Noisy Image Processing Using the Independent Component Analysis Algorithm AMUSE

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Abstract. In this article we investigate the performance of the ICA algorithm AMUSE when applied to images contaminated by noise. The classes of noise we are using have gaussian, multiplicative and impulsive distributions. We find that AMUSE copes surprisingly well with the different types of noise, including multiplicative noise.

1 Introduction

Currently signal processing and especially the processing of images are gaining more and more importance every day. To date, different investigations have been carried out in the field of image processing whose results were compared to the Human Visual System (HVS) [1], [2], [3], to model it's capacities to adept quickly to the hugh amount of data it is constantly receiving. In order to extract the desired information from these images multistep procedures are necessary. In the first steps, the data is transformed such that its underlying structure becomes visible. The obtained data is then subject to further analysis tools in order to detect elementary components like, e.g., borders, regions, textures etc. Finally, applications are developed which aim at solving the actual problems like, e.g. recognition tasks or 3D reconstruction, etc. [4].

The present article is structured as follows: Section 2 offers a brief review of Independent Componentes Analysis (ICA) and of its most important characteristics which are exploited in Blind Source Separation (BSS). Also a brief introduction to the algorithm AMUSE (Algorithm for Multiple Unknown Signals Extraction) is given. Section 3 evaluates the performance of the algorithm when applied to data of noisy images.

1.1 Relation between ICA and Images

In the left part of figure 1 the 256 x 256 pixel image "Lena" is displayed which we have analyzed by ICA in order to obtain its typical characteristics or filters. As can be seen in the right part of Fig. 1 these characteristics exhibit edges and other structures of interest. The characteristics were obtained by whitening the data first and by estimating afterwards the mixing-matrix **A** by means of the fastICA algorithm. The shown patches in the right part of Fig. 1 correspond to the columns $\mathbf{a}_l$ of the obtained mixing matrix **A** [5].

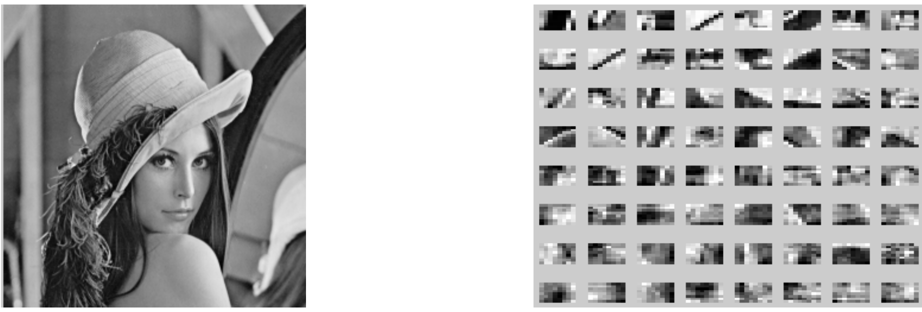

Fig. 1. Left: original image "Lena", 256 x 256 pixels. Right: Typical characteristics of the image, obtained applying ICA to blocks of 8 x 8 pixels.

For the processing of the image data two different approaches are usually used. The first alternative is like a local solution where the whitening-matrix $\mathbf{V}_{ZCA} = E\{\mathbf{x}\mathbf{x}^T\}^{-1/2}$ is used to identically filter certain local regions of the data, a procedure which is similar to that occuring in the receptive fields in the retina and the lateral geniculate nucleus (LGN). As second alternative Principal Component Analysis (PCA) can applied, so that orthogonal filters are produced that lead to uncorrelated sources. Here $\mathbf{V}_{PCA} = \mathbf{D}^{-1/2}\mathbf{E}^T$ where $\mathbf{E}\mathbf{D}\mathbf{E}^T = E\{\mathbf{x}\mathbf{x}^T\}$ is an eigen-system of the correlation-matrix $E\{\hat{\mathbf{x}}\hat{\mathbf{x}}^T\}$. In addition PCA allows to reduce to the dimension of the problem by only selecting a subgroup of the components $\mathbf{z} = \mathbf{V}_{PCA}\mathbf{x}$, which allows us, among other things, to reduce computational costs and execution time and to lower memory consumption, etc.

Once the data has been whitened, ICA (Independent Component Analysis) is used to find the separation- or demixing-matrix **W** such that the statistical dependence between the considered sources is minimal:

$$\hat{\mathbf{s}} = \mathbf{W}\mathbf{z} = \mathbf{W}\mathbf{V}_{PCA}\mathbf{x} = \mathbf{W}\mathbf{D}_n^{-1/2}\mathbf{D}_n^T\mathbf{x} \quad (1)$$

where $\mathbf{D}_n$ is a diagonal matrix that contains n eigenvalues of the correlation matrix $E\{\mathbf{x}\mathbf{x}^T\}$ and $\mathbf{E}_n$ is the matrix having the corresponding eigenvectors in its columns.

It is important to note the similarities between the characteristics or filters found by ICA and the receptive fields of the neurons in the primary visual cortex, a similarity which eventually leads to the suggestion that the neurons are able to carry out a certain type of independent component analysis and that the receptive fields are optimized for natural images [5] [6] [7] [8].

2 Independent Component Analysis (ICA)

The concept of Independent Component Analysis was introduced by Heroult, Jutten and Ans [9] as an extension of principal component analysis. The latter is a mathematical technique that allows to project a data set to a space of characteristics whose orthogonal basis is determined such that the variance of the projections of the data onto this basis is larger than that obtained by projecting onto any other orthogonal basis. The resulting signals of a PCA transform are uncorrelated which means that the covariance or the second order cumulants, respectively, are zero.

The signals resulting from an ICA are statistically independent while no assumptions on the orthogonality of the basis vectors are made. The goal of such an ICA is then to discover a new group of meaningful signals. In order to carry out this study three hypothesis are necessary: the sources are mutually statistically independent; at most one of them has a Gaussian distribution; and the mixing model (linear, convolutive or non-linear) is known *a priori*. [9]

A lineal mixture $x_1, x_2, \ldots, x_n$ of n independent components [10], [11], is expressed mathematically by:

$$x_j = a_{j1}s_1 + a_{j2}s_2 + a_{j3}s_3 + a_{jn}s_n \text{ for all} j \tag{2}$$

where each x_j represents a mixture and each s_k represents one of the independent components. These are random variables with zero mean.

This relation can also be expressed in matrix notation: Let $\mathbf{x}$ be the random vector having the mixtures $x_1, x_2, \ldots, x_n$ as its elements, and let $\mathbf{s}$ be the random vector consiting of the individual sources $s_1, s_2, \ldots, s_n$. Furthermore, consider the matrix $\mathbf{A}$ with elements a_{ij}. Following this notation the linear mixture model can be expressed as

$$\mathbf{x} = \mathbf{A}\mathbf{s} \tag{3}$$

The ICA model is a generative model, where the observed data originates from a mixture process of the hidden original components, which are mutually independent and cannot be observed directly. This means, that only the observed data is used to recover the mixing matrix $\mathbf{A}$ and the underlying sources $\mathbf{s}$.

2.1 AMUSE Algorithm

The AMUSE algorithm (Algorithm for Multiple Unknown Signals Extraction) uses temporal structures (the sources have to be uncorrelated and must have autocorrelation; no assumptions on statistical independence are necessary); it applies second order statistics with the purpose of obtaining independent components. The major motivation for the development of this algorithm was to surpass the difficulties many fourth order algorithms have when they are applied to problems with more than only one Gaussian source. [12].

The AMUSE algorithm can be formulated as follows:

1. Let $\mathbf{x}(t)$ be whitened and let $C_\tau^{\mathbf{x}}$ have n nondegenerated eigenvalues.

2. The eigenvalue decomposition of $C_\tau^{\mathbf{x}}$ is determined:

$$C_\tau^{\mathbf{x}} = \mathbf{W}^T D \mathbf{W} \tag{4}$$

whereas $\mathbf{W} \in O(n)$ and D is diagonal.
3. Then $\mathbf{W}$ is the separation matrix:

$$\mathbf{W}^T = \mathbf{W}^{-1} \sim \mathbf{A} \tag{5}$$

However, the condition that all n eigenvalues are different is often strict and are a problem in real-life applications. The eigenvalues of $\mathrm{Cov}(s_i(t), s_i(T-\tau))$ must differ significantly from each other, which is specially problematic with signals that have similar energy spectra.

3 Behavior of AMUSE when Applied to Noisy Image Data

In this section we investigate the behavior of the algorithm AMUSE when applied to the the images shown in the figure 2.

3.1 Method

As can be seen the set of images represent structures (mostly windows) which are displayed as grayscale pictures. These images consists of 256×256 pixels and each of them was previously contaminated by Gaussian, multiplicative and impulsive (also known as salt and pepper) noise. These types of noise can be seen as an own characteristic function on which the following studies have been based.

Fig. 2. Some images of structures used for the analysis.

Once having contaminated the original images with each type of noise the original and the noisy images were used to constitute the rows of the observation matrix $\mathbf{X}$, i.e. $\mathbf{X}$ consisted of 64 rows and 15360 columns. For them the results have been evaluated by means of the behaviour of the filters of the different mixing matrices $\mathbf{A}$ as well as by the typical distributions that must be preserved under the presence of noise.

Figure 3 depicts the general evaluation scheme that was used throughout this section. First, the images of the matrix **S** (see figure 3) are used and transformed into the matrix **X**. To these data the noise is added. From it four different observation matrices are obtained: the observation matrix of the original mixtures ($\mathbf{X}_{orig}$), the observation matrix contaminated with Gaussian noise $\mathbf{X}_{gau}$, the observation matrix contaminated with multiplicative noise ($\mathbf{X}_{mul}$) and the observation matrix contaminated with salt and pepper noise ($\mathbf{X}_{syp}$). Once the observations are created AMUSE is applied. Then the histograms are evaluated and the different results are compared with the goal to detect the filters which contain only noise.

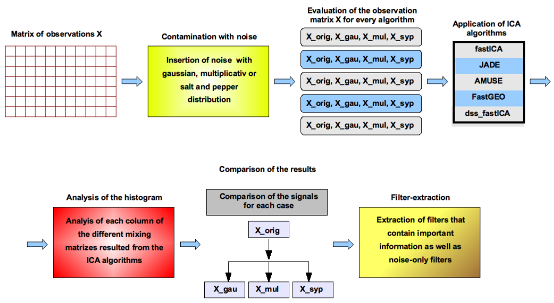

Fig. 3. General diagram. Scheme that describes the separate steps of the analysis.

3.2 Behavior of AMUSE

The results obtained from this analysis are comperatively extensive as 5 different algorithms and 3 classes of noise were used; its because of this reason that the algorithm AMUSE was used at this point of the study as it exhibits a series of particularities, especially in the context of impulsive noise. Apart from this, AMUSE was found stable no matter of the class of noise used in the data set.

First, the behavior of the bases of the mixing-matrix are investigated for each of the two cases (original and noisy signals) after dimension reduction by PCA. In this process the dimensions have been reduced to 49, 36, 25, 16, and 9 respectively. The results obtained after reducing the dimension are shown in 4 for the original signals, in 5 for the signals with Gaussion noise, in 6 for signals with multiplicative noise and in 7 for signals with salt and pepper noise.

Consider for example image 4 which presents a comparison between each of the filters while reducing the dimension, the purpose being to detect those filters which are stable and to find out if there could be a connection between the different classes of data. Independent of the class of noise, AMUSE found stable results in the filters, in where each iteration of the comparison between the different dimensional reductions also presented a concentration of stable filters in first 8 positions and mostly also in the last 3 or 4 filters. In these figures, the red frames show the stable filters (first and last) that stay throughout each reduction of dimension, arriving to obtain finally a reduction to 9 dimensions in that the filters appear clean from noise and clearly describe important information.

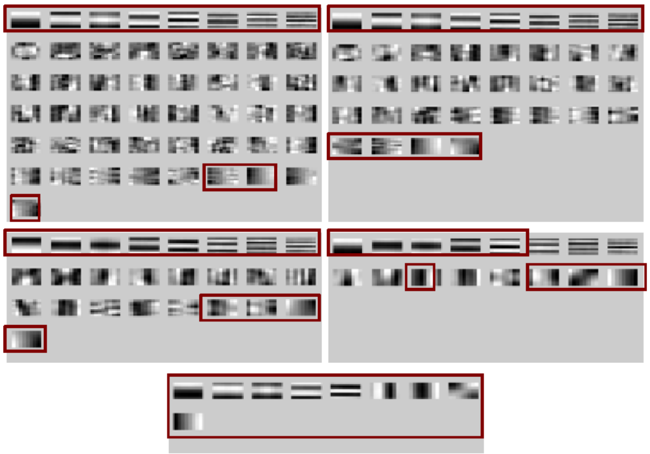

Fig. 4. Results of applying AMUSE with previous PCA to the original signals. Left superior part: reduction of dimensions to 49; right superior part: reduction of dimensions to 36; left central part: reduction of dimensions to 25; right central part: reduction of dimensions to 16; and inferior part: reduction of dimensions to 9.

4 Conclusion

In this article we have shown an analysis of the ICA algorithm AMUSE in digital images processing with noise. ICA has shown properties that allow to have a good model of the characteristics of the receivers of the cortical neurons in the human visual system. Here we have demonstrated the advantages of the ICA algorithm AMUSE that should allow investigators to choose the best algorithm according to the necessities and objectives that they have to consider.

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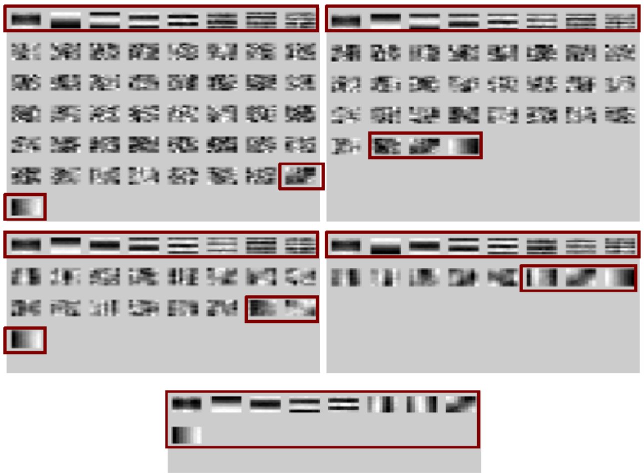

Fig. 5. Results of applying AMUSE with previous PCA to signals with gaussian noise. Left superior part: reduction of dimensions to 49; right superior part: reduction of dimensions to 36; left central part: reduction of dimensions to 25; right central part: reduction of dimensions to 16; and inferior part: reduction of dimensions to 9.

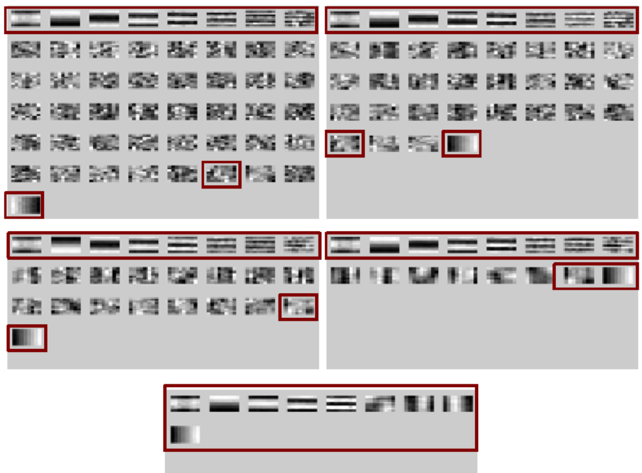

Fig. 6. Results of applying algorithm AMUSE with previous PCA to signals with multiplicative noise. Left superior part: reduction of dimensions to 49; right superior part: reduction of dimensions to 36; left central part: reduction of dimensions to 25; right central part: reduction of dimensions to 16; and inferior part: reduction of dimensions to 9.

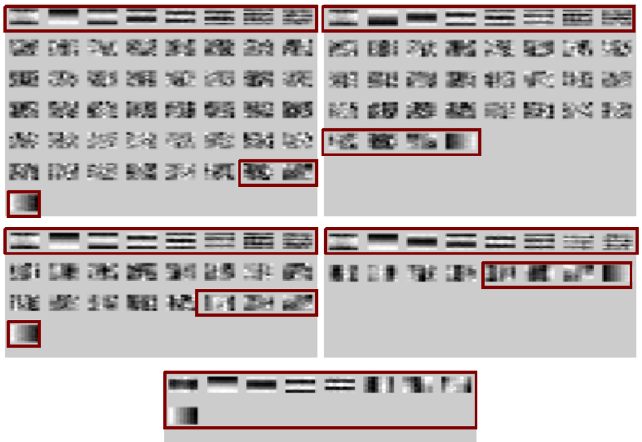

Fig. 7. Results of applying algorithm AMUSE with previous PCA to signals with salt and pepper noise. Left superior part: reduction of dimensions to 49; right superior part: reduction of dimensions to 36; left central part: reduction of dimensions to 25; right central part: reduction of dimensions to 16; and inferior part: reduction of dimensions to 9.

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