FAST ESTIMATION FOR RANGE IDENTIFICATION IN THE PRESENCE OF UNKNOWN MOTION PARAMETERS

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Abstract: A fast adaptive estimator is applied to the problem of range identification in the presence of unknown motion parameters. Assuming a rigid-body motion with unknown constant rotational parameters but known translational parameters, extraction of the unknown rotational parameters is achieved by recursive least square method. Simulations demonstrate the superior performance of fast estimation in comparison to identifier based observers.

1 INTRODUCTION

A variety of 3D motion estimation algorithms have been developed since 1970's, inspired by such disparate applications as robot navigation, medical imaging, and video conferencing. Even though motion estimation from imagery is not a new topic, continual improvements in digital imaging, computer processing capabilities, and nonlinear estimation theory have helped to keep the topic current. Assuming that the motion of the moving object follows certain structure, which can have parametric uncertainties, extended Kalman filter (EKF) has been used to estimate the states and parameters of the nonlinear system associated with the moving object dynamics. Application of EKF assumes linearization about the estimated trajectory. However, for the motion estimation from imagery the geometric structure of the perspective system can be lost during the linearization(Ghosh et al., 1994; Dixon et al., 2003). Refs. (Jankovic and Ghosh, 1995; Chen and Kano, 2002; Dixon et al., 2003; Karagiannis and Astolfi, 2005; Ma et al., 2005) have considered nonlinear observers for perspective dynamic systems (PDS) arising in visual tracking problems. In general, a PDS is a linear system, whose output is observed up to a homogeneous line(Chen and Kano, 2002). This class of nonlinear observers is referred to as perspective nonlinear observers.

Perspective nonlinear observers (Jankovic and Ghosh, 1995; Chen and Kano, 2002; Dixon et al., 2003; Karagiannis and Astolfi, 2005; Ma et al., 2005) are used quite often for determining the unknown states (i.e., the 3D Euclidean coordinates) of a moving object with known motion parameters. For example, an identifier-based observer was proposed in(Jankovic and Ghosh, 1995) to estimate a stationary point's 3D position using a moving camera. Another discontinuous observer, motivated by sliding mode and adaptive methods, is developed in(Chen and Kano, 2002) that renders the state observation error uniformly ultimately bounded. A state estimation algorithm with a single homogeneous observation (i.e., a single image coordinate) is presented in(Ma et al., 2005). A reduced-order nonlinear observer is described in(Karagiannis and Astolfi, 2005) to provide asymptotic range estimation. All these results are based on the assumption that the object is following a known motion dynamics in the 3D space.

In this paper, we discuss a situation when some of the motion parameters, more specifically, the rotational parameters, are unknown constants. The objective is to achieve fast state estimation and parameter convergence.

One model for the relative motion of a point in the camera's field of view is the following linear system(Jankovic and Ghosh, 1995; Chen and Kano, 2002; Dixon et al., 2003; Karagiannis and Astolfi, 2005):

$$\begin{bmatrix}\dot{X}(t)\\ \dot{Y}(t)\\ \dot{Z}(t)\end{bmatrix} = \begin{bmatrix}0 & w_1 & w_2\\ -w_1 & 0 & w_3\\ -w_2 & -w_3 & 0\end{bmatrix}\begin{bmatrix}X(t)\\ Y(t)\\ Z(t)\end{bmatrix} + \begin{bmatrix}b_1\\ b_2\\ b_3\end{bmatrix}, \quad (1)$$

where the matrix $[w_i]$ presents the rotational dynamics, the vector $[b_i]$ corresponds to the translational motion, while $[X,\ Y,\ Z]^\top$ are the coordinates of the point in the camera frame. From the 2D image plane, the homogeneous output observations are given by

$$x_1(t) = X(t)/Z(t), \quad x_2(t) = Y(t)/Z(t). \quad (2)$$

These equations might model either a stationary point's 3D position as observed from a moving camera (assuming that the moving camera's velocities can be measured(Jankovic and Ghosh, 1995)) or a moving point's 3D position as observed from a stationary camera(Tsai and Huang, 1981). In general, w_i can be time-dependent, but in this paper we limit the discussion to constant w_i's.

Let

$$\begin{aligned} x(t) &= [x_1(t), x_2(t), x_3(t)]^\top \\ &= [X(t)/Z(t), Y(t)/Z(t), 1/Z(t)]^\top. \end{aligned} \quad (3)$$

The system (1) with output observations (3) is equivalent to the system

$$\begin{cases} \begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}b_1 - b_3x_1\\ b_2 - b_3x_2\end{bmatrix}x_3 + \begin{bmatrix}w_2 + w_1x_2 + w_2x_1^2 + w_3x_1x_2\\ w_3 - w_1x_1 + w_2x_1x_2 + w_3x_2^2\end{bmatrix}, \\ \dot{x}_3(t) = (w_2x_1 + w_3x_2)x_3 - b_3x_3^2, \end{cases} \quad (4)$$

with the output

$$y(t) = [x_1(t), x_2(t)]^\top. \quad (5)$$

Estimation of $x_3(t)$ from the measurements $(x_1(t), x_2(t))$ constitutes the range identification problem. Refs. (Jankovic and Ghosh, 1995; Chen and Kano, 2002; Dixon et al., 2003; Karagiannis and Astolfi, 2005; Ma et al., 2005) have solved this problem assuming that the motion parameters w_i and b_i in (1) are known (where $i \in \{1,2,3\}$). Here, we assume that the parameters w_i are unknown. The objective, then, is to estimate $x_3(t)$ as well as the unknown parameters w_i. This problem can be formulated in a way such that an existing identifier-based observer (IBO), described in(Jankovic and Ghosh, 1995), can be applied, such that under certain assumptions, the approach provides exponential convergence of both the range and the parameter estimates. A more general case of the problem is discussed in(Ma et al., 2007), where the rotational matrix is represented by a 3×3 matrix instead of the skew-symmetric matrix as in (1).

In this paper, a recently-developed novel adaptive estimator is applied for the estimation of $x_3(t)$ along with the unknown parameters w_i. A numerical comparison of the performance of this adaptive estimator with the IBO observer is provided.

The paper is organized as follows. Range identification in the presence of unknown parameters via the IBO is presented in Sec. 2. A brief review of the fast estimator is given in Sec. 3. In Sec. 4, fast estimation for the range identification problem with unknown motion parameters is presented. Section 5 presents the simulation results. Section 6 extends the analysis to general affine motion. Finally, section 7 concludes the paper.

2 RANGE IDENTIFICATION IN THE PRESENCE OF UNKNOWN PARAMETERS VIA IBO

Consider the state estimation problem for the perspective dynamic system (7), where the motion parameters w_i (for $i = 1,2,3$) are assumed to be unknown constants. Let θ be a vector of these unknown constants defined as

$$\theta = [w_1, w_2, w_3]^\top. \quad (6)$$

The system (4) can be rewritten as

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = w_s^\top(x_1,x_2)\begin{bmatrix}x_3\\ \theta\end{bmatrix}, \quad (7a)$$

$$\begin{bmatrix}\dot{x}_3(t)\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}\underbrace{(w_2x_1 + w_3x_2)x_3 - b_3x_3^2}_{g_s(x_1,x_2,x_3,w_2,w_3)}\\ 0_{3\times1}\end{bmatrix}, \quad (7b)$$

with

$$w_s^\top(x_1,x_2) = \begin{bmatrix}b_1 - b_3x_1 & x_2 & 1 + x_1^2 & x_1x_2\\ b_2 - b_3x_2 & -x_1 & x_1x_2 & 1 + x_2^2\end{bmatrix}, \quad (8)$$

which fits into the form of the general nonlinear system to which IBO might be applicable, by regarding $\mathbf{x}_1 = [x_1, x_2]^\top$, $\mathbf{x}_2 = [x_3, \theta^\top]^\top$, and $\phi(\mathbf{x}_1, \mathbf{u}) = 0$ (please refer to(Jankovic and Ghosh, 1995) for details of the IBO).

To apply the IBO observer, we need the following assumption for the system in (7):

Assumption 2.1

1. *Let* $\mathbf{x}(t) = \left[x_1(t), x_2(t), x_3(t), \theta^\top\right]^\top$ *be bounded* $\|\mathbf{x}(t)\| < M$, $M > 0$ *for every* $t \geq 0$. *Let* $\Omega = \{\mathbf{x} \in \mathbb{R}^n : \|\mathbf{x}(t)\| < M\}$. *Further, for some fixed constant* $\gamma > 1$, *let* $\Omega_\gamma = \{\mathbf{x} \in \mathbb{R}^n : \|\mathbf{x}(t)\| < \gamma M\}$.
2. *The function* $w_s(x_1, x_2)$ *and its first time derivative are piecewise smooth and uniformly bounded.*

Suppose that there exist positive constants L_1, L_2 *such that*

$$\|w_s^\top(x_1,x_2)\| < L_1, \quad \left\|\frac{\mathrm{d}w_s^\top(x_1,x_2)}{\mathrm{d}t}\right\| < L_2. \quad (9)$$

Further, there do not exist constants κ_i *(for* $i = 1,2,3,4$*) with* $\sum_{i=1}^4 \kappa_i^2 \neq 0$ *such that*

$$\kappa_1 v_1(\tau) + \kappa_2 v_2(\tau) + \kappa_3 v_3(\tau) + \kappa_4 v_4(\tau) = 0, \quad (10)$$

for all $\tau \in [t, t+\mu]$*, where* $\mu > 0$ *is a sufficiently small constant, and* $v_i(\tau)$ *denotes the* i^{th} *column in* w_s *in (8).*

It is straightforward to verify that, under Assumption 2.1, the system in (7) verifies the assumptions required for the application of IBO. Estimation of $x_3(t)$, along with the unknown motion parameters θ, can be obtained via direct application of the IBO, as given below.

Letting $e_1 = \hat{x}_1 - x_1$, $e_2 = \hat{x}_2 - x_2$, $e_3 = \hat{x}_3 - x_3$, the following observer can be designed for the system (7)

$$\begin{cases} \begin{bmatrix}\dot{\hat{x}}_1\\ \dot{\hat{x}}_2\end{bmatrix} = GA\begin{bmatrix}e_1\\ e_2\end{bmatrix} + w_s^\top(x_1,x_2)\begin{bmatrix}\hat{x}_3\\ \hat{\theta}\end{bmatrix}, \\ \begin{bmatrix}\dot{\hat{x}}_3\\ \dot{\hat{\theta}}\end{bmatrix} = -G^2 w_s(x_1,x_2)P\begin{bmatrix}e_1\\ e_2\end{bmatrix} + \begin{bmatrix}g_s(x_1,x_2,\hat{x}_3,\hat{w}_2,\hat{w}_3)\\ 0_{3\times 1}\end{bmatrix}, \\ \hat{\mathbf{x}}(t_i^+) = M\dfrac{\hat{\mathbf{x}}(t_i^-)}{\|\hat{\mathbf{x}}(t_i^-)\|}, \end{cases} \quad (11)$$

where $\mathbf{x}$ denotes $[x_1, x_2, x_3, \theta^\top]^\top$, $\hat{\theta}$ denotes the estimation of θ, and the sequence of t_i is defined as

$$t_i = \min\{t : t > t_{i-1} \text{ and } \|\hat{\mathbf{x}}(t)\| \geq \gamma M\},\ t_0 = 0, \quad (12)$$

for some fixed constant $\gamma > 1$. The closed-loop error dynamics can be derived from (7) and (11) as

$$\begin{cases} \begin{bmatrix}\dot{e}_1\\ \dot{e}_2\end{bmatrix} = GA\begin{bmatrix}e_1\\ e_2\end{bmatrix} + w_s^\top(x_1,x_2)\begin{bmatrix}e_3\\ \tilde{\theta}\end{bmatrix}, \\ \begin{bmatrix}\dot{e}_3\\ \dot{\tilde{\theta}}\end{bmatrix} = -G^2 w_s(x_1,x_2)P\begin{bmatrix}e_1\\ e_2\end{bmatrix} \\ \qquad + \begin{bmatrix}g_s(x_1,x_2,\hat{x}_3,\hat{w}_2,\hat{w}_3) - g_s(x_1,x_2,x_3,w_2,w_3)\\ 0_{3\times 1}\end{bmatrix}, \end{cases} \quad (13)$$

where $\tilde{\theta} = \hat{\theta} - \theta$ and $\dot{\tilde{\theta}} = \dot{\hat{\theta}}$, since θ is assumed to be a constant vector. The main claim is that there exists a positive constant G_0, such that the estimation errors $[e_1, e_2, e_3, \tilde{\theta}^\top]^\top$ converge to zero exponentially if the constant G in (11) is chosen to be larger than G_0(Jankovic and Ghosh, 1995).

3 FAST ESTIMATOR

Range identification in the presence of unknown motion parameters is further pursued using a recently-developed fast adaptive estimator. The adaptive estimator enables estimation of the unknown time-varying parameters in the system dynamics via fast adaptation (large adaptive gain) and a low-pass filter. If the time-varying unknown signal is linearly parameterized in unknown constant parameters, the adaptive estimator can be further augmented by a recursive least-square algorithm (RLS) to estimate the unknown constant parameters asymptotically(Cao and Hovakimyan, 2007).

In the following, main results of the the adaptive estimator are given for the purpose of completeness. More details are presented in(Cao and Hovakimyan, 2007).

3.1 Preliminaries

Some basic definitions from linear system theory are given in this section.

Definition 3.1 *For a signal* $\xi(t),\ t \geq 0,\ \xi \in \mathbb{R}^n$*, its* $\mathcal{L}_\infty$ *norm is defined as*

$$\|\xi\|_{\mathcal{L}_\infty} = \max_{i=1,\ldots,n}\left(\sup_{\tau\geq 0}|\xi_i(\tau)|\right), \quad (14)$$

where ξ_i *is the* i^{th} *component of* ξ.

Definition 3.2 *The* $\mathcal{L}_1$ *gain of a stable proper single–input single–output system* $H(s)$ *is defined as:*

$$\|H\|_{\mathcal{L}_1} = \int_0^\infty |h(t)|dt, \quad (15)$$

where $h(t)$ *is the impulse response of* $H(s)$.

Definition 3.3 *For a stable proper m input n output system* $H(s)$ *its* $\mathcal{L}_1$ *gain is defined as*

$$\|H\|_{\mathcal{L}_1} = \max_{i=1,\ldots,n}\left(\sum_{j=1}^m \|H_{ij}\|_{\mathcal{L}_1}\right), \quad (16)$$

where $H_{ij}(s)$ *is the* i^{th} *row* j^{th} *column element of* $H(s)$.

3.2 Problem Formulation

Consider the following system dynamics:

$$\dot{x}(t) = A_m x(t) + \omega(t), \qquad x(0) = x_0, \quad (17)$$

where $x \in \mathbb{R}^n$ is the system state vector (measurable), $\omega(t) \in \mathbb{R}^n$ is a vector of unknown time-varying signals or parameters, and A_m is a known $n \times n$ Hurwitz matrix. Let

$$\omega(t) \in \Omega, \quad (18)$$

where Ω is a known compact set. The signal $\omega(t)$ is further assumed to be continuously differentiable with uniformly bounded derivative

$$\|\dot{\omega}(t)\| \leq d_\omega < \infty, \quad \forall\, t \geq 0, \quad (19)$$

where d_ω can be arbitrarily large. The estimation objective is to design an adaptive estimator that provides fast estimation of $\omega(t)$.

3.3 Fast Adaptive Estimator

The adaptive estimator consists of the state predictor, the adaptive law and a low-pass filter, which extracts the estimation information.

State Predictor: We consider the following state predictor:

$$\dot{\hat{x}}(t) = A_m\hat{x}(t) + \hat{\omega}(t), \quad \hat{x}(0) = x_0, \qquad (20)$$

which has the same structure as the system in (17). The only difference is that the unknown parameters $\omega(t)$ are replaced by their adaptive estimates $\hat{\omega}(t)$ that are governed by the following adaptation laws.

Adaptive Laws: Adaptive estimates are given by:

$$\dot{\hat{\omega}}(t) = \Gamma_c \text{Proj}(\hat{\omega}(t), -P\tilde{x}(t)), \quad \hat{\omega}(0) = \hat{\omega}_0, \quad (21)$$

where $\tilde{x}(t) = \hat{x}(t) - x(t)$ is the error signal between the state of the system and the state predictor, $\Gamma_c \in \mathbb{R}^+$ is the adaptation rate, chosen sufficiently large, and P is the solution of the algebraic equation $A_m^\top P + PA_m = -Q$, $Q > 0$.

Estimation: The estimation of the unknown signal is generated by:

$$\omega_e(s) = C(s)\hat{\omega}(s), \qquad (22)$$

where $C(s)$ is a diagonal matrix with its i^{th} diagonal element $C_i(s)$ being a strictly proper stable transfer function with low-pass gain $C_i(0) = 1$. One simple choice is

$$C_i(s) = \frac{\theta_a}{s + \theta_a}. \qquad (23)$$

3.4 Convergence Results

The fast adaptive estimator in Sec. 3.3 ensures that $\omega_e(t)$ estimates the unknown signal $\omega(t)$ with the final precision:

$$\|1 - C(s)\|_{\mathcal{L}_1} \|\omega\|_{\mathcal{L}_\infty} + \frac{\gamma_c}{\sqrt{\Gamma_c}}, \qquad (24)$$

where $\|\cdot\|_{\mathcal{L}_1}$ denotes the $\mathcal{L}_1$ gain of the system.

To quantify this performance bound between $\omega_e(t)$ and $\omega(t)$, an intermediate signal $\omega_r(t)$ is introduced as:

$$\omega_r(s) = C(s)\omega(s). \qquad (25)$$

The following theorem gives the performance bound between $\omega_e(t)$ and $\omega_r(t)$. Details of the proof can be found in(Cao and Hovakimyan, 2007).

Theorem 3.1 *For the system in (17) and the fast adaptive estimator in (20), (21) and (22), we have*

$$\|\omega_e - \omega_r\|_{\mathcal{L}_\infty} \le \frac{\gamma_c}{\sqrt{\Gamma_c}}, \qquad (26)$$

where

$$\gamma_c = \|C(s)H^{-1}(s)\|_{\mathcal{L}_1} \sqrt{\frac{\omega_m}{\lambda_{\min}(P)}}, \qquad (27a)$$

$$H(s) = (sI - A_m)^{-1}, \qquad (27b)$$

$$\omega_m = \max_{\omega\in\Omega} 4\|\omega\|^2 + 2\frac{\lambda_{\max}(P)}{\lambda_{\min}(Q)}\left(d_\omega \max_{\omega\in\Omega}\|\omega\|\right), \qquad (27c)$$

and $\|\cdot\|_{\mathcal{L}_\infty}$ denotes the $\mathcal{L}_\infty$ norm of the signal.

Corollary 3.1 *For the system in (17) and the fast adaptive estimator in (20), (21) and (22), we have*

$$\lim_{\Gamma_c \to \infty} (\omega_e(t) - \omega_r(t)) = 0, \quad \forall\, t \ge 0. \qquad (28)$$

We further characterize the performance bound between $\omega_r(t)$ and $\omega(t)$. For simplicity, we use a first order $C(s)$ as in (23). It follows from (25) that

$$\dot{\omega}_r(t) = -\theta_a\omega_r(t) + \theta_a\omega(t), \quad \omega_r(0) = 0. \qquad (29)$$

We note that $\omega_r(t)$ can be decomposed into two components:

$$\omega_r(t) = \omega_{r_1}(t) + \omega_{r_2}(t), \qquad (30)$$

where $\omega_{r_1}(t)$ and $\omega_{r_2}(t)$ are defined via:

$$\dot{\omega}_{r_1}(t) = -\theta_a\omega_{r_1}(t) + \theta_a\omega(t),\ \omega_{r_1}(0) = \omega(0) \qquad (31a)$$

$$\dot{\omega}_{r_2}(t) = -\theta_a\omega_{r_2}(t),\ \omega_{r_2}(0) = -\omega(0). \qquad (31b)$$

It follows from (31a) that

$$\|\omega_{r_1} - \omega\|_{\mathcal{L}_\infty} = \|1 - C(s)\|_{\mathcal{L}_1}\|\omega\|_{\mathcal{L}_\infty}. \qquad (32)$$

Since

$$\lim_{\theta_a \to \infty} \|1 - C(s)\|_{\mathcal{L}_1} = 0, \qquad (33)$$

the norm $\|\omega_{r_1} - \omega\|_{\mathcal{L}_\infty}$ can be rendered arbitrarily small by increasing the bandwidth of $C(s)$. Further, $\omega_{r_2}(t)$ decays to zero exponentially and the settling time is inverse proportional to the bandwidth of $C(s)$. Increasing the bandwidth of $C(s)$ implies that $\omega_{r_2}(t)$ decays to zero quickly.

From (26) and (32), when the transients of $C(s)$ due to the initial condition $-\omega(0)$ die out, $\omega_e(t)$ estimates $\omega(t)$ with the final precision given in (24). It is obvious that both the final estimation precision and the transient time can be arbitrarily reduced by increasing the bandwidth of $C(s)$, which leads to smaller $\mathcal{L}_1$ gain for $\|1 - C(s)\|_{\mathcal{L}_1}$. However, the large bandwidth of $C(s)$ leads to further increase of γ_c, which requires large Γ_c to keep the term $\frac{\gamma_c}{\sqrt{\Gamma_c}}$ small. We note that larger Γ_c implies faster computation and requires smaller integration step.

3.5 Extraction of Unknown Parameters

If the time-varying signal $\omega(t)$ can be linearly parameterized in unknown constant parameters and known nonlinear functions, extraction of the unknown parameters can be achieved by recursive least-square (RLS) algorithm under certain persistent excitation type of condition. The RLS algorithm is reviewed below.

Consider a linear scalar regression model denoted as:

$$\omega_k = \theta^\top \phi_k + e_k, \tag{34}$$

where

$$\theta = [\theta_1, \theta_2, \cdots, \theta_n]^\top \tag{35}$$

is the $n \times 1$ vector of the plant parameters, and

$$\phi_k = [\phi_{k,1}, \phi_{k,2}, \cdots, \phi_{k,n}]^\top \tag{36}$$

is the $n \times 1$ regressor vector at time instant k, while e_k is a zero-mean discrete white noise sequence with variance σ_k^2. When the observation of (ω_k, ϕ_k) has been obtained for $k = 1, \cdots, N$ (with $N > n$), the RLS estimate for θ, denoted by $\hat{\theta}$, can be obtained in the following discrete form(Verhaegen, 1989):

$$\begin{aligned} L_k &= \frac{P_{k-1}\phi_k}{\lambda + \phi_k^\top P_{k-1}\phi_k}, \\ \hat{\theta}_k &= \hat{\theta}_{k-1} + L_k(\omega_k - \phi_k^\top \hat{\theta}_{k-1}), \\ P_k &= \frac{1}{\lambda}\left(P_{k-1} - \frac{P_{k-1}\phi_k\phi_k^\top P_{k-1}}{\lambda + \phi_k^\top P_{k-1}\phi_k}\right), \end{aligned} \tag{37}$$

where $P_0 = pI_{p\times p}$ and $\lambda \in (0,1]$. Coefficients p and λ are design gains and need to be chosen appropriately. When ϕ_k is persistently exciting during the observation period, RLS algorithm ensures the convergence of $\hat{\theta}$ to θ. The convergence rate of RLS can be increased by choosing large λ.

The PE condition of the regressor vector is defined as(Verhaegen, 1989):

Definition 3.4 *The regressor vector ϕ_k is persistently exciting over the observation interval $k_0 \le k \le k_N$ with an exponentially forgetting factor $\lambda \le 1$, if the following condition is fulfilled:*

$$\alpha I \le \sum_{k=k_0}^{k_N} \phi_k \phi_k^\top \lambda^{k_N - k} \le \beta I \tag{38}$$

for some positive $\alpha > 0$ and $\beta > 0$.

4 FAST ESTIMATION FOR RANGE IDENTIFICATION IN THE PRESENCE OF UNKNOWN PARAMETERS

Denote

$$\eta(t) = \begin{bmatrix}\eta_1(t)\\ \eta_2(t)\end{bmatrix}, \tag{39}$$

and write equation (7a) as

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = w_s^\top(x_1, x_2)\begin{bmatrix}x_3(t)\\ \theta\end{bmatrix} = \eta(t). \tag{40}$$

From equations (6), (8), and (40), we have

$$\begin{bmatrix} b_1 - b_3x_1 & x_2 & 1+x_1^2 & x_1x_2 \\ b_2 - b_3x_2 & -x_1 & x_1x_2 & 1+x_2^2 \end{bmatrix} \begin{bmatrix}x_3(t)\\ w_1\\ w_2\\ w_3\end{bmatrix} = \eta(t). \tag{41}$$

Multiplying the first equation in (41) by $T_2 = b_2 - b_3x_2(t)$ and subtracting the second equation from it pre-multiplying it by $T_1 = b_1 - b_3x_1(t)$, we arrive at:

$$\begin{aligned} &\underbrace{\left[T_2x_2 + T_1x_1, T_2(1+x_1^2) - T_1x_1x_2, T_2x_1x_2 - T_1(1+x_2^2)\right]}_{\phi^\top(t)} \underbrace{\begin{bmatrix}w_1\\ w_2\\ w_3\end{bmatrix}}_{\theta(t)} \\ &= [T_2\eta_1 - T_1\eta_2]. \end{aligned} \tag{42}$$

Recursive least squares method can be used to extract w_i's according to (37), with ω replaced by $T_2\eta_1 - T_1\eta_2$. Once w_i (for $i = 1,2,3$) are available, equation (41) takes the form:

$$\begin{bmatrix}b_1 - b_3x_1\\ b_2 - b_3x_2\end{bmatrix} x_3 = \begin{bmatrix}\eta_1\\ \eta_2\end{bmatrix} - \begin{bmatrix} x_2 & 1+x_1^2 & x_1x_2 \\ -x_1 & x_1x_2 & 1+x_2^2 \end{bmatrix} \begin{bmatrix}w_1\\ w_2\\ w_3\end{bmatrix}, \tag{43}$$

where $x_3(t)$ can be extracted using pseudo-inverse.

Using the fast adaptive estimator described in Sec. 3, estimation of $\eta(t)$, denoted by $\eta_e(t)$, can be obtained via the following steps:

- **State Estimator:**

$$\begin{bmatrix}\dot{\hat{x}}_1\\ \dot{\hat{x}}_2\end{bmatrix} = A_m \begin{bmatrix}\tilde{x}_1\\ \tilde{x}_2\end{bmatrix} + \hat{\eta}(t), \quad \begin{bmatrix}\tilde{x}_1\\ \tilde{x}_2\end{bmatrix} = \begin{bmatrix}\hat{x}_1 - x_1\\ \hat{x}_2 - x_2\end{bmatrix}. \tag{44}$$

- **Adaptive Law (use large Γ_c):**

$$\dot{\hat{\eta}}(t) = -\Gamma_c P^\top \begin{bmatrix}\tilde{x}_1 & \tilde{x}_2\end{bmatrix}^\top. \tag{45}$$

- **Extraction:**

$$\eta_e(s) = C(s)\hat{\eta}(s), \quad C(s) = \frac{C}{s+C}. \tag{46}$$

According to Corollary 3.1, the final estimation precision $\eta_e(t) - \eta(t)$ and the transient time to achieve this can be arbitrarily reduced by increasing the bandwidth of $C(s)$. Increasing the bandwidth of $C(s)$ requires larger Γ_c.

The flow chart of state and parameter estimation of a rigid motion using the fast adaptive estimator is illustrated in Fig. 1. In the first step of estimating $\eta(t)$, both the estimation precision and transient time can be arbitrarily reduced by increasing the bandwidth of $C(s)$ and using larger Γ_c. In the second step of extracting $\hat{w}_i$'s from $\eta_e(t)$ using the recursive least square method, fast speed can be achieved by properly tuning the RLS gains. Estimation of $x_3(t)$, denoted by $\hat{x}_3(t)$, can be obtained from $\eta_e(t)$ and $\hat{w}_i$'s via pseudo-inverse. Since the fast adaptive estimator assumes minimization of the $\mathcal{L}_1$ gain of $1 - C(s)$ for performance improvement, it is referred to as $\mathcal{L}_1$ adaptive estimator.

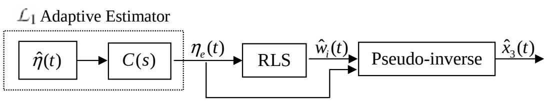

Figure 1: Flow chart of $\mathcal{L}_1$ adaptive estimator.

5 SIMULATION RESULTS

State estimation of $[x_3(t), \theta^\top]^\top$ using the IBO observer (11) and the fast adaptive estimator (44) (46) are implemented in Matlab, where the motion dynamics are selected to be

$$\begin{bmatrix}\dot{X}(t)\\ \dot{Y}(t)\\ \dot{Z}(t)\end{bmatrix} = \begin{bmatrix}0 & -4 & -0.8\\ 4 & 0 & -0.6\\ 0.8 & 0.6 & 0\end{bmatrix}\begin{bmatrix}X(t)\\ Y(t)\\ Z(t)\end{bmatrix} + \begin{bmatrix}10\\ 3\pi\sin(2\pi t)\\ 3\pi\sin(2\pi t + \pi/4)\end{bmatrix},$$
$$(X_0, Y_0, Z_0) = (1, 1.5, 2.5), \quad x_0 = (X_0/Z_0, Y_0/Z_0, 1/Z_0). \tag{47}$$

First, we present simulation results in the ideal case with no measurement noise. The parameters for the IBO observer and the fast adaptive estimator are chosen to be:

- IBO (referring to (11)):
  $G = 10$, $(\hat{x}_3(0), \hat{w}_1(0), \hat{w}_2(0), \hat{w}_3(0)) = (0, 0, 0, 0)$.
- Fast adaptive estimator (referring to (37), (45), (46)):
  $p = 100$, $\lambda = 0.99999$, $A_m = -I_2$,
  $(\hat{\eta}_1(0), \hat{\eta}_2(0)) = (0, 0)$, $\Gamma_c = 2 \times 10^8$, $C = 200$.

In both cases, we set $(\hat{x}_1(0), \hat{x}_2(0)) = (x_1(0), x_2(0))$, $M = 30$, $A = I_2$, $P = -1/2 \times I_2$, where I_2 denotes the 2×2 identity matrix.

Estimation of w_i (for $i = 1, 2, 3$) with the use of the IBO and the fast adaptive estimator is shown in Figures 2 and 3, respectively. Figure 4 shows the zoomed version of Figure 3 for the steady state error. State estimation error of x_3 is plotted in Figure 5 for comparison of both methods.

From Figures 2 and 3, it can be observed that the fast adaptive estimator achieves faster estimation of the motion parameters. The same is true for x_3.

Simulation results are also presented in Figs. 6∼9 when the output is noise-corrupted with uniform bound $\pm 10^{-2}$. The simulation parameters are the same as above. In this case, when extracting $\hat{x}_3(t)$, the output from the pseudo-inverse is further processed

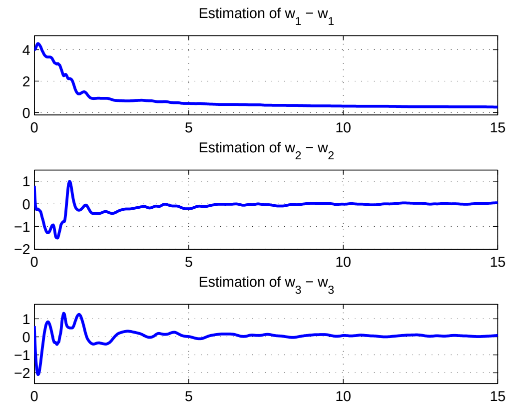

Figure 2: Estimation of motion parameters **using IBO** (without measurement noise).

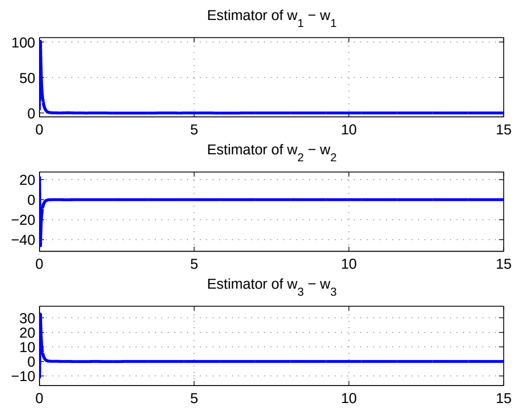

Figure 3: Estimation of motion parameters **using fast adaptive estimator** (without measurement noise).

using a low-pass filter $\frac{30}{s+30}$ to give the final state estimation. We observe that corresponding plots with or without measurement noise are very similar.

6 FURTHER EXTENSION

In this paper, rigid-body motion is considered that contains only three rotational parameters (w_1, w_2, w_3) as given in (1). For general affine motion described by

$$\begin{bmatrix}\dot{X}(t)\\ \dot{Y}(t)\\ \dot{Z}(t)\end{bmatrix} = \begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}\begin{bmatrix}X(t)\\ Y(t)\\ Z(t)\end{bmatrix} + \begin{bmatrix}b_1\\ b_2\\ b_3\end{bmatrix}, \tag{48}$$

the rotational matrix contains nine parameters. Assuming that the $[a_{ij}]$ (for $i, j = 1, 2, 3$) are unknown

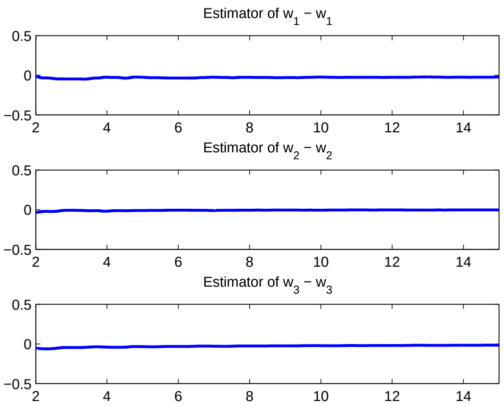

Figure 4: Enlarged view of Fig. 3 (without measurement noise).

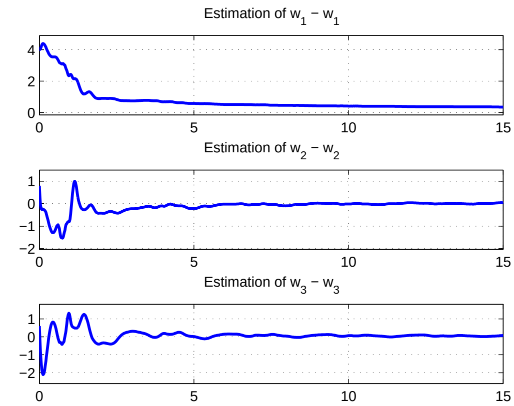

Figure 6: Estimation of motion parameters **using IBO** (with measurement noise).

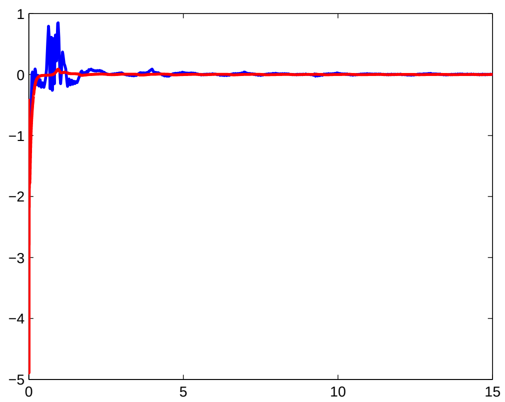

Figure 5: Comparison of state estimation errors (without measurement noise).

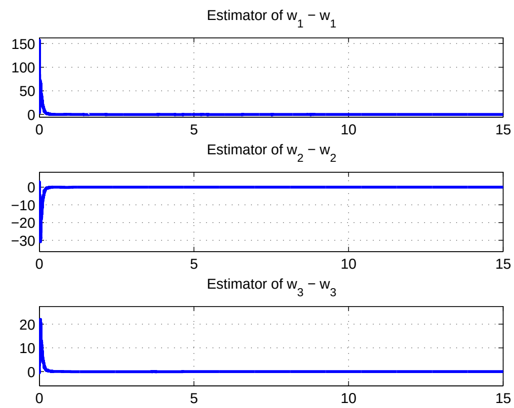

Figure 7: Estimation of motion parameters **using fast adaptive estimator** (with measurement noise).

constants, the method described in Sec. 4 cannot lead to extraction of the nine unknown parameters in a straightforward way.

The system (48) with output observations (3) is equivalent to the system

$$\begin{cases} \begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix} b_1-b_3x_1\\ b_2-b_3x_2\end{bmatrix} x_3 + \begin{bmatrix} a_{13}+(a_{11}-a_{33})x_1 \\ a_{23}+a_{21}x_1\end{bmatrix} \\ \qquad + \begin{bmatrix} a_{12}x_2 - a_{31}x_1^2 - a_{32}x_1x_2 \\ (a_{22}-a_{33})x_2 - a_{31}x_1x_2 - a_{32}x_2^2\end{bmatrix}, \\ \dot{x}_3(t) = -(a_{31}x_1+a_{32}x_2+a_{33})x_3 - b_3x_3^2, \end{cases} \tag{49}$$

with the output (5). The above system can also be rewritten in the form of (7a), where θ and $w_s^\top(x_1,x_2)$ take the forms

$$\theta = [a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33}]^\top, \tag{50}$$

and

$$w_s^\top(x_1,x_2) = \begin{bmatrix} b_1-b_3x_1 & x_1 & x_2 & 1 & 0 & 0 & 0 & -x_1^2 & -x_1x_2 & -x_1 \\ b_2-b_3x_2 & 0 & 0 & 0 & x_1 & x_2 & 1 & -x_1x_2 & -x_2^2 & -x_2 \end{bmatrix}, \tag{51}$$

respectively. Following the logic in Sec. 4, we can write the following system of algebraic equations

$$w_s^\top(x_1,x_2)\begin{bmatrix} x_3 & a_{11} & a_{12} & \cdots & a_{33}\end{bmatrix}^\top = \begin{bmatrix}\eta_1\\ \eta_2\end{bmatrix}, \tag{52}$$

with the $w_s^\top(x_1,x_2)$ given in (51). Again, multiplying the first equation in (52) by $T_2 = b_2 - b_3x_2$ and subtracting the second equation from it pre-multiplying it

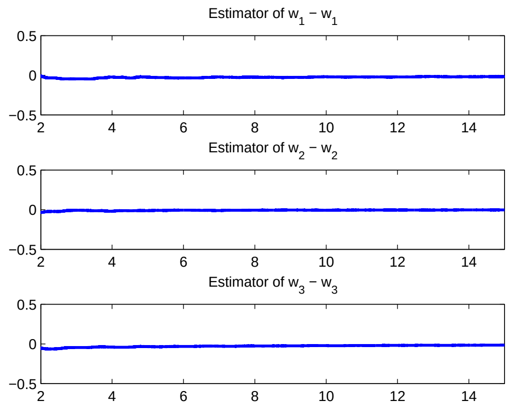

Figure 8: Enlarged view of Fig. 7 (with measurement noise).

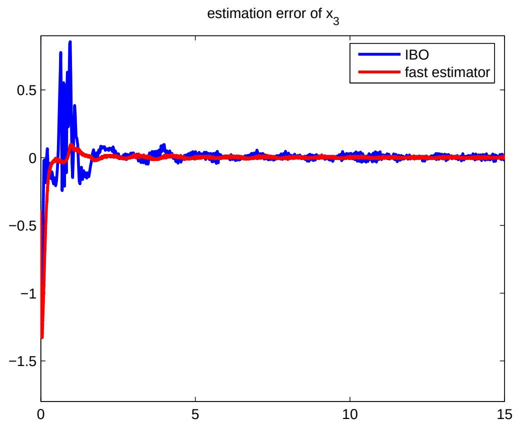

Figure 9: Comparison of state estimation errors (with measurement noise).

by $T_1 = b_1 - b_3 x_1$, we arrive at:

$$\underbrace{[T_2(x_1, x_2, 1),\, T_1(x_1, x_2, 1),\, (b_1 x_2 - b_2 x_1)(x_1, x_2, 1)]}_{\phi_{\text{affine}}(t)} \begin{bmatrix} a_{11} \\ a_{12} \\ \vdots \\ a_{33} \end{bmatrix} = [T_2 \eta_1 - T_1 \eta_2]. \tag{53}$$

The nine columns in $\phi_{\text{affine}}(t)$ in (53) are linearly dependent. It is obvious that the $7^{\text{th}}, 8^{\text{th}}$, and 9^{th} columns can be presented as linear combinations of the first six columns. For example, column_9 can be written as $\text{column}_9 = \text{column}_5 - \text{column}_1$. Thus, extraction of the nine unknown parameters cannot be performed by the recursive least square method since it violates the PE condition in (38). Further research will explore the use of adaptive observers for general affine motion identification.

7 CONCLUSION

A recently developed fast adaptive estimator is applied to the range identification problem of a rigid motion in the presence of unknown motion parameters. Fast convergence speed is achieved compared to existing nonlinear perspective observers.

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