MULTIPLE-MODEL DEAD-BEAT CONTROLLER IN CASE OF CONTROL SIGNAL CONSTRAINTS

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Abstract: The task of achieving a dead-beat control by a linear DB controller under control constraints is presented in this paper. Two algorithms using the concept of multiple-model systems are proposed and demonstrated - a multiple-model dead-beat (MMDB) controller with varying order using one sampling period and a MMDB controller with fixed order using several sampling periods. The advantages and disadvantages of these controllers are summarized.

1 INTRODUCTION

The Dead-Beat (DB) control problem in discrete time control theory consists of finding an input signal, which provides a transient response in a minimum number of sampling time steps. It has been studied by many researchers, e.g. (Jury, 1958), (Kucera, 1980), (Kaczorek, 1980), (Isermann, 1981), etc. If an n^{th} order linear system is null controllable, this minimum number of steps is n, as the applied feedback provides all poles of the closed-loop transfer function at the z-plane origin. The linear case is easy to solve, but DB control for non-linear systems is an open research problem (Nesic et al., 1998).

The DB controller of normal order (Isermann, 1981), denoted as DB(n,d), provides a constant control action after $n_s = (n+d)$ sampling steps, where d is the plant delay. For small sampling period the linear DB(n,d) controller forms extremely high control values at the first and second sampling steps after a step change of the system reference signal. In general, the control valve constrains the control signal, so these high amplitudes cannot be passed to the plant, thus making the system to be non-linear.

One way to solve the problem of constrained control signal, and still keeping the system as linear, is to prolong the transient response by increasing the controller order n_s. Isermann (1981) suggested increased by one order DB(n,d,1) controller, so the transient response takes $n_s = (n+d+1)$ sampling steps with decreased control value compared to the DB(n,d). This approach did not have essential practical application, but suggested two ideas:

- a higher controller order reduces the maximal amplitude of the control action;
- linear dead-beat control can be achieved by flexible tuning of the controller numerator coefficients.

In (Garipov and Kalaykov, 1991) an approach for design of adaptive DB(n,d,m) controller is presented, where the order increment m is sequentially changed until the control signal fits the control constraints. The reduction of the control magnitude pays off the prolongation of the transient response, as the signal energy distributes in more sampling time steps. Another approach is to increase the system sampling period without losing information. A control system with two sampling periods is proposed in (Garipov and Stoilkov, 2004) as a compromise solution.

These last two above mentioned approaches are useful for generalizing them by merging and involving various aspects of the multiple-model concept, as presented in (Murray-Smith and Johansen, 1997). In the present paper the task is solved by multiple-model dead-beat controller (MMDB) for one fixed and several sampling periods of the control system.

In Section 2 we present the theoretical base for design of DB controller of increased order. In Section 3 we describe the operation principle of DB control based on two sampling periods. In Section 4 the MMDB controller concept is developed in two variants. The first is based on a set of DB controllers of increased order in a system with one sampling period.

The second is utilizing a set of normal order DB controllers designed for several sampling periods. The concluding section summarizes the main properties of the proposed DB controllers.

2 DESIGN OF DB CONTROLLER OF INCREASED ORDER

Let the control plant description be:

$$W_o(z) = \frac{B(z)}{A(z)} z^{-d} =$$

$$= \frac{b_1 z^{-1} + b_2 z^{-2} + \ldots + b_n z^{-n}}{1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_n z^{-n}} z^{-d} \quad (1)$$

According (Garipov and Kalaykov, 1991) the designed DB(n,d,m) controller is

$$W_p(z) = \frac{Q(z)}{1 - z^{-d} P(z)} =$$

$$= \frac{q_0 + q_1 z^{-1} + \ldots + q_{n+m} z^{-(n+m)}}{1 - z^{-d}(p_1 z^{-1} + p_2 z^{-2} + \ldots + p_{n+m} z^{-(n+m)})} \quad (2)$$

The vector θ of (2n+2m+1) unknown coefficients of the DB controller can be determined from the following matrix equation

$$X\,\theta = Y, \quad (3)$$

$$X = \begin{bmatrix} & X^* & \\ \cdots & \cdots & \cdots \\ D_z & \vdots & Z \end{bmatrix}, Y = \begin{bmatrix} Y^* \\ \cdots \\ D_y \end{bmatrix}, \theta = \begin{bmatrix} p^{(1)} \\ p^{(2)} \\ q^{(1)} \\ q^{(2)} \end{bmatrix},$$

$$X^* = \begin{bmatrix} & E_1 & & \vdots & & D_e & \\ \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & \cdots \\ & A_1 & & \vdots & & -B_1 & \\ \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & \cdots \\ D_a & \vdots & A_2 & \vdots & D_b & \vdots & -B_2 \end{bmatrix},$$

$$\dim X^* = (2n+m+1) \times (2n+2m+1),$$

$$Y^* = \begin{bmatrix} 1 \\ 0 \\ . \\ 0 \end{bmatrix}, p^{(1)} = \begin{bmatrix} p_1 \\ p_2 \\ . \\ p_m \end{bmatrix}, p^{(2)} = \begin{bmatrix} p_{1+m} \\ p_{2+m} \\ . \\ p_{n+m} \end{bmatrix},$$

$$q^{(1)} = \begin{bmatrix} q_0 \\ q_1 \\ . \\ q_m \end{bmatrix}, q^{(2)} = \begin{bmatrix} q_{1+m} \\ q_{2+m} \\ . \\ q_{n+m} \end{bmatrix}.$$

$$\dim Y^* = (2n+m+1) \times 1,$$

$$A_1 = \begin{bmatrix} a_0 & 0 & . & . & . & . & 0 \\ a_1 & a_0 & 0 & . & . & . & 0 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ a_n & a_{n-1} & . & a_0 & 0 & . & 0 \\ 0 & a_n & . & . & a_0 & . & 0 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 0 & 0 & . & a_n & a_{n-1} & . & a_0 \end{bmatrix},$$

$$\dim A_1 = (n+m) \times (n+m),$$

$$B_1 = \begin{bmatrix} b_1 & 0 & . & . & . & . & 0 & 0 \\ b_2 & b_1 & 0 & . & . & . & 0 & 0 \\ . & . & . & . & . & . & . & . \\ b_n & b_{n-1} & . & b_1 & . & . & 0 & 0 \\ 0 & b_n & . & . & b_1 & 0 & . & 0 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ 0 & . & 0 & b_n & . & . & b_1 & 0 \end{bmatrix},$$

$$\dim B_1 = (n+m) \times (n+m+1),$$

$$A_2 = \begin{bmatrix} a_n & a_{n-1} & . & a_1 \\ 0 & a_n & a_{n-1} & a_2 \\ . & . & . & . \\ 0 & 0 & . & a_n \end{bmatrix},$$

$$B_2 = \begin{bmatrix} b_n & b_{n-1} & . & b_1 \\ 0 & b_n & . & b_2 \\ 0 & 0 & . & . \\ 0 & 0 & . & b_n \end{bmatrix}$$

$$\dim A_2 = n \times n, \dim B_2 = n \times n,$$

$$E_1 = [\,1\ 1\ 1\ \ldots\ 1\,],$$

D_e, D_a, D_z, D_y are matrices with zero elements, $\dim D_e = 1 \times (n+m+1)$, $\dim D_a = n \times m$, $\dim D_b = n \times (m+1)$, $\dim D_z = m \times (n+m), \dim D_y = m \times 1$.

The only solution of (3), which is the goal of dead-beat controller design task, is achieved when the rank of the linear system (3) is full. In fact this depends on the initially undetermined block matrix **Z,** $\dim Z = m \times (n+m+1)$. The z_{ij}values can be chosen in accordance with intention of the designer to guarantee desired control $u(k)$ such that additional m behavior conditions based on the following dependencies between parameters and signals:

a) When step change of the reference signal takes place at the k^{th} sampling step, the DB controller normally produces the largest positive amplitude $u(k)$ at k^{th} sampling step, followed by a smaller and negative value $u(k{+}1)$ at $(k{+}1)^{th}$ sampling step. Therefore, if the signal energy after the k^{th} sampling step is distributed over two or more sampling steps, holding the control signal, the large control magnitudes will be reduced (Isermann, 1981). This can be described by the inequality

$$Mod\{u(k+i)\}|_{u(k+i+1)=u(k+i)} < Mod\{u(k+i)\}|_{u(k+i+1)\neq u(k+i)}$$

$i = 0, 1, \ldots$, which should be related to the initially determined physical constraints on the control $u(k)$.

b) The matrix **Z** is needed only for dead-beat controllers of increased order, i.e. only when $m > 1$. Each row of it consists of one additional simple condition based on Isermann's idea for holding the previous value of the control signal

$$u(k+i+1) = u(k+i), \quad i = 0,\ 1,\ \ldots, \qquad (4)$$

for certain number of time steps. According (Garipov and Kalaykov, 1991), such behavior can be obtained by properly setting the coefficients of the polynomial $Q(z)$ of $(n+m)^{th}$ order. As always $q_0 \neq 0$ and $q_{n+m} \neq 0$, if we set $q_{i+1} = 0$ we obtain the desired condition $u(k+i+1) = u(k+i)$. Therefore the values z_{ij} play a special role of pointing which coefficient q_{i+1} is selected to be zero. When all values z_{ij} =0, it is assumed all coefficients $q_{i+1}$are nonzero. Therefore, first we have to zero the matrix **Z** and then set one unit value in the rows of **Z**. More details for how to select the values are given in (Garipov and Kalaykov, 1991).

c) If we want to hold the control signal longer time according condition (4), we have to zero more neighbor coefficients in $Q(z)$ by manipulating two or more neighbor rows of **Z**.

As an illustrative example let us take a plant with a continuous transfer function

$$W_o(s) = \frac{2s+1}{(10s+1)(7s+1)(3s+1)}e^{-4s}$$

For a sampling period T_o= 4 sec. we get

$$W_o(z) = \frac{0.06525z^{-1} + 0.04793z^{-2} - 0.00750z^{-3}}{1 - 1.49863z^{-1} + 0.70409z^{-2} - 0.09978z^{-3}}z^{-1},$$

$n_a = n_b = n = 3,\ d = 1$

Three dead-beat controllers with different structures: DB(3,1,0), DB(3,1,1) – three variants and DB(3,1,2) – six variants are designed according to the approach (Garipov and Kalaykov, 1991). It these variants some of the $Q(z)$ coefficients were zeroed. Obviously, the bigger is m the more variants of zeroing exist. Table 1 represents the maximum and minimum control values of the control signal during the transient response. The normal order DB controller (m=0) provides the largest values, while *variant1* when $m = 1$ and $m = 2$ provide significantly smaller values, which could fit to the control signal constraints.

Table 1: Max and min control values for the example.

m	Variant #	$u_{\max}$	$u_{\min}$
0		***9.46***	-4.71
1	variant1	***3.78***	-2.05
1	variant2	6.43	-0.18
1	variant3	8.28	-2.95
2	variant1	***2.34***	-0.83
2	variant2	3.01	0.28
2	variant3	3.49	-0.14
2	variant4	5.13	0.62
2	variant5	5.94	0.12
2	variant6	5.94	-2.27

3 DEAD-BEAT CONTROLLER IN A SYSTEM WITH TWO DIFFERENT SAMPLING PERIODS

The concept of DB controller of increased order, as described in the previous section, is one way of holding the control signal during more sampling steps of the transient response and consequently redistributing the signal energy in time. In this section we present an alternative approach employing nearly the same idea for redistributing the signal energy in time. To prolong the transient response and still keep the system null controllable, we can increase the sampling period for which we design a DB controller of normal order DB(n,d,0), but implement this controller in a system operating at smaller sampling rate. The concept (Garipov and Stoilkov, 2004) can be demonstrated by the discrete-continuous control system with two different sampling periods as shown on Fig.1. In fact this is a kind of internal model control (IMC) scheme, the inner loop of which is designed for a large sampling interval, and the outer loop is operating a small sampling interval. The main idea is that the main controller should work at the large sampling interval, thus redistributing the control signal energy in time and providing smaller control signal magnitude. But at the same time the entire system should operate at smaller sampling interval, therefore a correction signal from the plant-model difference should close the system.

The "Discrete Controller" block provides the control u to the "Continuous Plant" block (assumed to be linear with known time delay). Two different sampling periods are introduced:

- ***small sampling period*** T_0^{CS}, which is fundamental for the entire system, meaning that all signals are sampled and propagate at this period;
- ***large sampling period*** $T_0^{Reg} = l.T_0^{CS},\ l > 1$,used

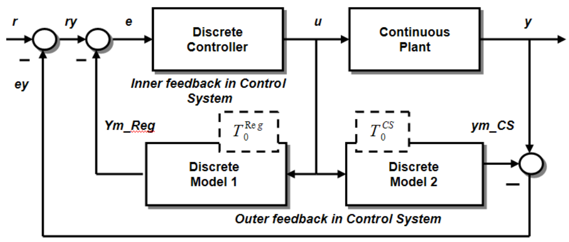

Figure 1: Discrete-continuous control system operating with two different sampling periods.

to define "Discrete Model 1" and respectively in the design of the "Discrete Controller" block.

In fact, the system contains two feedback loops:

- ***outer loop***, which forms corrected reference signal $ry = r - ey$ by the error $ey = y - ym\_CS$ between the measured output y of the "Continuous Plant" and the calculated output *ym_CS* of the "Discrete Model 2";
- ***inner loop***, forming the error $e = ry - ym\_Reg$ in the system between corrected reference *ry* and calculated output *ym_Reg* of "Discrete Model 1".

As an illustrative example let us take the same system given in Section 2. If we select a small sampling period T_o=0.1 sec, the normal order DB(n,d,0) controller produces extremely high control signal amplitude $u(0) = 216130$ after the unit step change of the reference signal. Obviously this value will be "clipped" by the control valve and the system performance will deteriorate. We decide to keep $T_0^{CS} = 0.1$ sec as a fundamental sampling period for the entire system, but introduce a second large sampling period $T_0^{Reg} = 8$ sec for which a DB controller is designed. Even $T_0^{Reg} = 8$ sec does not seem to be good choice, we intentionally use here for illustration. Hence, in the inner loop we have to use the "Discrete Model1", which is sampled at $T_0^{Reg} = 8$ sec, for providing proper control signal behavior. The outer loop is to correct the reference signal depending on the "Discrete Model2" operating at $T_0^{CS} = 0.1$ sec (nearly continuous-time control). The designed DB Controller for T_0^{Reg}=8 sec is:

$$W_o(z) = \frac{2.8653 - 2.4004z^{-1} + 0.5635{-}2 - 0.0285z^{-3}}{1 - 0.6045z^{-1} - 0.3991z^{-2} + 0.0036z^{-3}},$$

The first numerator coefficient q_o=2.8653 is equal theoretically to the control value $u(0)$. Fig. 2 demonstrates the controlled output (top) and the control signal (bottom), which has acceptable amplitude $u(0)$=2.8653 exactly as expected. The finite transient response takes 24 sec that is exactly three times T_0^{Reg}, as the system is of third order.

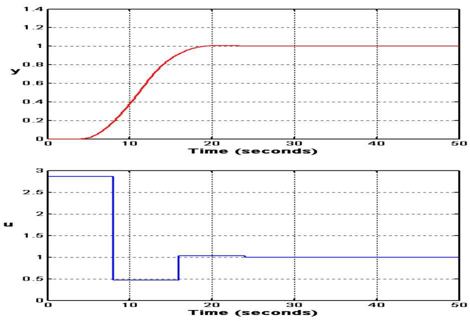

Figure 2: System with sampling period $T_0^{CS.} = 0,1$ sec and DB controller, designed for $T_0^{Reg} = 8$ sec.

4 MULTIPLE-MODEL DEADBEAT CONTROLLER

4.1 MMDB Controller with Varying Order using One Sampling Period

The existence of control signal constraints by the control valve clearly indicates the needs to guarantee a control magnitude that always fits within the control constraints for all operating regime of the system. The closer is the operating point to the constraints the bigger should be the DB controller order, as already clarified in Section 2. Obviously increasing the order the transient response becomes longer, but it is more important to keep the control signal within the constraints paying with the longer finite time of the response. As the plant operating point continuously changes, we should select the minimal order of the DB controller that satisfies the control signal constraints. So we came to the idea of building a MMDB controller that combines several DB controllers of different order running in parallel. The MMDB consists of two major parts:

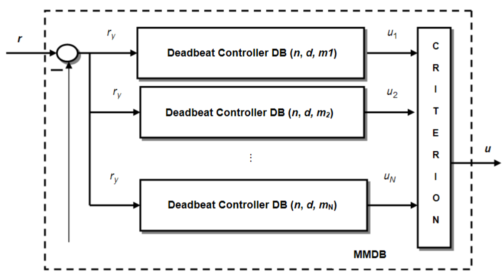

Figure 3: Structure of the MMDB.

- a set of N DB(n,d,m) controllers for the given model of the controlled plant, each of which is designed for different values of m, namely m_1, m_2, ..., m_N, such that all they provide constrained control signal within

the constraints of the control valve $[u_{\min}, u_{\max}]$for all possible variations of the reference signal; one sampling interval is assumed;
- a "criterion" block that switches the input of the plant to the output of one of the DB(n,d,m) controllers depending on a predefined set of conditions, in this case checking the output of which of the DB(n.d,m) controllers is within the constraints $[u_{\min}, u_{\max}]$. Additional criterion is to select the individual DB controller having the minimal value of m_i, because then the transient response is of minimum duration. As

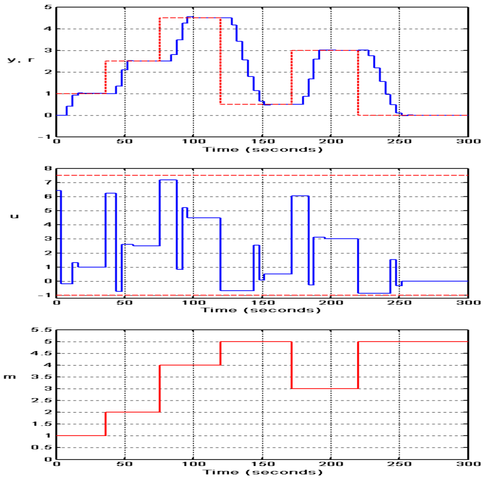

Figure 4: Reference signal and plant output (top); control signal within the constraints (middle); increment of the DB controller order (bottom).

an example we designed a MMDB controller for the plant described in Section 2 with sampling period $T_0^{Reg} = 4$sec. A set of DB controllers is included, namely DB(3,1,m), m = 0, 1, 2, 3, 4 and 5. On Fig. 4 the transient response of the plant follows the reference signal, but is stepwise as the sampling period is big. The control signal lies within the constraints. The "criterion" block decides to switch the appropriate DB(n,d,m_j) controller such that the constraints are satisfied, as seen on the bottom picture on Fig. 4. The "criterion" block is selecting an individual controller with higher or smaller order depending on the distance of the plant operating regime to the control constraints and the step change magnitude of the reference.

The important property of the proposed MMDB controller is the embedded flexibility to select the appropriate order of the DB controller. For comparison on Fig. 5 we present the performance of fixed DB(3,1,0) and DB(3,1,1) controllers at the same operating conditions. Obviously the transient response does not represent a deadbeat behavior as a result of applying too low DB controller order, which cannot bring the control signal within the constraints.

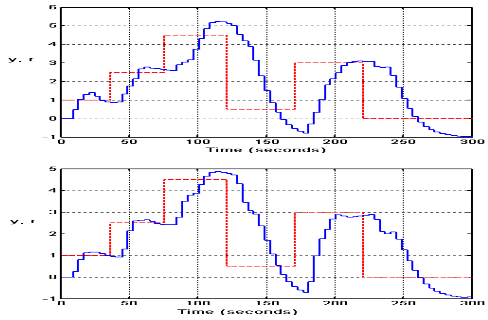

Figure 5: Plant output and reference signal for DB(3,1,0) (top) and DB(3,1,1) (bottom) controller.

4.2 MMDB Controller with Fixed Order using Several Sampling Periods

Contrary to the concept presented in Section 4, here we suggest a MMDB controller that contains a number of controllers, each of which is designed for different sampling periods $T_0^{Reg_i}$, i=1, 2, ..., N, assuming that the entire control system operates with a sampling period $T_0^{CS} << T_0^{Reg_i}$, as shown on Fig. 6.

The difference between this MMDB and the MMDB on Fig. 3 is the content of the individual DB controllers. Here they are assumed of DB(n,d,0) type (normal order DB controller), but they differ due to the different sampling period used for their design. Generally, there is no limitation to use DB(n,d,m) type controllers as well, but for simplicity m is not considered to be a parameter of choice. As an exam-

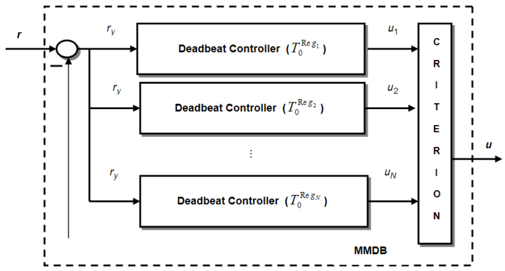

Figure 6: Structure of the MMDB.

ple we demonstrate a MMDB controller for the plant described in Section 2 with sampling period T_0^{CS}= 0.1 sec. A set of DB controllers is designed for T_0^{REG}= 4, 6, 8, 10, 12, 14 and 18 sec. The performance of the system is shown on Fig. 7. One can see that the transient response of the plant follows the reference signal and is rather smooth due to the small sampling period of the entire system. The control signal lies within the constraints. On the bottom picture on Fig.

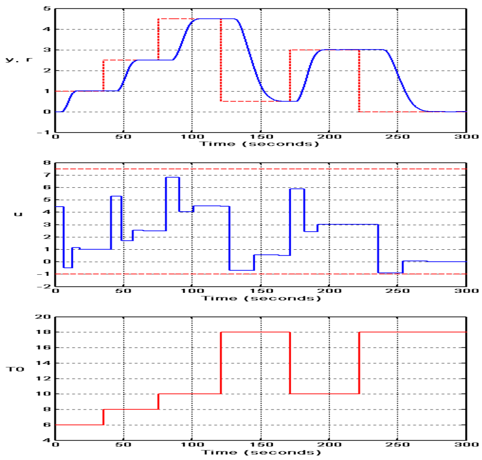

Figure 7: Reference signal and plant output (top); control signal within the constraints (middle); sampling period of the DB controller (bottom).

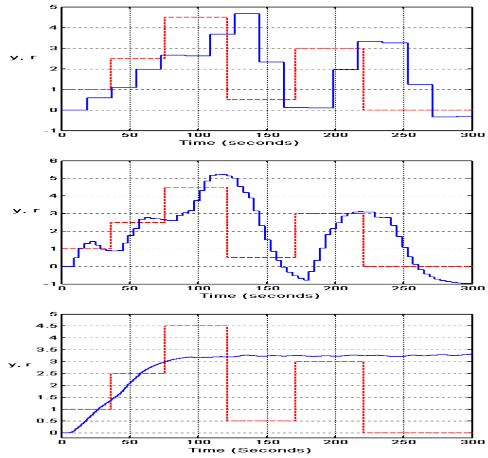

Figure 8: Plant output and reference signal for: $T_0^{Reg} = T_0^{CS}$=18 sec (top); $T_0^{Reg} = T_0^{CS}$=4 sec (middle); $T_0^{Reg} = T_0^{CS}$= 0.1 sec (bottom).

7 it can be seen that the “criterion” block is selecting an individual controller designed for bigger higher or smaller sampling period depending on the distance of the plant operating regime to the control constraints and the magnitude of the step change of the reference signal.

The important property of the proposed MMDB controller with fixed order is the possibility to select the appropriate sampling period of the DB controller that keeps the control signal within the constraints. For comparison on Fig. 8 we present the performance of fixed DB(3,1,0) controller designed and implemented at the same sampling period $T_0^{Reg} = T_0^{CS}$ and the same operating conditions. Obviously the transient response does not represent a deadbeat behavior as a result of applying too low DB controller order, which cannot bring the control signal within the constraints.

5 CONCLUSION

Two original ideas for solving the task of achieving a dead-beat control by a linear DB controller under control constraints were presented in this paper: for design of DB controllers of increased order and for implementation of a discrete-continuous control system, which operates with two different sampling periods. Two algorithms using the concept of multiple-model systems were proposed and demonstrated – a MMDB controller with varying order using one sampling period and a MMDB controller with fixed order using several sampling periods. Both algorithms provide normal operating of the control system and control signal does not leave the predefined constrains. Numerical simulations confirm the performance of the proposed algorithms.

The advantages and disadvantages of these controllers are summarized in Table 2, which can be a useful tool for selection of DB controllers in practical applications.

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Table 2: Basic properties of the Dead-beat controllers.

Controller	Advantages	Disadvantages
• DB controller of ***normal order***, system with one model and ***one sampling period***	• Easy tuning of the controller with small design efforts.	• Large control amplitudes for models of low order and small time delays for small sampling period. • Rough response to the reference signal when big sampling period is used. • No adaptive properties when changing the operating regimes of the control system.
• DB controller of ***increased order***, system with one model and ***one sampling period***	• Possibility of multi variant tuning. • Good control and significant reduction of the large control amplitudes at the first few sampling steps. • Smoother response even for small sampling period, due to the increased controller order.	• Relatively complex design algorithm. • Higher order of the controller needed to reduce the large control amplitudes. • No adaptive properties when changing the operating regimes of the control system.
• DB controller of ***normal order***, system with one model and ***two different sampling periods***	• Simple controller design algorithm. • Good control and significant reduction of the large control amplitudes at the first few sampling steps. • Smoother response to the reference signal even for small sampling period, due to the increased controller order.	• Complicated scheme of the control system. • No adaptive properties when changing the operating regimes of the control system.
• MMDB controller using ***increased order DB blocks***, system with ***one sampling period***	• Adaptation to changes in operating regimes of the control system in case of complex profile of the reference signal and controller output constraints. • Good control and significant reduction of the large control amplitudes at the first few sampling steps. • Smoother response even for small sampling period, due to the increased controller order.	• Relatively complex design algorithm. • Complicated scheme of the control system, as several DB controllers with different fixed structures but with one sampling period function at different operating points of the control system. • Need of supervisor for switching between various controllers.
• MMDB controller using ***normal order DB blocks***, system with ***several sampling periods***	• Adaptation to changes in operating regimes of the control system in case of complex profile of the reference signal and controller output constraints. • Good control and significant reduction of the large control amplitudes at the first few sampling steps. • Smoother response even for small sampling period, due to the increased controller order. • Simple algorithm for designing DB controller of normal order.	• Complicated scheme of the control system, as several DB controllers with different fixed structures but with one sampling period function at different operating points of the control system. • Need of supervisor for switching between various controllers.