ROBUST ADAPTIVE CONTROL USING A FRACTIONAL FEEDFORWARD BASED ON SPR CONDITION

Samir Ladaci, Jean Jacques Loiseau
IRCCyN, Ecole Centrale de Nantes, 1, rue de la Noë, BP: 92101 Nantes, 44321, France
{Samir.Ladaci,Jean-Jacques.Loiseau}@irccyn.ec-nantes.fr

Abdelfatah Charef
Département d'Electronique, Université Mentouri, Route de Ain Elbey, Constantine 25000, Algeria
afcharef@yahoo.com

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Abstract: This paper presents a new approach for robust adaptive control, using fractional order systems as parallel feedforward in the adaptation loop. The basic adaptive algorithm used here is Model Reference Adaptive Control (MRAC), which do not require explicit parameter identification. The problem is that such a control system may diverge when confronted with finite sensor and actuator dynamics, or with parasitic disturbances. One of the classical robust adaptive control solutions to these problems, makes use of parallel feedforward and simplified adaptive controllers based on the concept of positive realness.
This control scheme is based on the ASPR property of the plant. We show that this condition implies also robust stability in case of fractional order controllers. A simulation example of a SISO robust adaptive control system illustrates the interest of the proposed method in the presence of disturbances and noises.

1 INTRODUCTION

Adaptive control has proven to be a good control solution for the partially unknown systems or varying parameter systems. In this domain Model reference adaptive control (MRAC) became very popular since it presents a very simple algorithm with easy implementation and does not require identifiers or observers in the control loop (Astrom and Wittenmark, 1995; Landau, 1979). However such algorithm shows its limits in noisy or disturbed environment, which may make it inefficient or uncompetitive. Unfortunately very few industrial control processes are not subject to theses practical problems, which can damage the quality of product and the good process operating.
The use of simple parallel feedforward in the adaptation loop appeared as a robust solution since the 80's. Many works have used this approach towards robust control systems (Bar-Kana, 1987; Naceri and Abida, 2003). In the last decade a great interest was given to fractional order systems, which have shown good robustness performances, several robust control methods based on these systems have been developed, like CRONE Control (Oustaloup, 1991) and fractional adaptive control (Vinagre et al., 2002; Ladaci and Charef, 2006; Ladaci et al., 2007).
In this paper we present a fractional robust adaptive control solution for disturbed applications, based on the idea of Bar-kana (Bar-Kana, 1987), which uses the basic stabilizability property of the plant and simple parallel feedforward in order to satisfy the desired "almost positive realness" condition that can guarantee robust stability of the nonlinear adaptive controller.
The main contribution of this work is to improve the feedforward approach robust performances by using fractional order filters. This result is illustrated by a simulation example of a test in bad realistic conditions like finite bandwidth of actuators, input and output disturbances and no assumed natural damping.
This paper is structured as follows:
In section 2 definitions of fractional order systems are presented. Section 3 introduces the principles of robust adaptive control based on the concept of 'positive realness' condition and then the main result in fractional order case is presented in section 4. The implementation in Model Reference Adaptive Control scheme is introduced in section 5 and a simulation

example is given in section 6. The paper is concluded in section 7.

2 FRACTIONAL ORDER SYSTEMS

The analysis in Bode plot of many natural processes, like transmission lines, dielectric polarisation impedance, interfaces, cardiac rhythm, spectral density of physical wave, some types of noise (VanDerZiel, 1950; Duta and Hom, 1981), has allowed to observe a fractional slope. This type of process is known as $1/f$ process or fractional order system. During the last decade, a great interest was given by researchers to the study of these systems (Sun and Charef, 1990) and their application in control systems (Oustaloup, 1991; Hotzel and Fliess, 1997; Ladaci and Charef, 2006; Ladaci et al., 2007).
A SISO fractional order system can be represented by the following transfer function,

$$X(s) = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \ldots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \ldots + a_0 s^{\alpha_0}} \quad (1)$$

Where,

- α_i, β_j: real numbers such that,

$$\begin{cases} 0 \leq \alpha_0 < \alpha_1 < \ldots < \alpha_n \\ 0 \leq \beta_0 < \beta_1 < \ldots < \beta_m \end{cases}$$

- s: Laplace operator.

for the purpose of this work, let us introduce the following definitions,

Definition 1 *The fractional order transfer function $X(s)$ given in (1) is called proper if:* $\beta_m \leq \alpha_n$
It is called stricly proper if: $\beta_m < \alpha_n$

Definition 2 *(Desoer and Vidyasagar, 1975) The fractional order transfer function Matrix $M_X(s)$ whose elements are of the form (1) is proper (strictly proper) if and only if all elements of $M_X(s)$ are bounded at ∞ (tend to zero at ∞, resp.).*

We use in the sequel a description equation into frequency domain of a single pole fractional order process, given as follows:

$$Y(s) = \frac{1}{(s + p_T)^{\alpha}} \quad (2)$$

with

- α: fractional exponent, $0 \leq \alpha \leq 1$
- p_T: fractional pole which is the cut frequency.

Many previous works have shown that fractional systems present best qualities, in response time and in transition dynamic stability (Sun and Charef, 1990). All the control theory developed by Oustaloup especially on CRONE control was based on fractional order systems robustness in presence of uncertainties and perturbations (Oustaloup, 1991).

3 CONCEPT OF POSITIVE REALNESS CONDITION

Robustness is defined relatively to a certain property and a set of models. A property (generally stability or performance level) is said to be robust if all the models belonging to the set satisfy it. Robust adaptive stabilization means that all values involved in the adaptation process namely, states, gains and errors are bounded in the presence of any bounded input commands and input or output disturbances (Bar-Kana and Kaufman, 1985; Kwan et al., 2001).

In this paper we are interested by a particular configuration of feedforward controllers combined with MRAC control and fractional order systems giving a fractional robust adaptive control method.
The use of a simple feedforward in the adaptation loop (see Figure 4) improves the robust stability of the control system. This approach is based on the concept of the ”positive realness” condition (Bar-Kana, 1989); witch can guarantee stable implementation of adaptive control configuration. Let us present these definitions:

Definition 3 *The $m \times m$ transfer function matrix $G_s(s)$ is called strictly positive real (***SPR***) if (Landau, 1979; Bar-Kana, 1989):*

1. *All elements of $G_s(s)$ are analytic in $\Re(s) \geq 0$.*
2. *$G_s(s)$ is real for real s.*
3. *$G_s(s) + G_s^{T*}(s) > 0$ for $\Re(s) \geq 0$ and finite s.*

We also show that (Shaked, 1977) for a fractional order transfer function matrix $G_s(s)$,

$$G_s(s) \text{ is } \mathbf{SPR} \Leftrightarrow G_s^{-1}(s) \text{ is } \mathbf{SPR} \quad (3)$$

Indeed, by using the **SPR** property if we write (Bar-Kana, 1989),

$$G_s(s) = A + jB \Rightarrow G_s^{T*}(s) = A^T - jB^T$$

Since by definition

$$G_s(s) + G_s^{T*}(s) = A + A^T + j(B - B^T) > 0$$

we get $B = B^T$ and $A > 0$ (not necessary symmetric). Then whenever

$$\Re\left[G_s(s)\right] = A > 0$$

we get

$$G_s^{-1}(s) = (A + BA^{-1}B^T)^{-1} - jA^{-1}B(A + BA^{-1}B^T)^{-1}$$

and

$$\Re\left[G_s^{-1}(s)\right] = (A + BA^{-1}B^T)^{-1} > 0$$

which proves (3).

Definition 4 *(Bar-Kana, 1987)*
Let $G_a(s)$ be a $m \times m$ transfer matrix. Let us assume that there exists a positive definite constant gain matrix, $\tilde{K}_e$ such that the closed-loop transfer function

$$G_c(s) = \left[I + G_a(s)\tilde{K}_e\right]^{-1} G_a(s) \qquad (4)$$

is **SPR**. *$G_a(s)$ is called "almost strictly positive real (***ASPR***)".*

Now if we consider a fractional order proper or strictly proper **ASPR** transfer matrix $G_s(s)$.
Then the following statements are equivalent,

$$G_s(s) = \left[I + G_a(s)K_e\right]^{-1} G_a(s) \text{ is SPR} \qquad (5)$$

$$G_s(s) = \left[I + G_a(s)K_e\right]^{-1} \text{ is SPR} \qquad (6)$$

$$G_s^{-1}(s) = G_a^{-1}(s) + K_e \text{ is SPR} \qquad (7)$$

$$\Re\left[G_a^{-1}(s) + K_e\right]_{\Re(s)\geq 0} > 0 \qquad (8)$$

$$\begin{array}{ll} G_s^{-1}(s) & \text{is asymptotically stable and} \\ K_e & \text{is sufficiently large} \end{array} \qquad (9)$$

Because $\exists M$ such that $\Re\left[G_a^{-1}(s)\right]_{\Re(s)\geq 0} > M > -\infty$, and then any $K_e > -M$ will do (Bar-Kana, 1989).

$$\begin{array}{ll} G_a(s) & \text{is strictly minimum phase and} \\ K_e & \text{is sufficiently large} \end{array} \qquad (10)$$

All the above algebraic manipulation, as done to obtain (3) and definitions 3 and 4, apply to fractional systems as well. Here we can generalize as fellows the result of (Bar-Kana, 1989) to the fractional order case.

Lemma 1 *Let a fractional order transfer function matrix $G_a(s)$ be* **ASPR** *and let $\tilde{K}_e$ be any gain that satisfies (4). Then $G_a(s)$ is* **SPR** *for any gain K_e that satisfies $K_e > \tilde{K}_e$.*

It is obvious that **ASPR** fractional order systems, which are minimum phase proper systems maintain stability with high gains. The high gain stability is important when nonstationary or nonlinear (adaptive) control is used, because the robustness of the control system is maintained if, due to specific operational conditions, the time-varying gains become too large.

Remarks

1. The ASPR plant must also be proper.
2. The open loop is not necessarily stable (the plant will actually be stabilized by the fictitious gain K_e), however all the zeros must be placed in the left half plane. The plant must be minimum phase to obtain positivity.
3. We can easily show (Bar-Kana, 1987) that if a system is ASPR, then it can be stabilized by any constant or time variable output gain K_e, if it is large enough, i.e. $K_e > \tilde{K}_e$.

But in this method, instead of using high gain regulation we will use a simple parallel feedforward configuration which can by a similar way satisfy the positive realness conditions.
The idea of using feedforward in parallel with the controlled plant is based on the following Lemma of Bar-Kana,

Lemma 2 *(Bar-Kana, 1989) Let the plant be described by the $m \times m$ transfer function $G_p(s)$ of order n. Let $C(s)$ be any dynamic stability output feedback controller. Then*

$$G_a(s) = G_p(s) + C^{-1}(s) \qquad (11)$$

is **ASPR** *if $C^{-1}(s)$ is proper or strictly proper.*

We can adapt the proof of (Bar-Kana, 1989; Bar-Kana, 1986)) to the fractional case.

4 MAIN RESULT

At this stage we propose a fractional order feedforward configuration of the form:

$$F(s) = \frac{F_p}{\left(1 + \frac{s}{s_0}\right)^{\alpha}} \qquad (12)$$

with a real fractional power $0 < \alpha < 1$, to improve the robustness of the adaptive algorithm, in presence of perturbations, as such systems do not amplify much

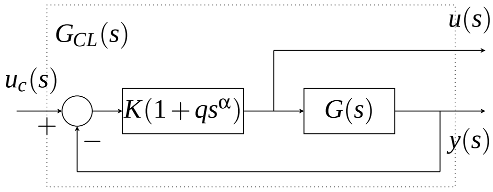


Figure 1: Closed-loop system.

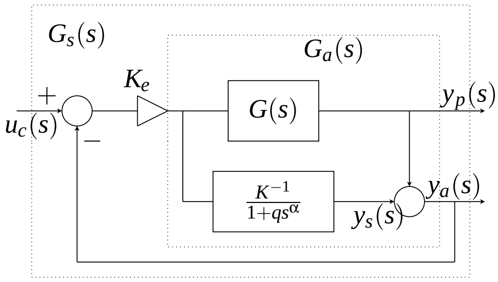


Figure 2: The fictitious SPR configuration.

these random signals. This configuration could be considered as the inverse of an improper fractional PD$^{\mu}$ controller, which was used in control systems with good proven performances (Oustaloup, 1983; Hotzel and Fliess, 1997; Podlubny, 1999).
We can formulate the main result of this paper in the following theorem.

Theorem 1 *Let $G(s)$ be any $m \times m$ strictly proper transfer matrix of arbitrary MacMillan degree. $G(s)$ is not necessarily stable or minimum phase. Let*

$$H_f(s) = K(1+qs^{\alpha}) \quad (13)$$

be some stabilizing controller for $G(s)$. Then the augmented controlled plant

$$G_a^f(s) = G(s) + H_f^{-1}(s) = G(s) + \frac{K^{-1}}{1+qs^{\alpha}} \quad (14)$$

is **ASPR**.

Proof of Theorem 1:
From definition 4, if $G_a(s)$ is **ASPR** then the closed-loop transfer function

$$G_c(s) = \left[I + G_a(s)\tilde{K}_e\right]^{-1} G_a(s)$$

is **ASPR**.

Since $H_f^{-1}(s)$ from (13) is strictly proper (relative degree $\alpha > 0$), then Lemma 2 implies that the augmented system $G_a^f(s)$ as defined in (14) is **ASPR**, which proves Theorem 1.

The stabilizing controller $H_f(s)$ can also be modelized as follows,

$$H_f(s) = K(1+qs)^{\alpha} \quad (15)$$

Figure 1 represents the feedback control system corresponding to the control (13).
From Definition 4 and the fact that the transfer function $G_a^f(s)$ is ASPR, we know that it can be stabilized by a gain $\tilde{K}_e$. Figure 2 illustrates the feedforward configuration. In addition, the stabilization is robust, it holds for any gain $K_e > \tilde{K}_e$.

Many previous works (Hotzel and Fliess, 1997; Podlubny, 1999) have proposed PD$^{\mu}$ improper controllers of the form (13):

$$C(s) = K_p + K_i s^{\alpha} \quad (16)$$

which can stabilize many realistic plants for sufficient high values of K.
A feedforward of equivalent effect is chosen as follows:

$$F(s) = C^{-1}(s) = \frac{F_p}{\left(1+\frac{s}{s_0}\right)^{\alpha}} \quad (17)$$

Where $F_p = K^{-1}$, such that the augmented plant becomes:

$$G_a(s) = G_p(s) + F(s) \quad (18)$$

As K should be very large, so F_p are small coefficients, guaranteeing that $G_a(s)$ be **ASPR**. And during the control design we can take $G_a(s) \approx G_p(s)$ as a practical approximation.

5 IMPLEMENTATION IN MRAC SCHEME

Model Reference Adaptive Control (MRAC) is one of the more used approaches of adaptive control, in which the desired performance is specified by the choice of a reference model. Adjustment of parameters is achieved by means of the error between the output of the plant and the model reference output. Let us introduce the basic ideas of this approach represented in Figure 3.

We consider a closed loop system where the controller has an adjustable parameter vector θ. A model which output is y_m specifies the desired closed loop response. Let e be the error between the closed loop system output y and the model one y_m, one possibility is to adjust the parameters such that the cost function:

$$J(\theta) = \frac{1}{2}e^2 \quad (19)$$

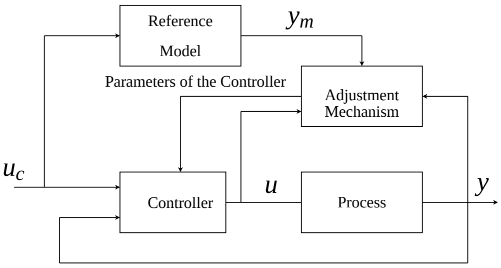


Figure 3: Direct Model Reference Adaptive Control.

be minimised. In order to make J small it is reasonable to change parameters in the direction of negative gradient J, so:

$$\frac{d\theta}{dt} = -\gamma \frac{\delta J}{\delta\theta} = -\gamma e \frac{\delta e}{\delta\theta} \tag{20}$$

or

$$\frac{d\theta}{dt} = \gamma\varphi e \tag{21}$$

where $\varphi = -\frac{\delta e}{\delta\theta}$ is the regression (or measures) vector and γ is the adaptation gain. This aproach is called *M.I.T. rule*.

The introduction of a simple feedforward in the MRAC adaptation loop us represented in figure 4 improves the robust stability performance against the controller gain fluctuations in presence of perturbation and noises (Naceri and Abida, 2003). Previous works (Sobel and Kaufman, 1986), showed that the **ASPR** property of a process, allows the implementation of very simple adaptive controllers that garantee robust stability of the closed loop in presence of bounded input or output disturbances.

The feedforward transfer function is choosen like in (12) where the gain F_p is a small coefficient.

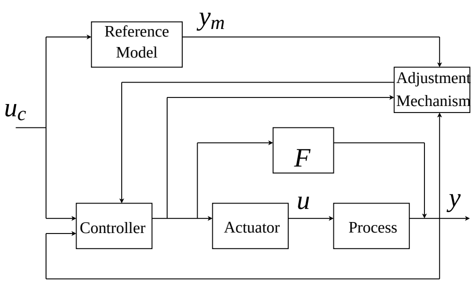


Figure 4: Simple feedforward in MRAC scheme.

6 SIMULATION EXAMPLE

Without any loss of generality we will apply this robust adaptive control method, both in the case of integer and fractional order feedforward, to a SISO model of a DC motor controlled in respect of velocity, given by:

$$G_p(z) = \frac{0.8513z + 5.099\ 10^{-6}}{z^2 + 2.442\ 10^{-7}z + 1.37\ 10^{-11}} \tag{22}$$

with a sampling period $T_s = 0.3sec$, and an actuator model of the form:

$$A(z) = \frac{0.007667z + 0.007049}{z^2 - 1.763z + 0.7772} \tag{23}$$

The plant is subject to random input and output perturbations of amplitudes 2 and 0.05 respectively.

The reference model G_m is given by:

$$G_m(z) = \frac{0.9411z + 0.1208}{z^2 + 0.05679z + 0.005092} \tag{24}$$

6.1 Integer Order Feedforward Case

The feedforward trunsfer fuction F is given by:

$$F(z) = \frac{3.2394\ 10^{-7}}{z - 0.9997} \tag{25}$$

with a regulation parameter $\gamma = 0.001$ we obtain the results of Figure 5.

6.2 Fractional Order Feedforward Case

The fractional order feedforward trunsfer function F is given in Laplace domain by:

$$F(s) = \frac{0.001}{(s+500)^{0.6}} \tag{26}$$

For the purpose of our approach we need to use an integer order model approximation of the fractional order feedforward model in order to implement the adaptation algorithm, for this aim we have used the so-called singularity function method (Charef et al., 1992).

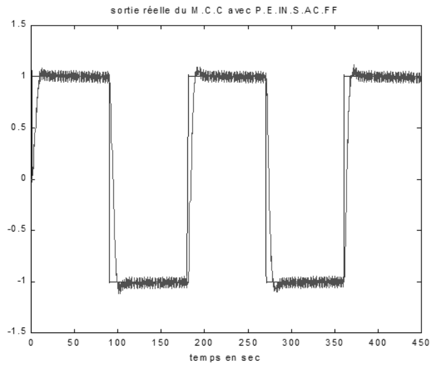
The fractional transfer function (26) is approximated to a linear transfer function and sampled to give the following formula:

$$\hat{F}(z) = \frac{0.001(z - 4.78\ 10^{-97})}{z^2 - 2.407\ 10^{-96}z + 1.001\ 10^{-207}} \tag{27}$$

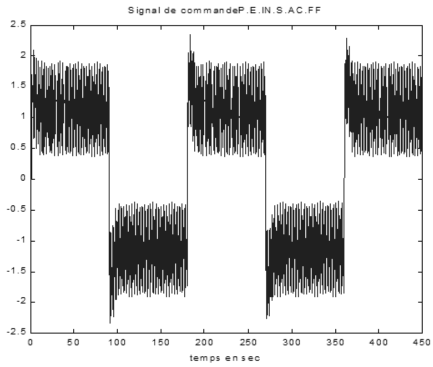
with a regulation parameter $\gamma = 0.005$, we obtain the results of Figure 6.

6.3 Remarks

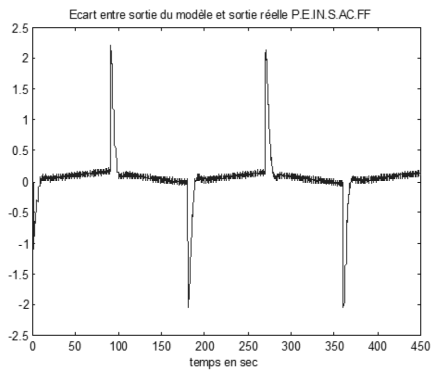
- The command signal u is more polish in the fractional case witch is a very useful property in regulation problem.



(a)

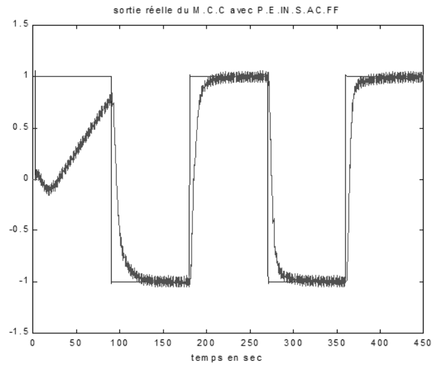


(b)

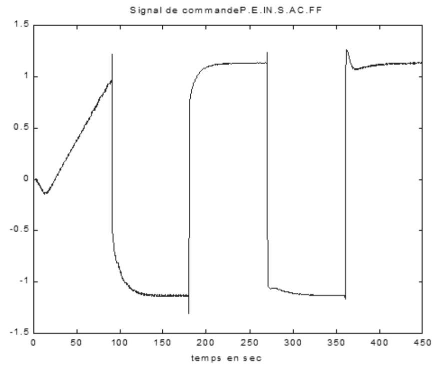


(c)

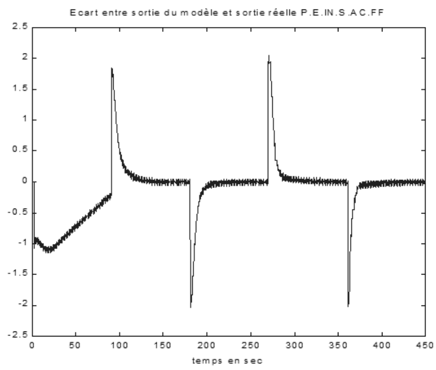
Figure 5: Process output with integer feedforward (a) Process output $y(t)$, (b) Control signal $u(t)$, (c) Error signal $e(t)$.



(a)



(b)



(c)

Figure 6: Process output with fractional feedforward (a) Process output $y(t)$, (b) Control signal $u(t)$, (c) Error signal $e(t)$.

- The proposed fractional order configuration of feedforward maintains stability and at less the same level of performances, witch confirms the interest of integrating fractional strategy in robust adaptive control.

7 CONCLUSION

In this paper we have presented a new robust adaptive control strategy, by introducing simple fractional feedforward configuration in the MRAC algorithm. The concept of positive realness condition which is the basis of this robust control strategy is extended to fractional order control systems. The idea was to take benefit of the high performance quality of fractional order systems confirmed in many precedent research works. The stability proofs of this adaptive control scheme developed for integer order filters in control literature still holds for such systems. Simulation results have shown a better filtering ability of command and output signals, and more robustness against additive perturbations, than in the integer order feedforward configuration case.

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