THE STRATEGIC GAMES MATRIX AS A FRAMEWORK FOR INTELLIGENT AUTONOMOUS AGENTS HIERARCHICAL CONTROL STRATEGIES MODELING

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Abstracts: This paper presents a framework for strategy formulation in multilevel multiple-agent control system architectures based on the Strategic Games Matrix (SGM), having game theory and control systems theory as basic concepts and models. New methodologies for analysis and for design of hierarchical control architectures with multiple intelligent autonomous agents, based on the SGM concept, are applied. Illustrative hierarchical control applications to system architectures analysis and synthesis based on the SGM are presented.

1 INTRODUCTION

The study of hierarchical multi-agent control systems is receiving growing attention within the control community. Driving applications of multiple agents control include: mobile robots coordination and control, satellite clusters, automated highways, unmanned aerial vehicles (UAV), distributed artificial intelligence, and strategic planning in general.

A wide diversity of multi-controller and coordination problems has been treated recently, e.g., multiple mobile agents moving coordination and control (Shi, Wang and Chu, 2005), traffic congestion control (Alpcan and Başar, 2002), multiple mobile robot control (Shao, Xie, Yu and Wang, 2005), collision avoidance scheme in navigation control (Dimaragonas and Kyriakopoulus, 2005), secure routing in communication networks (Bohacek, Hespanha and Obraczka, 2002), optimal bidding strategies in the electricity market (Rahimi-Kian, Tabarraei and Sadeghi, 2005), automa-teams coordination and control (Liu, Galati and Simaan, 2004), attack and deception strategies in military operations (Castañón, Pachter and Chandler, 2004), and intrusion detection in access control systems (Alpcan and Başar, 2004).

Mathematical approaches used in these papers treat the control problems as Nash, Pareto, Stackelberg, Minimax games, or some variations of them, in an insulated manner.

The formulation of optimal strategies in competitive and/or cooperative environments has constituted one of the main challenges for researchers and scholars (Schelling, 1960; Brandenburger and Nalebuff, 1995; and Bottura and Costa, 2004) and a wide variety of approaches has been proposed and used (Başar and Older, 1999; Costa F^{o}., 1992; and Cruz Jr., 1978). However, a structured combination of all these possible approaches on the *same* hierarchical architecture should be conceived, formulated, and should have its usefulness exhibited. Here, an integrated framework considering these classical games on the same analytical structure, by going a step further on the traditional approach used in papers like the above mentioned, is presented.

In this paper, an 'agent' represents a controller, a decision-maker, a commander, an autonomous robot, a player – person or team –, software, a policy-maker, a UAV, a stakeholder, or any human being. Our approach treats hierarchical, non-hierarchical, or heterarchical architectures as a structured collections of sub-games.

2 STRATEGIC GAMES MATRIX

The concepts, formulations and results from *non-cooperative dynamic game theory* (Başar and Olsder, 1999) open new possibilities as conceptual platform for optimal strategy formulation.

In generic conflict of interests' situations, the description and mapping of a particular cooperative or competitive confrontation between two or more *players* can be accomplished with only two dimensions: the '*player posture assumption'* and the 'player *power-ratio assumption*'. They are used to build a (3x3) matrix called *strategic games matrix* (SGM) (Costa and Bottura, 2006): The matrix horizontal axis represents the player postures assumptions: as *rival*, or *individualistic,* or *associative* and, on the vertical axis represents the *player power-ratio assumptions*: as *hegemonic*, or *balanced,* or *weak*, as shown in Figure 1.

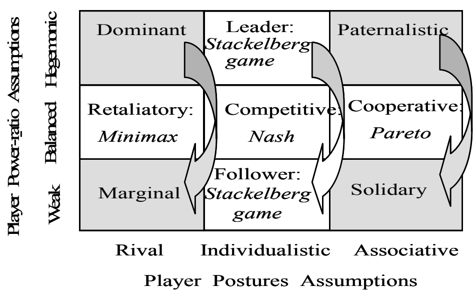

Figure 1: Typical strategic positions on the SGM highlighting, in gray, the two hierarchical limit-case strategic games.

These nine resulting strategic positions, at each of the nine matrix's cells, are named, respectively: *Dominant*, *Leader*, *Paternalistic*, *Retaliatory*, *Competitive*, *Cooperative*, *Marginal*, *Follower*, and *Solidary*, which are words that represent each one of the typical competitive confrontation strategic positions players may explicitly or implicitly adopt in a conflict of interests situation. In subsections 2.1 to 2.4 the five strategic positioning to which classic equilibrium strategies apply - *Minimax, Nash, Pareto*, for non-hierarchical games, and *Stackelberg,* for hierarchical games - and the respective situations where they normally occur, are described (Başar and Olsder, 1999; Costa F^o., 1992).

In subsections 2.5 and 2.6, the four special limit-cases strategic positions, representing two hierarchical games, not well covered by classic equilibrium strategies from game theory, here called *Dominant-Marginal*, and *Paternalistic-Solidary*, are presented in the next Sections. (The formal concept of dynamic games, of *equilibrium point* and of *equilibrium strategy* here used can be found in (Başar and Older, 1999)).

2.1 Retaliatory Games - Minimax

This strategic positioning applies to *lose-win* type games - at the left-center SGM cell -, where the players assume, explicit or implicitly, that a gain for one implies in losses to the remainder, characterizing a *retaliatory game*. For a zero-sum game, a solution, if it exists, for which each player acts towards what it understands as the most favorable to optimize its own objective function, considering all the possibilities the others could do, is called a *saddle-point*. This point has the peculiar characteristic that any deviation from it, by any of the players, makes its result worsen in relation to its objective function. For N players, a strategic decision $\hat{u}^i \in U^i$ by each player P_i is defined as a *saddle-point equilibrium solution* if, for every admissible set $\{u^1,\dots,u^i,\dots,u^N\} \in U$, the following relation is valid:

$$\max_{u^1,\dots,u^{i-1},u^{i+1},\dots,u^N} J_i(u^1,\dots,u^{i-1},\hat{u}^i,u^{i+1},\dots,u^N) \le .$$
$$\le \max_{u^1,\dots,u^{i-1},u^{i+1},\dots,u^N} J_i(u^1,\dots,u^N)$$

This strategy applies also to real situations where a player $\boldsymbol{P}_i$ can imagine that another player may have non-rational or erratic behavior, or even malicious, i.e., that an adversary may make moves to 'damage' $\boldsymbol{P}_i$'s objectives.

2.2 Competitive Games - Nash

The strategic position at the center-center SGM cell, named here as C*ompetitive*, describes situations of 'perfect competition', or 'free market', with many suppliers, where none of them is capable of dominating the remainders. In the non-cooperative variable-sum games, where a player decides to play a competitive strategic game, it seeks to optimize its objective function ignoring what the other players are doing or intending to do. If this solution exists, it is characterized by the situation where none of the players is able to improve its result by changing only its own decision-control. Such set of decisions is the *Nash equilibrium point*, defined below: A *Nash equilibrium point*

$$\hat{u}^* = (\hat{u}^1, . . . ,\hat{u}^i, . . . ,\hat{u}^N) \in U ,$$

if it exists, for a non-cooperative game, with K=1, and variable sum, with N players, is defined if, for all $u^i \in U^i$, $i \in N$, it obeys simultaneously the N following objective function inequalities:

$J_1(\hat{u}^1,\ldots,\hat{u}^i,\ldots,\hat{u}^N) \leq J_1(u^1,\ldots,\hat{u}^i,\ldots,\hat{u}^N)$, ... ,

$J_i(\hat{u}^1,\ldots,\hat{u}^i,\ldots,\hat{u}^N) \leq J_i(\hat{u}^1,\ldots,u^i,\ldots,\hat{u}^N)$, ... ,

$J_N(\hat{u}^1,\ldots,\hat{u}^i,\ldots,\hat{u}^N) \leq J_N(\hat{u}^1,\ldots,\hat{u}^i,\ldots,u^N)$.

2.3 Cooperative Games – Pareto

For variable-sum games - at the right-center SGM cell - the cooperation among players may lead to results - for all of them - that are better than those they would obtain if each one tries to optimize its objective function without an *a priori* knowledge of other's decisions. When players decide to share information on the respective constraints and conditions, alternative actions and objective functions, it is possible for them to find a point of equilibrium, the '*Pareto optimum*', which is 'the best' possible for all players. This point, if it exists, is characterized by the fact that none of the players can improve its result without, with its action, harming the other's results. These are the so called 'win-win games'. This type of game requires good faith and loyalty among all participants. For a variable-sum cooperative game (K=1) with **N** players, the point $\hat{u}^* = (\hat{u}^1, \cdots, \hat{u}^i, \cdots, \hat{u}^N) \in U$ is defined as a *Pareto optimum* if there is no other point

$u = (u^1,\ldots,u^i,\ldots,u^N) \in U$ such that

$J_i(u^i) \leq J_i(\hat{u}^i)$, $\forall i \in N$.

This condition requires that $J_i(u^i) \leq J_i(\hat{u}^i)$, $\forall i \in N$, only if $J_i(u^i) = J_i(\hat{u}^i)$, $\forall i \in N$, with a strict inequality for at least one $i \in N$.

2.4 Leader-Follower Stackelberg Games

The strategies applicable to hierarchical games with a strongest player, the *leader*, and another weaker player, the *follower*, are called *Stackelberg strategies* and correspond to two opposed positions: center-upper and center-lower SGM cells. Consider a simplified *hierarchical game* between a player *M*, called *leader*, and a player *P*, called *follower,* with strategic decisions λ and u , and objective functions $R(\lambda,u)$ and $J(\lambda,u)$, associated to players *M* and *P*, respectively (Haimes and Li, 1988; Costa F^o. and Bottura, 1990, 1991). Let us suppose also that, by the structure and rules of the game, player *M* selects first its strategic decision λ and, then, player *P* selects its strategic decision u , *knowing beforehand* the *M*'s decision. The pair $(\hat{\lambda},\hat{u}) \in (L,U)$, if it exists, defines a *Stackelberg equilibrium point* for which:

(a) There is a transformation $T : L \rightarrow U$ such that, for any given $\lambda \in L$, $J(\lambda, T\lambda) \leq J(\lambda, u)$ for every $u \in U$, and (b) There is a $\hat{\lambda} \in L$ such that $R(\hat{\lambda}, T\hat{\lambda}) \leq R(\lambda, T\lambda)$ for every $\lambda \in L$, where $\hat{u} = T\hat{\lambda}$. Note that, to obtain a *Stackelberg equilibrium point*, it is necessary that the *follower* be a rational agent, always making optimal decisions under its own game limitation. For this game structure, one can determine a pair of *Stackelberg* strategies - for the *leader* and for the *follower* - typically applied to situations of conflict of interests between a very strong player and another very weak, both with *individualistic* concurrent assumptions.

2.5 Dominant-Marginal Games

The Dominant-Marginal games are played by two players in two hierarchical antagonist strategic positions, both with rival posture assumption:

(1) Dominant strategic position: A *Dominant strategic position* - at the left-upper SGM cell - characterizes the player which has all strength and has the intention of destroying the smaller competitors. Its attitude may be of intimidation, blackmail, price war, for instance, to try to bankrupt the small ones. It may pressure its clients not to purchase from the small ones. A *Dominant equilibrium point* limit-case for this game can be obtained through the solution of a mono-criterion stochastic optimization problem in which the player in *Dominant position* ignores all the objective functions of its 'small' opponents and simply optimizes its own objective function. The player at a *Dominant* position could treat the possible actions of 'small' competitors simply as random noises.

(2) Marginal strategic position: Countering the *Dominant position* as described above, is the *marginal strategic position* - at the left-lower SGM cell -, where a weaker however courageous and competitive player in the game does everything it understands as necessary to survive, trying, as much as possible, to obtain some advantages upon causing losses to the major game dominator. A *marginal equilibrium point* limit-case for this game can be obtained through the solution of an optimization problem in which the *Marginal* position player, for instance, instead of minimizing, tries to *maximize* the main and stronger competitor's objective function

with the purpose of infringing upon it the maximum possible damage.

2.6 Paternalistic-Solidary Games

This game is played also by two players in two hierarchical antagonist strategic positions, both with associative posture assumption:

(1) Paternalistic strategic position: The *paternalistic strategic position* - at the upper-right SGM cell - occurs in games where a more powerful player, by its own decision, shapes its own actions and those of the remaining weaker players in the game, seeking preservation and development of the system as a whole. It is a game similar to the situation of a family father, supposed to have complete authority over the small children: he does all he comprehends to be necessary to promote the development, growth and harmony within his family, in a paternalistic way. A *paternalistic equilibrium point* limit-case game can be found as follows: Let $0 \leq \alpha_i \leq 1$ be a relative importance weight for the player $\mathbf{P_i}$ such that $\sum_{i=1}^{N} \alpha_i = 1$, and let

$z = \sum_{i=1}^{N} \alpha_i J_i(\ldots)$ be a multi-criteria objective function, encompassing all the objective functions of all the N players, the new function to be optimized. A *paternalistic equilibrium point* for this limit-case game can be found as a solution to a multi-criteria optimization problem (Bryson and Ho, 1975) where the new objective function is a linear combination of all the objective functions for all players. Otherwise, the *Paternalistic* player should take in account, on its decision, the 'risk' of a *Solidary* player decision for an alternative *solitary strategy*, leaving the game.

(2) Solidary strategic position: In opposition to the *paternalistic position* described above is the *Solidary position* - at the right-lower SGM cell -, that represents the situation of a player, in a game, in a weaker, however associative position which, without the power to impose its interests upon the others, seeks to follow the rules established by the 'ruling power', looking for some individual advantage. Otherwise it prefers to leave the game. This is how a member behaves in relation to its cooperative organization: it simply needs to decide whether it should join the 'collective' and obtain some advantage or, alternatively, it should rather act on its own. A *solidary equilibrium solution* for this limit-case game can be treated as a simple *decision tree problem* with only two branches, representing the alternative decisions: '*join the collective*', or '*work alone*'.

3 HIERARCHICAL GAMES

Departing from classic concepts and formulations from *dynamic game theory*, a formal conceptual platform for multilevel multiple decision-control problem formulation is built. A *deterministic dynamic game* (DDG) with several participants and multiple stages can be modeled as a systems optimization problem with multiple decentralized and autonomous decision-makers, called the '*players' –or intelligent autonomous agents.* From the point of view of *systems control theory*, a DDG is associated with a particular problem of *optimal control* with *multiple intelligent autonomous controllers, or agents* (Bryson and Ho, 1975).

In this type of games, each one of the N agents - or *players* - receiving information progressively disclosed by the structure of the game and considering the possible decisions of other agents, makes a sequence of decisions, stage by stage, attempting to optimize one's *objective function* - while obeying the game constraints. For a formal presentation of the optimization problem introduced above, let us adopt the notation derived from the terminology of *systems theory* (Başar and Olsder, 1999). Hierarchical architectures games with two levels, designed by HG2, and with three levels, designed by HG3, for multiple intelligent autonomous agents control strategies, are here described. A two-level hierarchical game, HG2, can be modeled through a similar process of forming a group of subsystems, each one representing a competing agent – for instance, a company. Each company - the i^{th} - here represented by a subsystem CS_i, vies in the market for raw materials, specialized production manpower, managerial resources, financial resources, technology, and other supplies. On the other hand, it also competes in the market for clients' preferences. The market, in the broader sense, also interferes in the game, acting upon prices and quantities transacted by the N agents with their clients and providers. The formulation of this concept can be obtained through a convenient partition and segmentation process of the DDG game: The HG2 is formed by two types of subsystems: the ***Companies Subsystems, CS_i,*** and the ***Market Coordinator Subsystems, MCS***. The CS_i modules communicate with the market coordinator subsystem, MCS, which informs to each one of them, at the beginning of each new period, its decision parameter. The CS_i, in turn, informs the MCS about their coordinated decisions for the next period. The dynamic hierarchical game HG-2 can be similarly expanded applying to each subsystem CS_i a segmentation process, where each i^{th} competing agent is assumed to consist of *G Managerial Units*, MU_{ij}, where $j \in \{1, 2, ..., G\}$, introducing *G* new intelligent autonomous agents for each company. These managerial units, MU_{ij}, represent the main functional or managerial areas of the company. In this sense, each MU_{ij}, as any intelligent autonomous agent, has its own state transition equation, information structure, strategy, decision, and specific objective function to be optimized. Therefore, the segmentation described produces a *three-level hierarchical game* HG-3 wherein the coordination, at the second level, is achieved by a new module called CSC_i, representing the coordination of all the MU_{ij}, by the i^{th} company's chief executive.

4 SGM APPLICATIONS

Let us apply, now, with illustrative purposes, the SGM methodology for a complex structure analysis to some HG-3 structured games.

4.1 Structure with One Coordinator

Suppose a complex business-economic structured system, with three decision hierarchical levels. Proceeding accord to this methodology the following results can be obtained:

(A) The four sub-games identified are: $\{CS_1,\ldots,CS_i,\ldots,CS_N\}$ competing - or cooperating - sub-game; $\{MU_{i1},\ldots, MU_{ij}, MU_{iG}\}$ competing - or cooperating - subgame; $\{MCS, CS_i\}$ hierarchical coordination sub-game; $\{CSC_i, MU_{ij}\}$ hierarchical coordination sub-game.

(B) The application of one or another equilibrium strategy on each specific sub-game depends on each particular situation of conflict of interests and on the postures and assumptions present in each case:

(i) The competitive sub-game among CS_i companies could be treated as a game where the agents are supposed to work in a *variable-sum* objective function environment, acting independently from each other and prevented from sharing information and from cooperating with each other. They are forbidden to make coordinated decisions to optimize together their objective functions; consequently, for this sub-game, the *Nash equilibrium strategy* is the applicable, as in subsection 2.2.

(ii) Among those responsible for the MU_{ij} Managerial Units on the same company, a sub-game is played where the agents aim to optimize a *variable-sum objective function* for which cooperation among the unit managers in charge is expected; hence, for this sub-game, the *Pareto equilibrium strategy* is the applicable, as in subsection 2.3.

(iii) The relationship between the agent MCS, the market coordinator, representing the market action, and each CS_i company could be interpreted as a sub-game with hierarchical coordination among them; therefore, the *Stackelberg equilibrium strategies* pair is applicable, considering the market coordinator as the *Leader* and each CS_i as a *Follower*, as in subsection 2.4;

(iv) The relationship between the agent CSC_i, internal coordinator of each company, and each MU_{ij} could be considered as a hierarchical coordination sub-game; so, the Stackelberg equilibrium strategy pair is applicable, considering the coordinator CSCi as the Leader and each MUij as a Follower, as in subsection 2.4.

(C) The structured mapping resulting from the fourth stage, easy to obtain in this case, is also indicated in Figure 2. Classic ways of solving these types of *optimal control problems* could use, for instance, *Pontryagin's Minimum Principle*, or *Calculus of Variations,* or *Dynamic Programming* (Bryson and Ho, 1975), depending on the case.

4.2 Structure with Two Coordinators

This subsection presents, in a summarized form, another illustrative application of this methodology for analysis of another type of hierarchic structure. Let us take the former HG-3 as a basis and introduce a second coordinator agent at the first level, as shown in Figure 2.

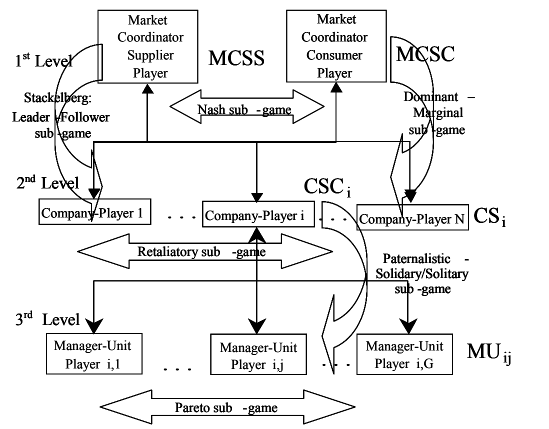

Figure 2: Game equilibrium strategies applied to a three-level multiple decision control architecture with two coordinators.

This structure has now two market coordinators, one representing the *market coordinator –supplier–*, MCSS, and another *market coordinator –consumer–*, MCSC**.** The resulting structural mapping obtained from a similar use of the four stages methodology, and the corresponding equilibrium strategies applicable to each sub-game identified, are shown in Figure 2.

5 FINAL CONCLUSIONS

In this paper the *strategic games matrix* (SGM) modeling framework is used as a tool for:

- Describing, characterizing, and mapping a wide variety of conflicts of interests situations among intelligent autonomous agents, both for hierarchical and for non-hierarchical games, in an integrated manner;
- Modeling, analysis and design of multilevel multiple-agent control architectures in an integrated manner, making explicit the obvious conflicts of interests possibilities;
- Establishing a useful two-way conceptual bridge between game theory and multiple-agent structures analysis and design.

The SGM permits to evidence that, for a specific real complex problem, we should be more concerned with the choice of the right game to model, than with the right way to solve the game, in spite of the importance of these techniques.

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