FAULT DETECTION ALGORITHM USING DCS METHOD COMBINED WITH FILTERS BANK DERIVED FROM THE WAVELET TRANSFORM

Oussama Mustapha 1,2, Mohamad Khalil 2,3, Ghaleb Hoblos 4, Houcine Chafouk 4 and Dimitri Lefebvre 1

1 University Le Havre, GREAH, Le Havre, France
2 Lebanese University, Faculty of Engineering, Section I- El Arz Street, El Kobbe, Lebanon
3 Islamic University of Lebanon, Faculty of engineering, Biomedical Department, Khaldé, Lebanon
4 ESIGELEC, IRSEEM, Saint Etienne de Rouvray, France
oussama_mustapha@hotmail.com, mkhalil@ieee.org, ghaleb.holos@esigelec.fr
houcine.chafouk@esigelec.fr, dimitri.lefebvre@univ-lehavre.fr

Keywords: Signal, Filters Bank, DCS, Fault, detection, wavelet transform.

Abstract: The aim of this paper is to detect the faults in industrial systems, such as electrical machines and drives, through on-line monitoring. The faults that are concerned correspond to changes in frequency components of the signal. Thus, early fault detection, which reduces the possibility of catastrophic damage, is possible by detecting the changes of characteristic features of the signal. This approach combines the Filters Bank technique, for extracting frequency and energy characteristic features, and the Dynamic Cumulative Sum method (DCS), which is a recursive calculation of the logarithm of the likelihood ratio between two local hypotheses. The main contribution is to derive the filters coefficients from the wavelet in order to use the filters bank as a wavelet transform. The advantage of our approach is that the filters bank can be hardware implemented and can be used for online detection.

1 INTRODUCTION

The fault detection and diagnosis are of particular importance in industry. In fact, the early fault detection in industrial machines can reduce the personal damages and economical losses. Many researchers have performed fault detection by using mechanical conditions such as vibration analysis. Recently the current or voltage signature analysis is used for the detection of electromechanical faults, such as a broken bar in electrical drives (Sottile and Kohler, 1993; Schoen et al., 1995; Kliman et al., 1996). Other researchers use the AI tools (Awadallah and Morcos, 2003) and frequency methods (Benbouzid, et al., 1999). The aim of this paper is to propose a method for the on-line detection of changes in the electric current feeding an induction motor due to a mechanical fault. The method is based on a filters bank, whose coefficients are derived from the wavelet, to decompose the signal in order to explore their frequency and energy components of the signal. Then, the Dynamic Cumulative Sum method is applied to the filtered signals in order to detect any change in the signal. The filters bank is derived from the wavelet transform, by using the Prony method, so the wavelet characteristics are approximately conserved and this allows both filtering and reconstruction of the signal. The main contributions are to derive the filters and to evaluate the error between filters bank and wavelet transform. This study continues our investigation concerning fault detection by means of wavelet transform and filters bank (Mustapha et al., 2006a, 2006b). Extraction and detection will be applied on simulated and real signals. The real signals are issued from long duration experiments, with GREAH, on asynchronous machines of 4kW. These signals are recorded when the machine is properly operating and then when a bar of the same machine is broken. This paper is decomposed as follows. First we present the wavelet transform (WT) and the filters bank technique. In section 3 we detail the derivation of filters from a WT. In section 4, the Cumulative Sum and the Dynamic Cumulative Sum methods are presented. In section 5, some results are discussed. Then, the choice of the suitable filter are discussed in section 6.
2 WT AND FILTERS BANK

The Fourier analysis is the most well known mathematical tool used for transforming the signal from time domain to frequency domain. But it has an important drawback represented by the loss of time information when transforming the signal to the frequency domain. To preserve the temporal aspect of the signals when transforming them to frequency domain, one solution is to use is the WT (Truchetet, 1998) which analyzes non-stationary signals by mapping them into time-scale and time-frequency representation. The Wavelet Transform is similar to the Short Time Fourier Transform but provides, in addition, a multi-resolution analysis with dilated and shifted windows. The multi-resolution analysis consists of decomposing the signal \( x(t) \) using the wavelet \( \psi(t) \) and its scale function \( \phi(t) \) (Flandrin, 1993; Krim, 1995):

\[
T_x^\psi(a,b) = \int_{-\infty}^{+\infty} x(t) \psi_{ab}(t) dt, \quad \psi_{ab}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right)
\]  

(1)

where \( a \) and \( b \) are respectively the dilation and translation parameters. The filter associated with the scale function \( \phi(t) \) is a low pass filter and the filter associated to the wavelet \( \psi(t) \) is a band pass filter.

The following formulas can be used to calculate detail and approximation coefficients (Truchetet, 1998):

\[
a_x(n,m) = \int_{-\infty}^{+\infty} x(t) \phi_{mn}(t) dt
\]  

(2)

\[
d_x(n,m) = \int_{-\infty}^{+\infty} x(t) \psi_{mn}(t) dt
\]  

(3)

where \( m \) and \( n \) are integers.

In this way, the relevant events, to be detected, can be shown as details on specific scale levels. In Discrete Wavelet Transform (DWT), the multi-resolution analysis uses a scaling function and a wavelet to perform successive decomposition of the signal into approximations and details (figure 1: a and b).

At each time \( t \), the signal is first decomposed by using an N-channels band-pass filters bank whose central frequency moves from lowest frequency \( f_1 \) up to the highest frequency \( f_N \). Each component \( m \in \{1, ..., N\} \) is the result of filtering the original signal \( x \) by a band-pass filter centered on \( f_m \). The frequency response of the filters bank is shown in (figure 2).

For each component \( m \), the sample \( y^{(m)}(t) \), is on-line computed according to the original signal \( x(t) \) and using the parameters \( a_i^{(m)} \) and \( b_j^{(m)} \) of the corresponding band-pass filter according to (4):

\[
y^{(m)}(t) = \sum_{j=0}^{q} b_j^{(m)} x(t - j) - \sum_{p=1}^{p} a_i^{(m)} y(t - i)
\]  

(4)

where \( x \) is the original signal, \( f_s \) is the sampling frequency of the original signal \( x, f_N \) must satisfy the condition \( f_N \leq f_s / 2 \), \( N \) is the number of channels used, \( p \) and \( q \) are the orders of the filter at level \( m \). The choice of the filters bank depends on the original signal and its frequency band. The number of filters \( N \) depends on the details that we have to extract from the signal and on the events that must be distinguished. In our case we will use \( N = 3 \) filters.

The procedure of decomposing \( x(t) \) into signals \( y^{(m)}(t), m=1...N \), allows us to explore all frequency components of the signal. \( y^{(0)}(t) \) gives the low
frequency components and \( y^{(N)}(t) \) gives the high frequency ones. Therefore, the points of change of each component give information about the frequency and energy contents and will be used to detect any changes in frequency and energy in the original signal.

3 PRONY'S METHOD

In the present work, the main objective is to derive the filters coefficients of a filters bank from a wavelet in order to use the filters bank as a WT. The filters bank is derived from the WT, by using the Prony’s method, so the wavelet characteristics are approximately conserved and this allows both filtering and reconstruction of the signal.

For a given wavelet, we can use the approximation coefficients of the wavelet function \( \psi(t) \) to extract the coefficients \( a_i \) and \( b_j \) in order to design an IIR filter that behaves as the wavelet. The extraction of the filter coefficients can be done by using the Prony’s method. The main advantage of the wavelet-derived filter is that it can be used instead of the wavelet and can be hardware implemented in order to be used for online signal filtration. Figure 3 shows the response curves \( (h_{wav}) \) of the wavelet function 'db3' and the response curves \( (h_{filt}) \) of the derived filter.

Prony’s method is an algorithm that can be used to find an IIR filter with a prescribed time domain impulse response. According to the time domain impulse response \( h_{wav} \) of the wavelet function \( \psi(t) \), the numerator order \( p \) and the denominator order \( q \) of the desired filter, Prony’s method is used to compute the filter’s coefficients \( a_i \) and \( b_j \) \( j=1 \ldots q \) if the length of \( h \) is less than the largest order \( p \) or \( q \), \( h \) is padded with zeros. It is fundamentally based on signal approximation with a linear combination of adjustable exponentials.

The impulse matching problem for modeling an entire causal signal \( x(t) \), \( t=0,1,\ldots,\infty \), produces an infinite number of equations. The problem is to find the parameters \( a_i \) and \( b_j \) such that the equation \( (x) \) is satisfied:

\[
\begin{align*}
\begin{bmatrix}
x(0) & 0 & \cdots & 0 \\
x(1) & x(0) & \cdots & 0 \\
x(2) & x(1) & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
x(p) & x(p-1) & \cdots & x(0)
\end{bmatrix}
\begin{bmatrix}
a_1 \\
a_2 \\
a_3 \\
\vdots \\
a_p
\end{bmatrix}
= \begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
\vdots \\
x_q
\end{bmatrix}
\end{align*}
\]

where \( X_j \) is the top part of matrix \( X \), \( a \) is a \( p \) dimensional vector of parameters \( \{a_i\} \), \( b \) is a \( q+1 \) dimensional vector of parameters \( \{b_j\} \), \( x \) is the first column of bottom part of matrix \( X \), and \( X_2 \) is the \( p \) last columns of matrix \( X \) bottom part.

The equation \( x+X_2\tilde{a}=0 \) contains an infinite number of equations to be solved for \( \tilde{a} \). This linear equation is usually over determined and no exact solution exists. This means that since the vector \( \tilde{x} \) can only be approximated by the columns of matrix \( X_2 \), it's necessary to choose \( \tilde{a} \) to minimize the equation error defined by the equation (6):

\[
\varepsilon = \sum_{i=q+1}^{\infty} \varepsilon^2(i) = \sum_{i=q+1}^{\infty} x(t) + \sum_{i=1}^{p} x(t - i)^2
\]

with \( x = x + X_2 \tilde{a} \). The error is minimized by using partial differentiation with respect to parameters \( \{a_i\} \):

\[
\varepsilon_{\min} = \tilde{X}_2 \tilde{a} = -X_2 x
\]

In order that the orthogonality condition \( \varepsilon_{\min} \tilde{X}_2 = 0 \) is satisfied, (7) provides a solution for the optimum vector \( \tilde{a} \), which can then be used to find the solution to vector \( b \) by simple matrix multiplication in (5).

4 CUMSUM AND DCS

The Cumulative Sum algorithm (CUMSUM) algorithm is based on a recursive calculation of the logarithm of the likelihood ratios. This method can
be considered as a sequence of repeated tests around the point of change \( t_M \) (figure 4) (Nikiforov, 1986; Basseville and Nikiforov, 1993). For the seek of simplicity \( x(t) \) will be referred as \( x_t \) in the following. Let \( x_1, x_2, x_3, \ldots, x_t \) be a sequence of observations. Let us assume that the distribution of the process \( X \) depends on parameter \( \theta_0 \) until time \( t_M \) and depends on parameter \( \theta_1 \) after the time \( t_M \). At each time \( t \) we compute the sum of logarithms of the likelihood ratios as follows:

\[
S_1^{(t,m)} = \sum_{i=1}^{t} s_{i}^{(m)} = \sum_{i=1}^{t} \ln \left( \frac{f_{\theta_1}(x_i / x_{i-1}, \ldots, x_1)}{f_{\theta_0}(x_i / x_{i-1}, \ldots, x_1)} \right) \quad (8)
\]

The importance of this sum comes from the fact that its sign changes after the point of change. The detectability (Basseville and Nikiforov, 1993) is due to the fact that the expectation \( E_{\theta_0}[s_i] < 0 \) and \( E_{\theta_1}[s_i] > 0 \). We, then, calculate the following detection function \( g^{(t,m)} = S_1^{(t,m)} - \min_{2 \leq s \leq t} S_1^{(t,m)} \). This function compares, at any time \( t \), the difference between the value of the sum of the logarithm of the likelihood ratio and its minimal current value. The point of change can be defined as follows \( t_M = \max \{ t > 1 : g^{(t,m)} = 0 \} \).

Figure 4: CUNSUM algorithm (a) Signal (b) CUNSUM (c) Detection function.

At any time \( t \) and for the observation vector \( X = X_t = (x_1, \ldots, x_t) \), suppose that the distribution of the process \( X \) depends on parameter \( \theta \). A change can affect the frequency distribution of the signal. The Dynamic Cumulative Sum method (DCS) is a repetitive sequence around the point of change \( t_M \). It is based on the local cumulative sum of the likelihood ratios between two local segments estimated at the current time \( t \). These two dynamic segments \( S_a^{(t)} \) (« after ») and \( S_b^{(t)} \) (« before ») are estimated by using two windows of width \( W \) (figure 5) before and after the time instant \( t \) as follows:

\[
\begin{align*}
S_a^{(t)} : x_i; i = [t+1, t + W] & \text{ follows a probability density function } f_{\theta_1}(x_i) \\
S_b^{(t)} : x_i; i = [t - W, t - 1] & \text{ follows a probability density function } f_{\theta_0}(x_i)
\end{align*}
\]

Figure 5: DCS algorithm (a) Signal; (b) Dynamic cumulative sum Cumulative sum; (c) Detection function.

The parameters \( \theta_b \) of the segment \( S_b^{(t)} \), are estimated using \( W \) points before the time instant \( t \) and the parameters \( \theta_a \) of the segment \( S_a^{(t)} \), are estimated using \( W \) points after the time instant \( t \). At a time \( t \), the DCS is defined as the sum of the logarithm of likelihood ratios from the beginning of the signal up to the time \( t \):

\[
DCS^{(m)}(S_a^{(t)}, S_b^{(t)}) = \sum_{i=1}^{t} \ln \left( \frac{f_{\theta_1}(x_i)}{f_{\theta_0}(x_i)} \right) \sum_{i=1}^{t} \hat{s}_i 
\]

(Khalil, 1999) proves that the DCS function reaches its maximum at the point of change \( t_M \). The detection function used to estimate the point of change is

\[
g^{(t,m)} = \max_{1 \leq s \leq t} \left( DCS^{(m)}(S_a^{(t)}, S_b^{(t)}) - DCS^{(m)}(S_a^{(s)}, S_b^{(s)}) \right) \quad (10)
\]

The instant at which the procedure is stopped is \( t_a = \inf \{ t : g^{(t,m)} \geq h \} \), where \( h \) is the detection threshold. The point of change is estimated as \( t_M = \max \{ t > 1 : g^{(t,m)} = 0 \} \). The DCS is a method that
can be used when the parameters of the signal are unknown.

5 RESULTS

The algorithm is first applied to simulated signals and then to real signals (figure 6). The simulated signal is generated by concatenating two random signals of different variances \( (\sigma_0=1 \text{ et } \sigma_1=3) \), and two sinusoidal signals of different frequencies \( (f_0=150\text{Hz et } f_1=600\text{Hz}) \). Real and simulated signals are decomposed into 3 scales before applying the DCS method. These scales are computed by using the ARMA coefficients calculated by Prony’s method and corresponding to the ‘db3’ wavelet. The coefficients of the derived filter of order 5 from the wavelet ‘db3’ for scale level 3 are detailed in the next table:

<table>
<thead>
<tr>
<th>ai</th>
<th>1.000</th>
<th>-1.247</th>
<th>0.527</th>
<th>-0.165</th>
<th>0.604</th>
<th>-0.409</th>
</tr>
</thead>
<tbody>
<tr>
<td>bi</td>
<td>0</td>
<td>0.0110</td>
<td>0.0130</td>
<td>0.0144</td>
<td>0.0203</td>
<td>0.0252</td>
</tr>
</tbody>
</table>

The results lead us to determine the point of change of statistical parameters of these signals.

<table>
<thead>
<tr>
<th>Expected Time of change</th>
<th>1st comp.</th>
<th>2nd comp.</th>
<th>3rd comp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st simulated signal</td>
<td>1000</td>
<td>1006</td>
<td>1005</td>
</tr>
<tr>
<td>Real signal</td>
<td>4000</td>
<td>4107</td>
<td>4097</td>
</tr>
</tbody>
</table>

Note that the third component, which is filtered by a highest central frequency band-pass filter, presents the closest point of change to the real one as shown in the table 2.

6 FILTER'S ORDER

In the wavelet theory, the choice of the wavelet is a critical problem. To extract the specific events in a signal, the choice of the wavelet is important to be adapted to the event to be detected. Many researchers have performed the detection by using the wavelet in different domains of application: in image edge detection (Mallat, 2000), for compression (Benbouzid et al., 1999), for signal denoising in speech processing (Misiti et al.). In biomedical applications, the quadratic spline wavelet is used by Li (Li et al., 1995) and the complex wavelet is used by Shenhadji (Shenhadji et al., 1995) to process the ECG signal.

In our work, filters derived from many wavelets such as the Daubechies, the coiflet and the symlet wavelets are tested and according to the results obtained in figure 7, the filter derived from the wavelet ‘db3’ at level 3 has been used because it presents the minimum error and then it is chosen.

Note that the error is defined as follows:

\[
\text{error} = \sum_{i=1}^{k} (h_{\text{wav}} - h_{\text{filt}})^2
\]

Where, \( h_{\text{wav}} \) and \( h_{\text{filt}} \) are the impulse responses of the wavelet and the derived filter respectively.

The orders of the filter \( (p \text{ and } q) \) are very important parameters and can affect the error due to the application of Prony’s method to extract the filter coefficients from the wavelet. As shown in figure 8, we can see that if the order of the filter becomes 30 and above, the error due to the derivation becomes negligible for filter derived from db3.
Figure 8: Error due to the order of the filter derived from different types of wavelets (scale 2 and $p=q=30$).

7 CONCLUSIONS

This article has proposed a method to detect the point of change of statistical parameters in signals issued from industrial machines. This method uses a band-pass filters bank, derived from a wavelet transform, to decompose the signal and the DCS algorithm to characterize and classify the parameters of a signal in order to detect any variation of the statistical parameters due to any change in frequency and energy. The main contribution of the work is to find a filters bank that approximates a wavelet. The filters bank derivation is done by using the Prony's method. After the calculation of the resulting error, between the derived filters bank and the correspondent wavelet, the wavelet ‘db3’ has been selected. In order to reduce the error due to the order of the derived filter, the order is taken to be beyond 30. This on-line algorithm is developed and tested and it gives good results for the detection of changes in the signals. It is necessary to test the algorithm with other types of wavelets, to explain the error depending on the scale levels, and to implement the whole algorithm in a DSP. The detectability of DCS must be proved after decomposing the signal, especially after using the ARMA decomposition. Another perspective is to complete the filters design by determining the optimum orders $p$ and $q$.

REFERENCES


Nikiforov I. Sequential detection of changes in stochastic systems. Lecture notes in Control and Information Sciences, NY, USA, 1986, pp. 216-228.


Khalil M. Une approche pour la détection fondée sur une somme cumulée dynamique associée à une décomposition multiéchelle. Application à l'EMG utérin. 17ème Colloque GRETSI sur le traitement du signal et des images, Vannes, France,1999.


