1-bit quantizers, via replication of the devices associated in a parallel array where N independent noises are added at the input of each device or, as proposed in (Gammaitoni, 1995) for a constant signal, by collecting the output of a single device at N distinct instants. Similarly to what is found in Eq. (2), the input–output characteristic of the process of Eq. (3) is shaped by the presence of the N noises η_i. Because of Eq. (3), one has first

$$\mathrm{E}[y] = \mathrm{E}[y_i]\ , \qquad (4)$$

and also one has for any i

$$\mathrm{E}[y_i] = \int_{-\infty}^{+\infty} g(s+u) f_\eta(u) du\ . \qquad (5)$$

The N noises η_i bring fluctuations which can be quantified by the nonstationary variance $\mathrm{var}[y] = \mathrm{E}[y^2] - \mathrm{E}[y]^2$, with $\mathrm{E}[y^2] = \mathrm{E}[y_i^2]/N + \mathrm{E}[y]^2(N-1)/N$ and

$$\mathrm{E}[y_i^2] = \int_{-\infty}^{+\infty} g^2(s+u) f_\eta(u) du\ . \qquad (6)$$

For large values of N, $\mathrm{var}[y]$ tends to zero. In these asymptotic conditions where N tends to infinity, the process constituted by the device $g(\cdot)$, the N noises η_i and the averaging of Eq. (3), becomes a deterministic equivalent device with input–output characteristic given by Eq. (2). For finite values of N, the presence of N noises η_i will play a constructive role if the improvement brought to the transmission or processing of the input signal s by the modification of the device characteristic is greater than the nuisance due to the remaining fluctuations in y.

The process of Eq. (3) delimits a general problem: given a device characteristic $g(\cdot)$ and a number N of averaging samples, how can one choose the probability density function of the noises η_i to obtain a targeted characteristic response. This inverse problem is, in general, difficult to solve. A pragmatic solution, inspired from the studies on stochastic resonance, consists in fixing a probability density function for the noises η_i and to act only on the rms amplitude σ_η of these identical independent noises.

For illustration, we now give an example of application of the process of Eq. (3). We consider devices with input–output characteristic $g(u)$ presenting a linear regime limited by a threshold and a saturation

$$g(u) = \begin{cases} 0 & \text{pour} \quad u \leq 0 \\ u & \text{pour} \quad 0 < u < 1 \\ 1 & \text{pour} \quad u \geq 1\ . \end{cases} \qquad (7)$$

The possibility of shaping the response of such devices by using the process of Eq. (3) is shown in Figure 1. The noises η_i injected in Eq. (3), arbitrarily chosen Gaussian here, tend to extend the amplitude range upon which the effective input–output characteristic $g_{\text{eff}}(\cdot)$ of Eq. (2) is linear.

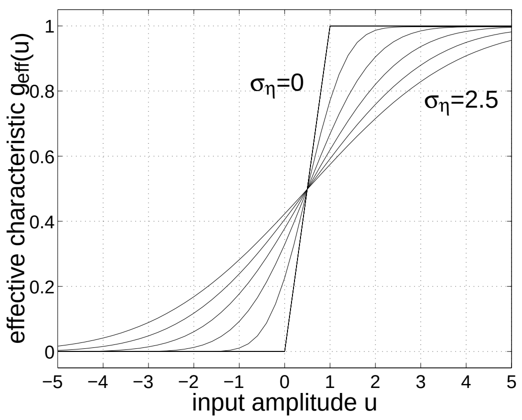

Figure 1: Effective input–output characteristic $g_{\text{eff}}(\cdot)$ of Eq. (2) for the device of Eq. (7) in presence of Gaussian centered noise with various rms amplitude $\sigma_\eta = 0, 0.5, 1, 1.5, 2.5$.

The input–output characteristic $g(u)$ of Eq. (7) can constitute a basic model for measurement sensors. In the domain of instrumentation and measurement, a quasi-linear behavior associated to perfect reconstruction is sought. Nevertheless, sensor devices are usually linear for moderate inputs but can present saturation at large inputs or/and a threshold for small inputs. Such behaviors at large and small inputs induce distortions degrading the quality of the signal transmitted by these sensor devices. Therefore, the linear regime of the input–output characteristic of a saturating sensor usually sets the limit of the signal dynamic to be transmitted with fidelity. In this measurement framework, the process of Eq. (3) can be used to widen the dynamic of sensors presenting threshold and saturation like in Eq. (7).

For further illustrations, we consider the case where Eq. (7) models the response of an imaging sensor (CCD, retina, or even photographic films). We assume that, although all physical parameters (for example luminance of the scene, time exposure, imaging sensor sensibility) have been adjusted to their best, the image s submitted to Eq. (7) still undergoes saturation by $g(\cdot)$ and is therefore over-exposed. The possibility of a noise–widened dynamic of this imaging sensor can be visually appreciated in Figure 2. Fluctuations due to the presence of noises η_i in Eq. (3) are decreasing with increasing N. Nevertheless, in some cases, as perceptible in Figure 2c, the sensor device can benefit from the presence of the noise even with $N = 1$ in the process of Eq. (3). Also, the noise–widened visual improvement of the transmitted image, can be quantitatively assessed, in Figure 3, by the normalized cross-covariance between the original image and the transmitted image.

3 NOISE-ASSISTED INFORMATION TRANSMISSION

We now move to a higher information-processing level, with statistical quantification of information, and possible connection to pattern recognition tasks, for another example of a constructive role of noise. Consider a binary image with values $s(x_1,x_2) \in \{0,1\}$ at spatial coordinates (x_1,x_2). The detector $g(\cdot)$ is taken as a hard limiter with threshold θ,

$$g(u) = \begin{cases} 0 & \text{for} \quad u \leq \theta \\ 1 & \text{for} \quad u > \theta\,, \end{cases} \qquad (8)$$

and delivers the output image $y(x_1,x_2) = g[s(x_1,x_2) + \eta(x_1,x_2)]$, with independent noise $\eta(x_1,x_2)$ at distinct pixels $(x_1,,x_2)$. When the detection threshold θ is high relative to the values of the input image $s(x_1,x_2)$, i.e. when $\theta > 1$, then $s(x_1,x_2)$ in absence of the noise $\eta(x_1,x_2)$ remains undetected as the output image $y(x_1,x_2)$ remains a dark image; thus, with no noise, no information is transmitted from $s(x_1,x_2)$ to $y(x_1,x_2)$. From this situation, addition of the noise $\eta(x_1,x_2)$ allows a cooperative effect where $s(x_1,x_2)$ and $\eta(x_1,x_2)$ cooperate to overcome the detection threshold. This translates into the possibility of increasing and maximizing the information shared between $s(x_1,x_2)$ and $y(x_1,x_2)$ thanks to the action of the noise $\eta(x_1,x_2)$ at a nonzero level which can be

Figure 2: Noise–widened dynamic of an imaging sensor with response given by Eq. (7). (a) a 8-bit version of the "lena" image correctly exposed with 256 grey-levels coded between 0 and 1; (b) over-exposed transmitted image in absence of noise. The over-exposure is controlled by an exposure parameter k_{ex}, which is a constant added to all the pixel value of the original image before the process of Eq. (7); (c) transmitted image with the same exposure parameter $k_{ex} = 0.75$ but with the presence of zero-mean Gaussian noise of rms amplitude $\sigma_\eta = 0.3$ with $N = 1$ acquisition averaged; (d),(e),(f) same conditions but with $\sigma_\eta = 0.1, 0.2, 0.4$ with $N = 3, 7, 63$.

optimized. The effect can be precisely quantified by means of a Shannon mutual information $I(s,y)$ between $s(x_1,x_2)$ and $y(x_1,x_2)$, definable as

$$I(s,y) = H(y) - H(y|s)\,. \qquad (9)$$

With the function $h(u) = -u\log_2(u)$, the entropies in Eq. (9) are

$$H(y) = h(p_{00}p_0 + p_{01}p_1) + h(p_{10}p_0 + p_{11}p_1) \quad (10)$$

and

$$H(y|s) = p_0[h(p_{00}) + h(p_{10})] + p_1[h(p_{01}) + h(p_{11})]\,. \qquad (11)$$

In Eqs. (10)–(11) one has the probabilities $\Pr\{s = 1\} = p_1 = 1 - p_0$ determined by the input image

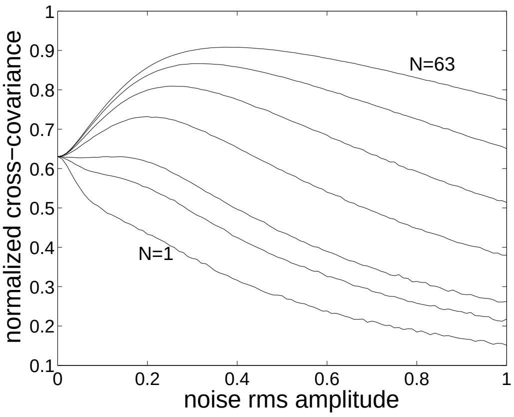

Figure 3: Normalized cross-covariance of original "lena" image correctly exposed with the image transmitted by the process of Eq. (3) as a function of the level of the noise σ_η for various number of acquisitions $N = 1, 2, 3, 7, 15, 31, 63$. The exposure parameter $k_{ex} = 0.75$ is the same as in Figure 2.

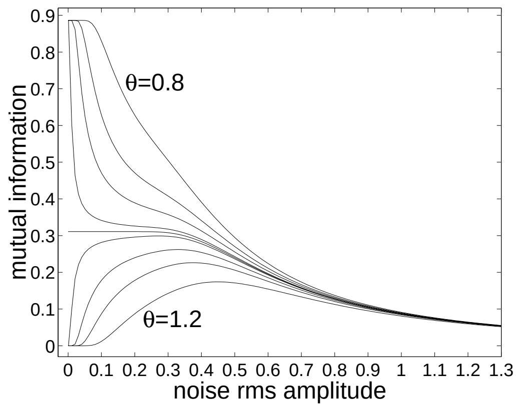

Figure 4: Mutual information $I(s,y)$, as a function of the noise rms amplitude σ_η, in succession for the threshold $\theta = 0.8, 0.9, 0.95, 0.99, 1, 1.01, 1.05, 1.1$ and 1.2.

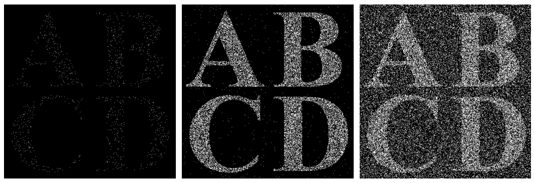

Figure 5: Output image $y(x_1,x_2)$ with threshold $\theta = 1.1$ and noise level $\sigma_\eta = 0.05$ (left), 0.4 (middle) and 1.3 (right).

$s(x_1,x_2)$, and

$$p_{0s} = 1 - p_{1s} = \Pr\{y = 0 \,|\, s\} = F_\eta(\theta - s) \qquad (12)$$

determined by the noise $\eta(x_1,x_2)$ via its cumulative distribution function $F_\eta(\cdot)$.

For a 410×415 binary image with $p_1 = 0.3$, Figure 4 shows typical evolutions for information $I(s,y)$ as a function of the rms amplitude σ_η of the noise $\eta(x_1,x_2)$ chosen zero-mean Gaussian. In Figure 4, when $0 < \theta < 1$ the noise is felt only as a nuisance, and information $I(s,y)$ is maximum at zero noise and decreases when σ_η grows. Meanwhile, when $\theta > 1$ no information is transmitted in the absence of noise, and it is the increase of the noise level σ_η above zero which authorizes information $I(s,y)$ to grow in order to culminate at a maximum for a nonzero optimal amount of noise maximizing information transmission.

This noise-aided information transmission quantified in Figure 4 can be visually appreciated in Figure 5, with the optimal noise configuration in the middle image.

This example illustrates one basic form of noise-aided information processing, in which the noise has a constructive influence in a binary decision in the presence of a fixed discrimination threshold. This basic form can find applicability in many areas, and can be elaborated upon in many directions. It can be related to the effect of dithering known in imaging at low processing levels (Gammaitoni, 1995). It can also be related to threshold nonlinearities found in biophysical sensory processes, for instance originating in the retina for visual perception (Patel and Kosko, 2005). At high processing levels, the effect is relevant to pattern recognition in human vision (Piana et al., 2000).

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