Codesign Strategy based upon Takagi Sugeno Control Design and Real-Time Computing Integration

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Abstract. Nowadays the idea of network control systems design considering the restriction results from schedulling analysis becomes a challenge based upon the persepctive of codesign point view since both analytic tools are pursued. A clear strategy is to define in cascade mode the scheduling analysis and afterwards the stability analysis of the respective control strategy. However, any modification in both structures has an integrated impact which is necessary to measure. In that respect the use of time delay impact is a suitable strategy to be followed and is explored in this paper. The use of codesign is to pursuit as a two objective strategy the definition of a valid metric that represents the effects in both, following the idea that stability analysis is affected according to the schedullability analysis. In both analysis a relaxation at the local conditions is feasible but it will have a global cost giving a non valuable approximation.

1 Introduction

Nowadays, the use of multiple tools for complex systems design like Real-Time distributed systems need any background in terms of design, for instance, the relation between analysis constrains expresed in different metrics like reliability, schedullability, safety, stability and so on. The need to relate these measures can be pursued in terms of a hollistic design or codesign sctrategy [4]. In order to define this kind of strategy it is necessary to determine the effects of each technique over the rest. Several approaches can be persued like decision trees, common metrics definitions, stochastic processes and others. However, this problem remains open in terms of a standard approximation amongst the complexity of the goal. One interesting approximation is based upon the codesign way of thinking, by choosing one specific aspect from each technique. Therefore, the individual achievemnt of every technique considering its effects over the rest should be pursuit. The objective of this paper is to review this approximation over a Real-Time Distributed System considering its effects over a specific application such as dynamic system. Specifically, this is studied by the use of schedulling and control design, where schedulability and strability analysis are reviewed to guarantee the feasability of this strategy.

Although this is dependant on the specific strategy to be followed, global characteristics such as the respective analysis can be pursued.

Further on, the union of different techniques allows a holistic view of the problem, although, the result can be restricted to specific algorithms and the inherent restriction of the case study.

Section 2 presents a review of structural codesign based upon the schedulling approximation. Section 3 presents control codesign where fuzzy logic control law is used in order to incorporate scheduler information onto stability analysis. Section 4 presents some concluding remarks of thi approximation.

2 Structural Codesign

The codesign proposal is based upon the iteration between schedulability and stability analysis following online approximation as shown in Fig. 1.

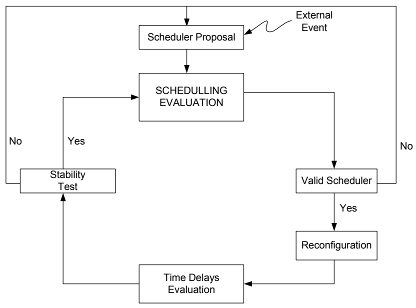

Fig. 1. Dynamic Reconfiguration in terms of Codesign strategy.

Any classical scheduller [1] bounds the time behaviour of the tasks in terms of their own priority where certain modification amongst them produces important differences. For instance, consumption time from tasks named as sensors (t_{sj}), actuators (t_{aj}) and controllers (t_{cj}) can be seen as follows:

$$\begin{aligned} t_{sj} &= t_{sj}^{*} + \Delta t_{sj}^{*} \\ t_{aj} &= t_{aj}^{*} + \Delta t_{aj}^{*} \\ t_{cj} &= t_{cj}^{*} + \Delta t_{cj}^{*} \end{aligned} \tag{1}$$

Where the total time spent by the tasks is equal to t_t^j

$$t_t^j = \sum_{i=1}^{N} t_{cj}^i + \sum_{i=1}^{M} t_{sj}^i + \sum_{i=1}^{P} t_{aj}^i \tag{2}$$

Index i is the number of tasks involved per structural elements like sensors (M), actuators (P) and controllers (N). Index j is the current scenario defined by the scheduller. These time delyas that are the result of priority modification on the peripheral elements as individual manner should change the design parameters at the control law. At least these time delays provide enough information to perform an adecuate control law design. Fig. 2 shows how t_t^j should be bounded to Control period of time.

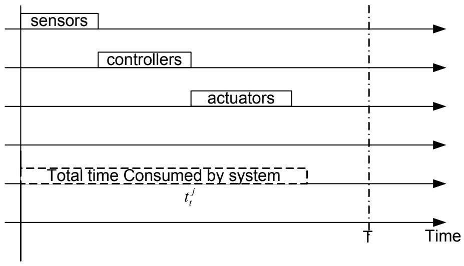

Fig. 2. Bounded Time Inherent to Control Period of Time.

Where T is a long enough time window where t_t^j should take place. Δt is the variation presented per element [12] during element actuations according to the pursued scheduler algorithm such as EDF, RM or FTT [1] [2] [3] [15].

To guarantee schedulability is necessary an effective performance from the control law [4]. This can only be pursued if only if the time delays exist within the bounded time delays used to design a suitable control law as a classical gain schedulling strategy. When task schedulling is performed, it implies a variation Δt giving a modification to the control law. Therefore the classical scehdulability analysis [1] can be modified in order to incorporate this kind of uncertainty giving the following result

$$U = \sum_{i=1}^{N} \frac{c_i + \Delta c_i}{P_i} \leq 1 \tag{3}$$

Where c_i represents the consumption time of each task, Δc_i is the related uncertainty, P_i is the related period, N is the number of tasks and U is the total relation between cosumptions and periods. This last value should be less than one in order to guarantee schedulability. It is important to deploy that any classical schedulling algorithm should fit into this condition as longs as the tasks are periodicals (which is the case herein) and the inherent uncertaintes should be fit into the same condition.

In fact, these time delays can be seen like a phase modification within the communication period from the involved processes. This scenario presents a complete phase modification at the entire system.

The communication network plays a key role in order to define the behaviour of the dynamic system in terms of time variance giving a nonlinear behaviour. In order to understand such a nonlinear behaviour, time delays are incorporated by the use of real-time system theory that allows time delays to be bounded even in the case of causal modifications due to external effects. In order to model this behaviour a reconfigurable real time schedulling algorithm is proposed, named Structural reconfiguration algorithm (SRA).

This algorithm bounds Time delays through a real-time scheduling algorithm within communication network. According to Figure 1, structural reconfiguration takes place as a result of Earliest Deadline First (EDF) Schedulling algorithm and the associated user request. This reconfiguration causes a control law modification [1] which is the actual control law reconfiguration.

Schedulling approach pontentially modifies frequency execution and communication of tasks [5] in order to give certain priority to some of them during a bounded time as shown in Fig. 3. Furthermore, in this kind of strategy Tasks modifies their priority, it does not imply that neither the period nor the consumtion times are modified. Therefore the tasks would have a bounded delay within the sampling time wich is reflected as changing on the phase.

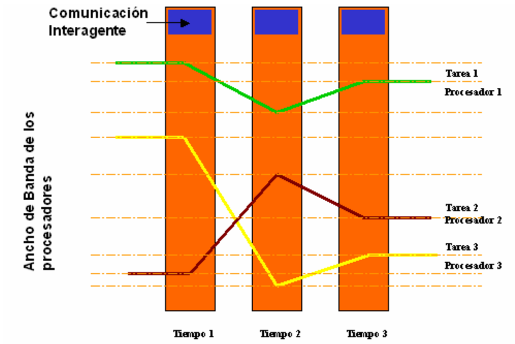

Fig. 3. Task Frequency modification as result of Scheduller.

Potential modifications onto schedulling approach deploy change in the priorities that affects time delays and the respective control law. The delays are measured as Δt [14] and bounded into the inherent control period of time [6] [7].

Now by taking partial results from schedulling algorithm like t_{sj} and the related Δt, the actual time delays are used at the control law for parameters design as shown in following section. The involved time delays are depicted as τ_j^i and come from this scheduling design. Other delays like actuators and control delays are not used in the design of the control law, although play an important role.

Therefore schedulling and control analysis merge together when time delays are complete bounded even in the case of time variance. The main restriction is in terms of predictable time delays.

The approach followed at the control reconfiguration does not take into account scheuller decision in a direct manner. It takes the time delyas as bounded values already defined and used to design a suitabe control law. Therefore, acording to current state plant values, the related fuzzy rule is selected.

3 Control Reconfiguration Approach

The control law is defined as a group of Fuzzy TKS [8] [9] [10] control law related to each local linear system. At the beginning the general structure of each fuzzy rule is:

$$r_i \text{ if } x_1 \text{ is } A^c_{1i} \text{ and } x_2 \text{ is } A^c_{2i} \text{ and} \ldots \; x_l \text{ is } A^c_{li} \text{ then } f(k) = Q_i x(t) \tag{4}$$

where $i = \{1,....,N\}$, N is the number of fuzzy rules, $\{x_1,....,x_l\}$ are current states of the plant, A^c_{ij} are the gaussians membership functions like:

$$A^c_{ij} = \exp \frac{\left(x_i - c^c_{ij}\right)^2}{\sigma^{c\,2}_{ij}} \tag{5}$$

where: c^c_{ij} and σ^c_{ij} are constants to be tuned.

Similar to fuzzy system plant [9], fuzzy control representation is integrated as:

$$w_i = \prod_{j=1}^{l} A^c_{ij}\left(x_j\right) \tag{6}$$

And

$$u(k) = \frac{\sum_{i=1}^{N} w_i\left(Q_i x(t)\right)}{\sum_{i=1}^{N} w_i} \tag{7}$$

Wher Q_i is the related i'th control gain. The configuration of the fuzzy logic control (FLC) integrated to the plant, epxressed as well in terms of Fuzzy Takagi Sugeno approach is represented in Fig. 4 [11]. The closed loop system is pursued in terms of local plant and related control gain per rule. In order to pursue this strategy, plant model is shown in terms of its state space representation.

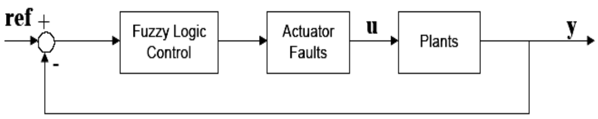

Fig. 4. Plant configuration using FLC control.

Using the proposed dynamic plant based on state space representation, see [11]:

$$\begin{aligned} x(k+1) &= A^P x(k) + B^P u(k) \\ y &= c^P x(k) \end{aligned} \tag{8}$$

Specially B^P is stated as

$$B^p = \sum_{i=1}^{N} \rho_i B_i \sum_{j=1}^{M} \int_{t^i_j}^{t^i_{j+1}} e^{A^P_i (t-\tau)} d\tau \tag{9}$$

where $\rho_i = 1$ and $\sum_{i=1}^{N} \rho_i = 1$ taking into account that *N* are the total number of possible faults and *M* are the involved time delays from each fault. Current communication time delays are expressed as τ_{j-1}^{i} and τ_{j}^{i} remember that $\sum_{j=1}^{M} \tau_j^i \leq T$ (total period of time inherent from control law design) and B_i in general terms is integrated as

$$B_i = \begin{bmatrix} b_1 \\ b_2 \\ 0_i \\ \vdots \end{bmatrix} \rightarrow \text{, i fault element}$$

where $b_1 \rightarrow b_N$ are the elements conformed at the input of the plant (such as actuators) and 0_i is the lost element due to local sensor fault where B_i^P represents only one scenario. Remember that the only considered faults are sensor faults. Therefore one input signal is measured. This can lose its confidence but not current value [10]. Since this approximation current B_i^p considers local sensor faults and related time delays of

$$B_i^p = B_i \sum_{j=1}^{M} \int_{\tau_j}^{\tau_{j-1}^i} e^{A_i^P (t-\tau)} d\tau \tag{10}$$

Remember that the related time delays are the result of structural reconfiguration (SRA) explained before are calculated according to eqns. 4 and 5.

Back to the controller definition where *N* is the number of possible scenarios, therefore, number of rules.

$$Q_z = \sum_{j=1}^{N} \left[Q_j h_j (x(t)) \right] \tag{11}$$

$$h_j(x(t)) = \frac{w_i}{\sum_{i=1}^{N} w_i} \tag{12}$$

as for the plant integrated to the controller in closed loop, this is expressed as:

$$A_z^p x + B_z^p u \Rightarrow A_z^p x - B_z^p Q_z x \tag{13}$$

$$\Rightarrow A_z^p x - x \left\{ B_i \sum_{j=1}^{N} \int_{\tau_{j-1}^i}^{\tau_j^i} e^{\left[A_i^p (t-\tau)\right]} d\tau \right\} \left\{ \sum_{j=1}^{N} \left[Q_j h_j (x(t)) \right] \right\}$$

$$\Rightarrow \left[A_z^p - \left\{ \sum_{i=1}^{N} h_i B_i \sum_{j=1}^{N} \int_{\tau_{j-1}^i}^{\tau_j^i} e^{A_i^p (t-\tau)} d\tau \right\} \left\{ \sum_{j=1}^{N} \left[Q_j h_j (x(t)) \right] \right\} \right] x(t)$$

$$A_z^p = \sum_{i=1}^{N} A_i^p \{h_i(x(t))\}$$

Then the proposed Lyapunov function is:

$$v(x(t)) = x^T(t) P_z x(t) \tag{14}$$

And its derivative is expressed in eqn 14 as a necessary condition for stability

$$\dot{v}(x(t)) = \dot{x}^T(t) P_z x(t) + x^T(t) P_z \dot{x}(t) \leq 0 \tag{15}$$

In terms of the plant integrated to the control law this is expressed as follows:

$$\dot{v} = x^T \left(A_z^p - B_z^p Q_z\right)^T P_z x + x^T P_z \left(A_z^p - B_z^p Q_z\right) x + x^T \dot{P}_z x \tag{16}$$

remember that

$$B_z^p = \sum_{i=1}^{N} \; B_i \; \sum_{j=1}^{M} \int_{\tau_{j-1}^i}^{\tau_j^i} e^{A_i(t-\tau)} d\tau \;\; h_i$$

where M is the number of time delays per scenario within the control law inherent period.

$$Q_z = \sum_{j=1}^{N} \{Q_j h_j(x(t))\}$$

Therefore the core of lyapunov function is given as :

$$\dot{x} = \left(A_z^p - B_z^p Q_z\right) x \tag{17}$$

Therefore the derivative of the energy as expressed in 15 can be expressed as:

$$= x^T \left\{ \left(A_z^p - B_z^p Q_z\right)^T P_z + P_z^T \left(A_z^p - B_z^p Q_z\right) + \dot{P}_z \right\} x \tag{18}$$

$$P_z = \sum_{i=1}^{N} \left(g_i(x(t)) P_i\right)$$

$$\dot{P}_z = \sum_{i=1}^{N} \left(\dot{g}_i(x(t)) P_i\right)$$

and

$$g_z = A_z^p - B_z^p Q_z \tag{19}$$

Now by expressing the same energy function in terms of an inequality relation in a relaxed manner, considering all the possible P_z matrices equals in terms of the same matrix ($g_z = A_z^p - B_z^p Q_z$) for any condition considered along the N fuzzy rules, energy function can be expressed as:

$$\dot{v} \leq \sum_{i=1}^{N} \left(\left(h_i^2(x) x^T \left(\left(A_i^P - B_i^P Q_i\right)^T P_z + P_z \left(A_i^P - B_i^P Q_i\right) \right) x \right) \right) + \tag{20}$$

$$\sum_{i=1}^{N} \sum_{i<j} 2h_i(x) h_j(x) x^T \quad \frac{A_i^P \quad B_i^P Q_j + A_j^P \quad B_j^P Q_i}{2}^{T} P_z + P_z \quad \frac{A_i^P \quad B_i^P Q_j + A_j^P \quad B_j^P Q_i}{2} \quad x$$

Therefore, the controller design can be expressed as:

$$-P_z A^{P^T}_i - A_i^P P_z + P_z Q_i^T B^{P^T}_i + B^P{}_i Q_i P_z > 0 \tag{21}$$

Remember that i has a value between 1 to N. Therefore for every given plant and the respective controller by decomposing this equation, the P_z matrix should be bounded as:

$$-P_z A^{P^T}_i - A^P{}_i P_z - P_z A^{P^T}_j - A^P{}_i P_z + P_z Q_j^T B^{P^T}_i + B^P{}_i Q_j P_z + P_z Q_i^T B^{P^T}_j + B^P{}_j Q_i P_z > 0 \tag{22}$$

Remember that that also j has a value between 1 to N related to the number of rules. Therefore in terms of Linear Matrix Inequality [9] is given by

$$\begin{matrix} P_z & P_z A_i^T - Q_i^T P_z^T B_j^T \\ A_i P_z - B_i Q_j P_z & P_z \end{matrix} > 0 \tag{23}$$

This condition is given for every single time delay and local fault appearance. Furthermore the stability and the convergence of states should be assured by the adequate selection of matrices P_z and the related parameters from both fuzzy systems. In this case a recommendable procedure to follow is multi-objective optimization in order to define those suitable values [12].

The whole system considering this codesign strategy, has been implemented in several environments such as simulation based [12] using True-Time [16] and real-life using CANBUS [13]. Although this approximation is out of the scope of the paper, these implementations have given enough information in terms of the practical experience for current approach. Moreover, related strategies for codesign control theory have been reviewed with preliminar succesful results in [6].

4 Conclusions

The use of codesign as a suitable strategy for networked control and scheduling analysis is a real possibility as explored in this paper. Although it is restricted to the fesability of both techniques, this can be approximated as an iteractive procedure where both techniques need to achieve an aggrement.

In this case time delays are approximated and bounded through a suitable scheulling policy which affects the results of current selected controller. The exploration followed in here is based upon classical schedulling algorithms and fuzzy takagi sugeno approach. The key characteristic of last approach is design of local control law considering bounded time delays per valid scenario from schedulling results.

Several results need to be highlighted such as the convergence of variable time delays due to the use of schedulling approximation and the restricted and known modification onto control law design. Furthermore, bounded time delays as long as they are from the same source, like sensor delays, they modifiy similar control paramters, therefore, control structure does not need to be modified on a large scale.

Future work need to be focus onto strucutral modification from the control law, as well as a deeper study from time delays source. For instace, the complex computing relationship stablished through the operating system, middleware transactions, interprocees communications, communication network protocols, and others.

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