

# GPC BASED ON OPERATING POINT DEPENDENT PARAMETERS LINEAR MODEL FOR THERMAL PROCESS

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**Abstract:** This paper presents the application of generalized predictive control strategy (GPC) based on an OPDPLM (Operating Point Dependent Parameters Linear Model) structure to a heating and ventilation nonlinear-subsystem of a complex passive air-conditioning unit. For this purpose, several discrete-time models were identified with respect to measurable exogenous events. The parameters of the identified models change according operating conditions (sliding opening window). The objective of the studied subsystem was to guarantee a microclimate with controlled temperature set-points. The on line adaptive strategy was implemented to compute the controller parameters in order to adapt to the operating conditions variations. Efficiency of the resulting algorithm is illustrated by a real experiment.

## 1 INTRODUCTION

The consumption of energy by Heating, Ventilating, and Air Conditioning (HVAC) equipments in industrial and commercial buildings constitutes 50% of the world energy consumption (Arguello-Serrano and Vélez-Reyes 1999). Growing crop in greenhouse is one of important branch of agriculture industry and, it is labour intensive and technically challenging business. Optimized control helps to increase production despite saving precious sources (Young and Lees 1994). Standard air-conditioning units which are used in environment control for the growth chambers are usually composed of heating elements, a cooling system with compressor and evaporator techniques (Albright 2001), (Jones *et al.*, 1984), (Hanan 1997). The air-conditioning unit studied is passive and does not use the more typical compression system or absorption-refrigeration cycle (Tawegoum *et al.*, 2006a, Riadi *et al.*, 2006). The specificity of the system is to produce a variable microclimate with variable temperatures and variable relative humidity set-point values.

A complete physical model of this plant developed in (Riadi *et al.*, 2006), showed that the

global system is complex and composed of three HVAC nonlinear subsystems. Therefore the implementation of centralized control strategy is cumbersome and decreases reliability. For these reasons, a typical local-loop control configuration for each subsystem of this air conditioning unit will be more efficient. For such control loops, self tuning controller parameters is usually considered, and the present study is focused only on one single-input-output (SISO) non linear subsystem, with multiple operating modes. Many adaptive strategies using recursive estimator are generally applied on thermal process (Arguello-Serrano and Vélez-Reyes 1999), (Landau and Dugard 1986), (Ljung 1999), especially, efficient, when parameters values are slowly varying. In our case, an idea about the model structure is possible and the parametric disturbance factor is measurable.

A generalized predictive control strategy, based on online controller parameters adaptation, was used to ensure stability and desired performances. Subsystems operating points were modeled by a linear structures. These models should be fairly close in their structure but with different parameters values. The different models were identified for the main

operating points and the outdoor disturbance variations were taken into account and were parameterized using polynomial function interpolation in order to provide a single structure, called Operating Point Dependent Parameters Linear Model (OPDPLM) (Lakhdari et al., 1994), (Landau et al. 1987).

This paper proceeds as follows. The problem statement is presented in section II. In section III, the process control problem in presence of multiple operating modes is formulated and the proposed GPC (Generalized Predictive Control) based on OPDPLM is presented. The last section shows the real-time implementation and the experimental results are discussed.

## 2 SYSTEM DESCRIPTION AND PROBLEM STATEMENT

Figure 1 outlines the overall design of the proposed air-conditioning unit. The main features of the unit are: a humid climate, operation without a freezing unit, equilibrium of head losses and minimum energy consumption. Moisture removal is not required.

The unit is composed of two flows: a non-saturated flow (or non-saturated duct) and a saturated flow (saturated duct). As shown in Figure 1, in the saturated air flow, fresh air is saturated on with water after being heated by a coil resistor. Saturation operates at constant enthalpy (Chraibi et al., 1997). The saturated duct subsystem consists of a closed system, including a suction pump, a water tank and cross-corrugated cellulosic pads of the type using in cooling. The suction pump carries water from the tank to the top of the pads. Once the saturation steady state is reached, the pads contain a constant mass of water with a given water output and a given temperature.

In the unsaturated duct subsystem, fresh air is only heated by another resistor coil to a desired temperature

The proportional mixing of the two air flows is carried out by a sliding window driven by a DC motor.

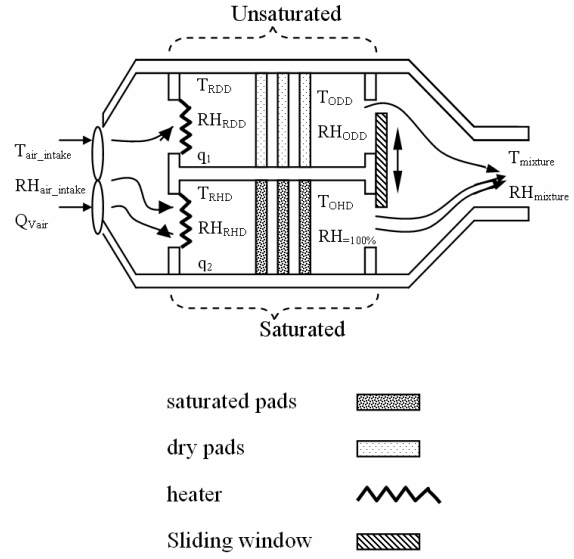


Figure 1: Complete air-conditioning system.

In this paper, we are interested in the unsaturated duct subsystem. The purpose control strategy is to regulate the temperature  $T_{ODD}(t)$  ( $^{\circ}\text{C}$ ) of the outgoing air at constant temperature reference  $T_r$  ( $^{\circ}\text{C}$ ), in spite of air flows variations through the unsaturated duct and in spite of air intake temperature behaviour  $T_{air\_intake}$  ( $^{\circ}\text{C}$ ). The air flow varied by changing the sliding window percentage,  $x$  (%). This latter is close to air flow by Eq(1).

$$q_1 = \alpha(x)Q_{Vair} \quad (1)$$

The heat balance in the unsaturated duct is given by the following equation:

$$\frac{dT_{ODD}}{dt} = \frac{\alpha(x)Q_{Vair}}{V_{DD}} [T_{air\_intake} - T_{ODD}] + \frac{k_{RDD}}{\rho_{air}C_{air}V_{DD}} U_{DD} \quad (2)$$

with  $U_{DD}$  the applied voltage (V), proportional to the resistor heating in the dry duct,  $k_{RDD}$  the proportional coefficient between the voltage and the heating-power (J/sV),  $V_{DD}$  the volume of the dry duct ( $\text{m}^3$ ).

### 3 GENERALIZED PREDICTIVE CONTROL OF NONLINEAR CLASS SYSTEM

#### 3.1 The Operating Point Dependent Parameters-linear Model

The existence of a system with parameters depending on the operating point (the particular case of affine systems) means that one or more parameters of a linear differential equation vary according to an auxiliary variable  $\zeta$ , which represents the operating point (Landau et al., 1987). An application to the temperature identification of a helium bath cryostat is presented in (Lakhdari et al., 1994), and it is also used in (Tawegoum et al., 2006b) for climate identification nearly steady weather conditions. This variable can be calculated via the input or output process, or via another measurable variable related to the operating point. The Operating Dependent Parameter Linear Model (OPDPLM) has the following properties:

- It allows the description of the non-linear phenomena with regard to the operating point  $\zeta$ .
- It makes it possible to extend the linear formalism to systems that are not linear.

The system input-output form given by “(3), (4), and (5)” is as follows:

$$A(\zeta(t), d, q^{-1}) \cdot \Delta Y(t) = q^{-d} \cdot B(\zeta(t), d, q^{-1}) \cdot \Delta U(t) \tag{3}$$

$A(\zeta(t), d, q^{-1})$  is polynomial in  $q^{-1}$ , depending on the delay  $d$ , nonlinear with respect to  $\zeta(t)$ , and defined by:

$$A(\zeta(t), d, q^{-1}) = 1 + \sum_{i=1}^{na} a_i(\zeta(t-d-i)) \cdot q^{-i} \tag{4}$$

The polynomial  $B(\zeta(t), q^{-1})$  is non-linear in  $\zeta(t)$ , polynomial in  $q^{-1}$ , and is defined by

$$B(\zeta(t), q^{-1}) = \sum_{i=0}^{nb} b_i(\zeta(t-i)) \cdot q^{-i} \tag{5}$$

$na, nb$  are respectively the polynomial degrees of

$A(\zeta(t), d, q^{-1})$  and of  $B(\zeta(t), q^{-1})$ , issued from the identification process.

The parameters  $a_i(\zeta(t))$  and  $b_i(\zeta(t))$  can be modeled by polynomial functions of order  $\eta_1$  and  $\eta_2$  as follows:

$$a_i(\zeta(t)) = \sum_{j=0}^{\eta_1} a_{ij} \zeta^j(t) \tag{6}$$

$$b_i(\zeta(t)) = \sum_{j=0}^{\eta_2} b_{ij} \zeta^j(t) \tag{7}$$

In our case, the percentage of the window opening represents the operating point ( $\zeta = x$ ).

#### 3.2 GPC Design based on OPDPLM

The basic idea of the GPC (Clarke et al., 1987 a), (Clarke et al., 1987 b), (Camacho and Bordons 1998) is to calculate a sequence of future control signals in such way that it minimizes a multistage cost function defined over a control horizon. The index to be optimized is normally the expectation of a function measuring the distance between the predicted system output and some predicted references sequence over the control horizon plus a function measuring the control effort on the same horizon.

Consider the plant described by CARIMA (Controlled Auto-Regressive Integrated Moving Average) model in OPDPLM case:

$$A(\zeta, q^{-1})y(t) = B(\zeta, d, q^{-1})u(t-1) + C(\zeta, q^{-1}) \frac{\varepsilon(t)}{\Delta(q^{-1})} \tag{8}$$

The optimal  $j$ -step predictor defined between  $N_1$  and  $N_2$  is given by:

$$\hat{y}(t+j) = F_j(\zeta, q^{-1})y(t) + H_j(\zeta, q^{-1})\Delta u(t-1) + G_j(\zeta, q^{-1})\Delta u(t+j-1) \tag{9}$$

Where polynomials  $F_j, G_j, H_j$  are solutions of the following Diophantine equations:

$$\Delta(q^{-1})A(\zeta, q^{-1})J_j(\zeta, q^{-1}) + q^{-j}F_j(\zeta, q^{-1}) = 1 \tag{10}$$

$$G(\zeta, q^{-1}) + q^{-j}H_j(\zeta, q^{-1}) = B(\zeta, q^{-1})J_j(\zeta, q^{-1}) \tag{11}$$

The cost function is given by

$$J(N_1, N_2, N_u, \lambda) = E \left\{ \sum_{j=N_1}^{N_2} [(y(t+j) - w(t+j))]^2 + \sum_{j=N_1}^{N_u} \lambda [\Delta u(t+j-1)]^2 \right\} \quad (12)$$

with  $N_1$  the minimum prediction horizon;  $N_2$  the maximum prediction horizon,  $N_u$  the control costing horizon and  $\lambda$  a control-weighting.

Minimizing the cost function yields the control law

$$\Delta U_{opt} = (G^T(\zeta, d) \cdot G(\zeta, d) + \lambda I)^{-1} G^T(\zeta, d) \cdot (W - Y_1) \quad (13)$$

The previous equation can be written as:

$$\Delta U_{opt} = M(\zeta, d) \cdot (W - Y_1) \quad (14)$$

Where

$$M = (G^T(\zeta, d) \cdot G(\zeta, d) + \lambda I)^{-1} G^T(\zeta, d) \quad (15)$$

With  $G$  a  $(N_2 - N_1 + 1) \times N_u$  matrix.

Its elements are the coefficients of step response depending on the operating point.

$W = [w(t + N_1), w(t + N_1 + 1), \dots, w(t + N_2)]^T$  is the reference signal within the prediction horizon;

$Y_1 = [y_1(t + N_1), y_1(t + N_1 + 1), \dots, y_1(t + N_2)]^T$  is the prediction based on the past measurements.

Notice that  $\Delta U$  is not a scalar but a vector which can be written as:

$$\Delta U_{opt} = [\Delta u(t)_{opt} \quad \Delta u(t+1)_{opt} \quad \dots \quad \Delta u_{opt}(t + N_u)]^T \quad (17)$$

In real time control, only the first value of Eq (14) is finally applied to the system, according to the receding horizon strategy.

$$\Delta u_{opt} = m_1(\zeta, d) \cdot (W - Y_1) \quad (16)$$

With  $m_1(\zeta, d)$  is the first line of the matrix  $M$

The RST polynomial controller structure given by (Dumur et al., 1997) can be extended for OPDPLM formalism (figure. 2) as:

$$S(\zeta, q^{-1}) \Delta(q^{-1}) u(t) = -R(\zeta, q^{-1}) y(t) + T(\zeta, q) w(t) \quad (18)$$

With,

$$S(\zeta, q^{-1}) = 1 + m_1(\zeta) H(\zeta, q^{-1}) q^{-1} \quad (19)$$

$$R(\zeta, q^{-1}) = m_1(\zeta) F(\zeta, q^{-1}) \quad (20)$$

$$T(\zeta, q) = m_1(\zeta) [q^{N_1} \dots q^{N_2}] \quad (21)$$

Where

$$F = [F_{N_1}(\zeta, q^{-1}) \dots F_{N_2}(\zeta, q^{-1})]^T \quad (22)$$

$$H = [H_{N_1}(\zeta, q^{-1}) \dots H_{N_2}(\zeta, q^{-1})]^T \quad (23)$$

The adapting phase of regulator parameters can be performed according stability and robustness, by considering the updating the controller parameters as:  $N_1 = 1$ , and  $\lambda_{opt}(\zeta) = \text{trace}(G^T(\zeta) \cdot G(\zeta))$ .

The closed loop stability using the equivalent RST controller structure was studied in (Dumur et al, 1997), (M'saad and Chebassier.)

## 4 EXPERIMENTAL RESULTS

A set of electronic units was used to apply heating voltage on the resistors or to control the DC motor and thus, the window opening rate. Measurements were carried out using Pt100 sensors for temperature, and encoder sensors for position window. A sampling interval of  $T_e=30$  sec was chosen to satisfy the predominant time constant, and data acquisition time varied from two to four hours, depending on the operating point values  $\zeta = x \in ]0\%, 100\%[$  for a large interval variation .

### 4.1 Discrete Model Identification for Different Operating Modes

The air-flow measurements for the main window positions indicate a nonlinear relationship between the air-flow percentage and the window opening percentage (Tawegoum et al., 2006c). Therefore, in the identification process, the parameters of the model describing the output temperature behavior of the conditioning unit were assessed using the ARX model for each window position (*i.e.* for each operating point). A linear difference equation of the type of structure case is given in (Landau et al., 1987):

$$Y_{Mf}(k) + \sum_{i=1}^{nj} a_{ji}(x) Y_{Mf}(k-i) = \sum_{s=1}^m \sum_{i=1}^{nj} b_{jsi}(x) U_s(k-i-r_{js}) \quad (24)$$

The choice of ARX structures was based on their advantageous application in digital models, i.e. use of simpler and effective estimation algorithms and because of their easy and flexible usage in computer software (Borne et al., 1990).

The ARX model obtained for the temperature model, in the non-saturated flow, is given in (Riadi et al., 2006):

$$(1 + a1(x)q^{-1} + a2(x)q^{-2}) \cdot T_{ODD}(t) = b1(x)q^{-1} \cdot U_{DD} + b2(x)q^{-1} T_{air\_intake} + e(t) \quad (25)$$

where  $a_i(x)$  and  $b_i(x)$  are four-degree polynomials, depending on  $x$ , the window opening percentage:

$$\begin{aligned} a1(x) &= 13.6201x^4 - 34.3282x^3 + 30.6083x^2 - 11.0663x \\ a2(x) &= -3.4418x^4 + 8.8203x^3 - 8.2317x^2 + 3.0954x \\ b1(x) &= -1.6383x^4 + 3.6847x^3 - 2.7780x^2 + 0.7962x \\ b2(x) &= -0.4537x^4 + 1.0765x^3 - 0.8865x^2 + 0.3385x \end{aligned} \quad (26)$$

### 4.2 Results of Strategy Control

The control parameters were chosen as follows: the minimum prediction horizon  $N_1 = 1$ , the maximum prediction horizon  $N_2 = 14$ , the control costing horizon  $N_u = 7$ , and the control weighting  $\lambda(\zeta) = 0.93 \text{ trace}(G^T(\zeta) \cdot G(\zeta))$ .

Figure 3 illustrates the temperature response over a ten hours period, when applying the GPC strategy based on the OPDPLM elaborated in (13), subject to external temperature disturbances and to parameters disturbances by varying the window opening.

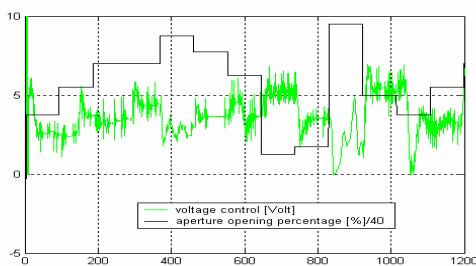


Figure 2: The window opening rate and voltage control.

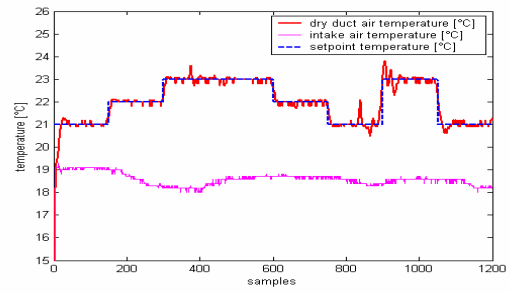


Figure 3: Regulation of temperature of non saturated using GPC based on OPDPLM.

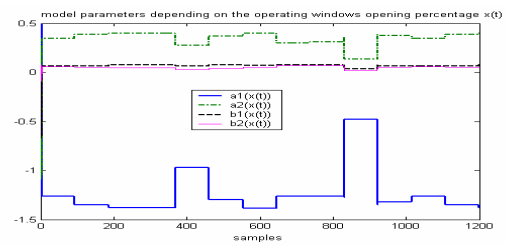


Figure 4: Parameter evolution of the model system.

The objectives of stability and energy minimization are reached (disturbances rejection and robustness stability).

As it can be seen the temperatures reach their setpoints in a very short time, exhibiting a small overshoot. It can also be observed that an interaction exists between the variation of the aperture position and the output temperature.

Figure.3 shows that, consecutive to the switching parameters (window moving), oscillatory behavior appears on the output temperature response, due to the control input discontinuities. The GPC algorithm shows robustness in spite of these disturbances.

## 5 CONCLUSION

This paper has presented an application of the generalized predictive control using the OPDPLM structure of nonlinear thermal process. Stability is maintained with an adequate choosing of controller parameters values. The performances are maintained in spite of parameters system variation and controller disturbance rejection is capable to reduce the effect of thermal loads, with a simple updating of the regulator parameters depending on operating points.

The control strategies will be performed with an introduction of an overshoot constraint on the output temperature and with robust techniques of the GPC

algorithm.

Further investigations on the decentralized architecture make it possible to extend this local control strategy to other part of the complex air conditioning unit.

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