A New Model of Associative Memories Network

Roberto A. Vázquez and Humberto Sossa

Centro de Investigación en Computación-IPN
Av. Juan de Dios Batíz, esquina con Miguel Othón de Mendizábal
Mexico City, 07738, Mexico
ravem@ipn.mx, hsossa@cic.ipn.mx

Abstract. An associative memory (AM) is a special kind of neural network that only allows associating an output pattern with an input pattern. However, some problems require associating several output patterns with a unique input pattern. Classical associative and neural models cannot solve this simple task. In this paper we propose a new network composed of several AMs aimed to solve this problem. By using this new model, AMs can be able to associate several output patterns with a unique input pattern. We test the accuracy of the proposal with a database of real images. We split this database of images into four collections of images and then we trained the network of AMs. During training we associate an image of a collection with the rest of the images belonging to the same collection. Once trained the network we expected to recover a collection of images by using as an input pattern any image belonging to the collection.

1 Introduction

An associative memory AM is a special kind of neural network that allows recalling one output pattern given an input pattern as a key that might be altered by some kind of noise (additive, subtractive or mixed). Several models of AMs are described in [1], [2], [3], [5], [9], [10], [11] and [12]. In particular, models described in [5], [9] and [10] cannot handle with mixed noise. Associative model presented in [11] and [12] is robust to mixed noise.

An association between input pattern $\mathbf{x}$ and output pattern $\mathbf{y}$ is denoted as $\left(\mathbf{x}^k,\mathbf{y}^k\right)$, where k is the corresponding association. AM $\mathbf{W}$ is represented by a matrix whose component w_{ij} can be seen as the synapse of the neural network. Operator $\mathbf{W}$ is generated from a finite a priori set of know associations, known as the fundamental set of association and is represented as: $\left\{\left(\mathbf{x}^k,\mathbf{y}^k\right)\middle|k=1,\ldots,p\right\}$ where p is the number of associations. If $\mathbf{x}^k=\mathbf{y}^k\forall k=1,\ldots,p$ then $\mathbf{W}$ is auto-associative, otherwise it is hetero-associative. A distorted version of a pattern $\mathbf{x}$ to be restored will be denoted as $\tilde{\mathbf{x}}$. If an AM $\mathbf{W}$ is fed with a distorted version of $\mathbf{x}^k$ and the output obtained is exactly $\mathbf{y}^k$, we say that recalling is perfect.

In this paper we present how a network of AMs can be used to recall nor just one pattern but several of them given an input pattern. In this proposal an association

between input pattern $\mathbf{x}$ and a collection of output pattern $\mathbf{Y}$ is denoted as, $\left\{ \left(\mathbf{x}^k, \mathbf{Y}^k \right) \middle| k = 1, \ldots, p \right\}$ where p is the number of association, $\mathbf{Y}^k = \left\{ \mathbf{y}^1, \ldots, \mathbf{y}^r \right\}$ is a collection of output patterns and r is the number of patterns belonging to collection $\mathbf{Y}$.

The remaining of the paper is organized as follows. In section 2 we describe the associative model used in this research. In section 3 we describe the proposed network of AMs. In section 4 we present the experimental results obtained with the proposal. In section 5 we finally present the conclusions and several directions for further research in this direction.

2 Dynamic Associative Model

The brain is not a huge fixed neural network, as had been previously thought, but a dynamic, changing neural network that adapts continuously to meet the demands of communication and computational needs [8]. This fact suggests that some connections of the brain could change in response to some input stimuli.

Humans, in general, do not have problems to recognize patterns even if these are altered by noise. Several parts of the brain interact together in the process of learning and recalling a pattern. For example, when we read a word the information enters the eye and the word is transformed into electrical impulses. Then electrical signals are passed through the brain to the *visual cortex*, where information about space, orientation, form and color is analyzed. After that, specific information about the patterns passes on the other areas of the *cortex* that integrate visual and auditory information. From here information passes through the *arcuate fasiculus*, a path that connects a large network of interacting brain areas; paths of this pathway connect language areas with other areas involving in cognition, association and meaning, for details see [4] and [7].

Based upon the above example we have defined in our model several interacting areas, one per association we would like the memory to learn. Also we have integrated the capability to adjust synapses in response to an input stimulus.

As we could appreciate from the previous example, before an input pattern is learned or processed by the brain, it is hypothesized that it is transformed and codified by the brain. In our model, this process is simulated using the following procedure recently introduced in [11]:

Procedure 1. Transform the fundamental set of associations into codified patterns and de-codifier patterns:

```
Input: FS Fundamental set of associations:
```
{1. Make $d = const$ and make $\left(\overline{\mathbf{x}}^1, \overline{\mathbf{y}}^1 \right) = \left(\mathbf{x}^1, \mathbf{y}^1 \right)$
```
 2. For the remaining couples do {
```
   For $k = 2$ to p {
```
        For $i = 1$ to $n$ {
```
$$\overline{x}_i^k = \overline{x}_i^{k-1} + d \; ; \quad \hat{x}_i^k = \overline{x}_i^k - x_i^k \; ; \quad \overline{y}_i^k = \overline{y}_i^{k-1} + d \; ; \quad \hat{y}_i^k = \overline{y}_i^k - y_i^k$$
```
}}} Output: Set of codified and de-codifying patterns.
```

This procedure allows computing *codified patterns* from input and output patterns denoted by $\bar{\mathbf{x}}$ and $\bar{\mathbf{y}}$ respectively; $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are *de-codifying patterns*. Codified and de-codifying patterns are allocated in different interacting areas and *d* defines of much these areas are separated. On the other hand, *d* determines the noise supported by our model. In addition a simplified version of $\mathbf{x}^k$ denoted by s_k is obtained as:

$$s_k = s\left(\mathbf{x}^k\right) = \mathbf{mid}\ \mathbf{x}^k \tag{1}$$

where **mid** operator is defined as $\mathbf{mid}\ \mathbf{x} = x_{(n+1)/2}$.

When the brain is stimulated by an input pattern, some regions of the brain (interacting areas) are stimulated and synapses belonging to those regions are modified.

In our model, we call these regions *active regions* and could be estimated as follows:

$$ar = r(\mathbf{x}) = \arg\left(\min_{i=1}^{p}\left|s(\mathbf{x}) - s_i\right|\right) \tag{2}$$

Once computed the *codified patterns*, the *de-codifying patterns* and s_k we can build the associative memory.

Let $\left\{\left(\bar{\mathbf{x}}^k, \bar{\mathbf{y}}^k\right) \middle| k = 1, \ldots, p\right\}, \bar{\mathbf{x}}^k \in \mathbf{R}^n, \bar{\mathbf{y}}^k \in \mathbf{R}^m$ a fundamental set of associations (codified patterns). Synapses of associative memory $\mathbf{W}$ are defined as:

$$w_{ij} = \bar{y}_i - \bar{x}_j \tag{3}$$

After computed the *codified patterns*, the *de-codifying patterns,* the reader can easily corroborate that any association can be used to compute the synapses of $\mathbf{W}$ without modifying the results. In short, building of the associative memory can be performed in three stages as:

```
1.  Transform the fundamental set of association into codified and de-
    codifying patterns by means of previously described Procedure 1.
2.  Compute simplified versions of input patterns by using equation 1.
3.  Build W in terms of codified patterns by using equation 3.
```

2.1   Modifying Synapses of the Associative Model

As we had already mentioned, synapses could change in response to an input stimulus; but which synapses should be modified? For example, a head injury might cause a brain lesion killing hundred of neurons; this entails some synapses to reconnect with others neurons. This reconnection or modification of the synapses might cause that information allocated on brain will be preserved or will be lost, the reader could find more details concerning to this topic in [6] and [13].

This fact suggests there are synapses that can be drastically modified and they do not alter the behavior of the associative memory. In the contrary, there are synapses that only can be slightly modified to do not alter the behavior of the associative memory; we call this set of synapses *the kernel* of the associative memory and it is denoted by $\mathbf{K_W}$.

In the model we can find two types of synapses: synapses that can be modified and do not alter the behavior of the associative memory; and synapses belonging to the kernel of the associative memory. These last synapses play an important role in recalling patterns altered by some kind of noise.

Let $\mathbf{K_W} \in \mathbf{R}^n$ the kernel of an associative memory $\mathbf{W}$. A component of vector $\mathbf{K_W}$ is defined as:

$$kw_i = \mathbf{mid}\left(w_{ij}\right), j = 1,\ldots,m \tag{4}$$

According to the original idea of our proposal, synapses that belong to $\mathbf{K_W}$ are modified as a response to an input stimulus. Input patterns stimulate some *active regions*, interact with these regions and then, according to those interactions, the corresponding synapses are modified. Synapses belonging to $\mathbf{K_W}$ are modified according to the stimulus generated by the input pattern. This adjusting factor is denoted by Δw and can be computed as:

$$\Delta w = \Delta\left(\mathbf{x}\right) = s\left(\overline{\mathbf{x}}^{ar}\right) - s\left(\mathbf{x}\right) \tag{5}$$

where ar is the index of the *active region*.

Finally, synapses belonging to $\mathbf{K_W}$ are modified as:

$$\mathbf{K_W} = \mathbf{K_W} \oplus \left(\Delta w - \Delta w_{old}\right) \tag{6}$$

where operator $\oplus$ is defined as $\mathbf{x} \oplus e = x_i + e \; \forall i = 1,\ldots,m$. As you can appreciate, modification of $\mathbf{K_W}$ in equation 6 depends of the previous value of Δw denoted by Δw_{old} obtained with the previous input pattern. Once trained the $\mathbf{AM}$, when it is used by first time, the value of Δw_{old} is set to zero.

2.2   Recalling a Pattern using the Proposed Model

Once synapses of the associative memory have been modified in response to an input pattern, every component of vector $\overline{\mathbf{y}}$ can be recalled by using its corresponding input vector $\overline{\mathbf{x}}$ as:

$$\overline{y}_i = \mathbf{mid}\left(w_{ij} + \overline{x}_j\right), j = 1,\ldots,n \tag{7}$$

In short, pattern $\overline{\mathbf{y}}$ can be recalled by using its corresponding key vector $\overline{\mathbf{x}}$ or $\tilde{\mathbf{x}}$ in six stages as follows:

```
1. Obtain index of the active region ar by using equation 2.
2. Transform  x^k  using de-codifying pattern  x̂^ar  by applying the
   following transformation: x̂^k = x^k + x̂^ar .
3. Compute adjust factor Δw = Δ(x̂) by using equation 5.
4. Modify synapses of associative memory  W  that belong to  K_W  by
   using equation 6.
5. Recall pattern ŷ^k by using equation 7.
```

6. Obtain $\mathbf{y}^k$ by transforming $\widehat{\mathbf{y}}^k$ using de-codifying pattern $\hat{\mathbf{y}}^{ar}$ by applying transformation: $\mathbf{y}^k = \widehat{\mathbf{y}}^k - \hat{\mathbf{y}}^{ar}$.

The formal set of prepositions that support the correct functioning of this dynamic model can be found in [14].

3 Architecture of the Network

Classical AMs (see for example [1], [2], [3], [5], [9], [10], [11] and [12]) are able to recover a pattern (an image) from a noisy version of it. In their original form classical AMs are not useful when image is altered by image transformations, such as translations, rotations, and so on.

The network of AMs proposed in this paper is robust under some of these transformations. Taking advantage of this fact, we can associate different versions of an image (rotated, translated and deformed) to an image.

Our task is to propose a network of AMs aimed to associate an image with other images belonging to the same collection. In order to achieve this, first suppose we want to associate images belonging to a collection with an image of the same collection using an AM. A good solution could be to compute the average image of whole images belonging to the collection and then associate the average image with any image that belongs to the collection. The same solution can be applied to other collections. Once computed the average images from different collections and chosen the images to be associated, we can train the AM as was described in section 2.

Until this point the AM only can recover an association between a collection of input patterns $\mathbf{X}$ and output pattern $\mathbf{y}$ denoted as, $\left\{ \left(\mathbf{X}^k, \mathbf{y}^k \right) \middle| k = 1, \ldots, p \right\}$ where p is the number of association, $\mathbf{X}^k = \left\{ \mathbf{x}^1, \ldots, \mathbf{x}^r \right\}$ is a collection of input patterns and r is the number of patterns belonging to collection $\mathbf{X}$. This means that it can only be recovered the associated image using any image from a collection. However, we would like to get the inverse result; instead of recovering the associated image using any image from a collection, we would like to recover all the images belonging to the collection using any image of the collection.

To achieve this goal we will train a network of AMs built as in previous sections. Each AM will associate all the images of a collection with one image of this collection. This implies that for recovering all images of a given collection, we would need r AMs, where r is the number of images belonging to the collection. The network architecture of AMs needed for recovering a collection of images is shown in Fig. 1.

In order to train the network of r AMs, first of all we need to know the number of collections we want to recover. Training phase is done as follows:

1. Transform each image into a vector.
2. Build n collections of images $\left[\mathbf{CI}^n \right]_{q \times r}$ where q is the number of pixels of each image and r the number of images.

3. Let $\left[\mathbf{AI}\right]_{q \times n}$ a matrix of average images. For $k = 1$ to n compute the average image as:

$$\mathbf{AI}_k = \frac{\sum_{s=1}^{r} \mathbf{CI}_s^k}{r} \tag{8}$$

4. For $s = 1$ to r build an $\mathbf{AM}^s$ as described in Section 2.2. For $k = 1$ to n $\mathbf{x}^k = \mathbf{AI}_k$ and $\mathbf{y}^k = \mathbf{CI}_s^k$

Once trained the network of AMs, when is fed with any image of a collection, each AM will respond with an image that belongs to the collection. To recover a collection of images we just operate each AM with the input image as described in Section 2.3.
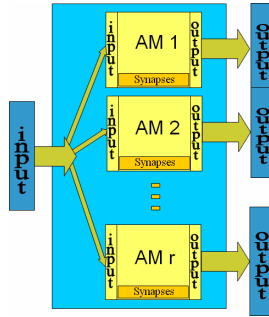
Fig. 1. Architecture of a network of AMs for recalling a collection of patters using an input pattern.

4 Experimental Results

In this section the accuracy of the proposal is tested using different collections of images shown in Fig. 2.

| (a) | (b) | (c) | (d) | (e) |
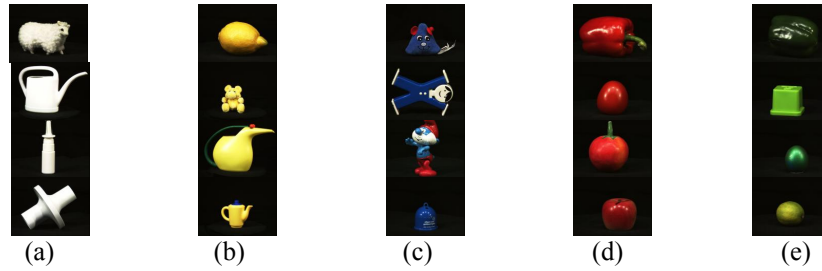
Fig. 2. (a-e) Collections of images taken from the Amsterdam Library of Objects Images (ALOI).

Twenty complex images were grouped into five collections composed by four images, see Fig. 2. After that, we proceeded to train the network of AMs as was

explained in Section 3. For this problem our network is composed of four AMs. It is important to say that the number of images composing a collection could by any, the only restrictions to guaranty perfect and robust recall is that patterns (images) satisfy propositions described in [14].

Once trained the network, four experiments were performed to test the accuracy of the proposal. The first experiment consisted on recovering a collection of images using any image of the collection in order to verify how much robust is the proposal under image deformations. Second experiment consisted on recovering a collection of images using any image of the collection altered by mixed noise in order to verify how much robust is the proposal under deformations and noisy version of the images. In the third experiment each image of the training set was rotated (from 0 to 360). We then used them to fed the network of AMs in order to verify how much robust is the proposal under deformations and rotations. Finally for forth experiment the images of the training set were rotated (from 0 to 360) and translated, and then used them to fed the network of AMs in order to verify how much robust is the proposal under deformations, rotations and translations.

The accuracy of the proposal was of 100% in the first experiment. The five collections of images where perfectly recovered by using any image of the collection (20 images), in Table 1 are shown some results obtained in this experiment. Remember that we train the network of AMs with average images, so then; when we fed the network with an image of any collection this image could be seen as a deformed version of the average images. The results provided by our proposal in this experiment show that the associative model used to train the network of AMs is robust under deformations. Something important to say is that if we use other associative models for training the network of AMs such as morphological or median AMs, the collections might not be correctly recovered due to they are not robust under these kind of transformations or deformations.

Table 1. Some results obtained for the first experiment. As you can appreciate all sets of images were perfectly recovered.

Input image	Image recovered by each AM.			

The accuracy of the proposal, in the second experiment, was of 100%. The five collections of images where recovered perfectly by using any image of the collection even when this images were altered by noise (200 images). As you can see in Table 2, despite of the level of noise add to the images, the collections were correctly

recovered. Despite of other associative models are robust to this kind of noise, they might not recover the all collections due to they are only robust under additive, subtractive and mixed noise but not to deformations.

**Table 2.** Some results obtained for the second experiment. Despite of the noise added to the images, all sets of images were again correctly recovered.

Input image	Image recovered by each AM							

The accuracy of the proposal, in the third experiment, was also of 100%. The five collections of images where recovered perfectly even when rotated version of the images were used (700 images), see in Table 3. Some important to say is, to our knowledge, neither morphological AMs nor other classical models are robust under rotations. Due to we used simplified patterns using **mid** operator and due to this operator is invariant to rotations the accuracy of the proposal was of 100%.

Table 3. Some results obtained for the third experiment. Despite of the noise added to the images and rotations, all sets of images were again correctly recovered.

Input image	Image recovered by each AM							

Finally, the accuracy of the proposal, in the forth experiment was of 40%. In this experiment with 700 images; with some images we recall a collection, but with some other images (when patterns do not satisfied the proposition which guarantee robust recall) the collections were not recalled, see Table 4. However, the results obtained by our proposal are acceptable if they are compared with the results provided by order associative models (less of 10% of accuracy).

Table 4. Some results obtained for the forth experiment. With some images we recall a collection, but with some other images (when patters do not satisfied the proposition which guarantee robust recall) the collections was not recalled.

Input image			Image recovered by each AM						

In general, the accuracy of the proposal with different banks of images (altered by noisy and rotated) was of 100%. This was due to the input patterns (the images) satisfy the propositions presented in [14]. If these patterns do not satisfy these propositions, as images used in experiment four (translated and rotated images), the accuracy of the proposal diminish. However, the results provided by our proposal up-performed the results provide by other associative models.

5   Conclusions and Directions for Further Research

In this paper we have proposed a network of AMs. This network is useful for recalling a collection of output patterns using an input pattern as a key. The network is composed by several dynamic associative memories (DAM). This DAM is inspirited in some aspects of human brain. The model, due to plasticity of its synapses and functioning, is robust under some transformations as rotation, translation and deformations.

In addition, we describe an algorithm for training a network of AMs codifying the images of a collection by using an average image. Once computed the average images we proceed to training the network of AMs.

The network is capable to recall a collection of images (patterns) even if images are altered by noise or suffer some deformations, rotations and translations.

Through several experiments we have shown the efficiency of the proposal. In the first three experiments the proposal provided an accuracy of 100%. Even when the images were altered with mixed noise and rotated, the network of AMs recovered the corresponding collection. When object in images suffer translations, the accuracy of the proposal diminish. This is because most of the patterns under this transformation do not satisfied the propositions that guarantee robust recall. However, the results provided by our proposal up-performed the results provide by other associative models.

The performed experiments could be seen as an application for image retrieval problems. We could say that we have developed a small system able to recover a

collection of images (previously organized), even in the presence of altered versions of the images.

Nowadays we are working and directed this research to solve real problems in images retrieval system. We are focusing our efforts to propose new associative models able to associate and recall images under more complex transformations. Furthermore, this new models have to work with images of much more complicated objects such as flowers, animals, cars, faces, etc.

Acknowledgements

References

1. K. Steinbuch. 1961. Die Lernmatrix. *Kybernetik,* 1(1):26-45.
2. J. A. Anderson. 1972. A simple neural network generating an interactive memory. *Mathematical Bioscien*ces, 14:197-220.
3. J. J. Hopfield. 1982. Neural networks and physical systems with emergent collective computational abilities. *Proceedings of the National Academy of Sciences,* 79: 2554-2558.
4. M. Kutas and S. A. Hillyard. 1984. Brain potentials during reading reflect word expectancy and semantic association. *Nature,* 307:161–163.
5. G. X. Ritter el al. 1998. Morphological associative memories. *IEEE Transactions on Neural Network*s, 9:281-293.
6. Iva Reinvan. 1998. Amnestic disorders and their role in cognitive theory. *Scandinavian Journal of Psychology,* 39(3): 141-143.
7. C. J. PRICE. 2000. The anatomy of language: contributions from functional neuroimaging. *Journal of Anatomy,* 197(3): 335-359.
8. S. B. Laughlin and T. J. Sejnowski. 2003. Communication in neuronal networks. *Science,* 301:1870-1874.
9. P. Sussner. 2003. Generalizing operations of binary auto-associative morphological memories using fuzzy set theory. *Journal of mathematical Imaging and Vision*, 19(2):81-93.
10. G. X. Ritter, G. Urcid, L. Iancu. 2003. Reconstruction of patterns from noisy inputs using morphological associative memories. *Journal of mathematical Imaging and Vision,* 19(2):95-111.
11. H. Sossa, R. Barrón, R. A. Vázquez. 2004. Transforming Fundamental set of Patterns to a Canonical Form to Improve Pattern Recall. *LNAI,* 3315:687-696.
12. H. Sossa, R. Barrón, R. A. Vázquez. 2004. New associative memories for recall real-valued patterns. *LNCS,* 3287:195-202.
13. K. D. Jovanova-Nesic and B. D. Jankovic. 2005. The Neuronal and Immune Memory Systems as Supervisors of Neural Plasticity and Aging of the Brain: From Phenomenology to Coding of Information. *Annals of the New York Academy of Sciences,* 1057: 279-295
14. R. A. Vázquez and H. Sossa. 2007. A new associative memory with dynamical synapses. Submitted to *IJCNN* 2007.