

# A NOVEL PARTICLE SWARM OPTIMIZATION APPROACH FOR MULTI-OBJECTIVE FLEXIBLE JOB SHOP SCHEDULING PROBLEM

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**Abstract:** Because of the intractable nature of the flexible job shop scheduling problem and its importance in both fields of production management and combinatorial optimization, it is desirable to employ efficient metaheuristics in order to obtain a better solution quality for the problem. In this paper, a novel approach based on the vector evaluated particle swarm optimization and the weighted average ranking is presented to solve flexible job shop scheduling problem (FJSP) with three objectives (i) minimize the makespan, (ii) minimize the total workload of machines and (iii) minimize the workload of critical machine. To convert the continuous position values to the discrete job sequences, we used the heuristic rule the Smallest Position Value (SPV). Experimental results in this work are very encouraging since that relevant solutions were provided in a reasonable computational time.

## 1 INTRODUCTION

Solving a NP-hard scheduling problem with only one objective is a difficult task. Adding more objectives obviously makes this problem more difficult to solve. In fact, while in single objective optimization the optimal solution is usually clearly defined, this does not hold for multiobjective optimization problems. Instead of a single optimum, there is rather a set of good compromises solutions, generally known as Pareto optimal solutions from which the decision maker will select one. These solutions are optimal in the wider sense that no other solution in the search space is superior when all objectives are considered. Recently, it was recognized that Particle Swarm Optimization (PSO) was well suited to multiobjective optimization mainly because of its fast convergence.

The Particle Swarm Optimization (PSO) is a population based search algorithm developed by Kennedy and Eberhart in 1995 (Kennedy, 1995) (Abraham, 2006, pp. 3-15) (Clerc, 2005) inspired by social behaviour of bird flocking or fish schooling. Unlike Genetic Algorithms (GA), PSO has no evolution operators such as crossover and mutation. In PSO, the

population is initialized randomly and the potential solutions, called particles (Hu, 2004) fly through the search space with velocities which are dynamically adjusted according to their historical behaviors. In PSO, each particle is influenced by both the best solution that it has discovered so far and the best particle in its neighbors (local variant of PSO) or in the entire population (global variant of PSO).

Figure 1 shows the general flow chart of PSO. At each time step, the behaviour of a given particle is a compromise between three possible choices:

- to follow its own way
- to go towards its best previous position
- to go towards the best neighbour

This compromise is formalized by equations (1) and (2) and illustrated by figure 2.

$$v_i^{\rightarrow}(t+1) = c_1 * v_i^{\rightarrow}(t) + c_2 * (p_i^{\rightarrow}(t) - x_i^{\rightarrow}(t)) + c_3 * (p_g^{\rightarrow}(t) - x_i^{\rightarrow}(t)) \quad (1)$$

$$x_i^{\rightarrow}(t+1) = x_i^{\rightarrow}(t) + v_i^{\rightarrow}(t+1) \quad (2)$$

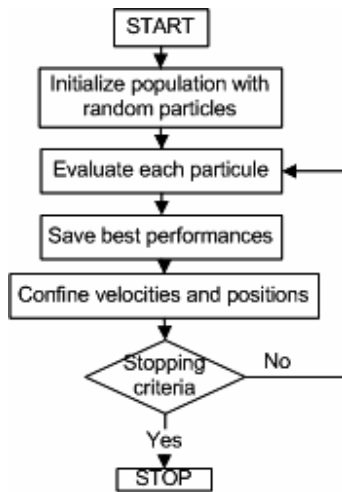


Figure 1: The mapping between particle and FJSP.

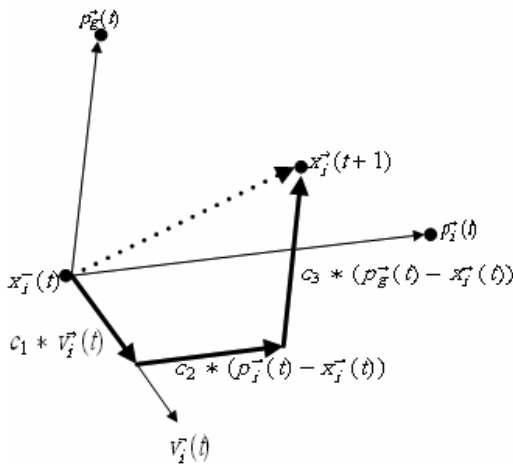


Figure 2: An illustration of particle's move.

with

- $v_i^-(t)$ : velocity of particle  $i$  at iteration  $t$
- $p_i^-(t)$ : best previous position of particle  $i$  at iteration  $t$
- $p_g^-(t)$ : best neighbour of particle  $i$  at iteration  $t$
- $x_i^-(t)$ : position of particle  $i$  at iteration  $t$
- $c_1, c_2, c_3$ : positive random numbers. These numbers are social-confidants coefficients

Although PSO is still new in evolutionary computation field, it has been applied to a plethora of problems in science and engineering. Multiobjective optimization problem (MOO) has been one of the most studied application areas of PSO algorithms (Coello, 2002), (Coello, 2004), (Hu, 2002), (Parsopoulos, 2002), (Hu, 2003). Number of approaches

have been used and/or designed to manage MOO problems using PSO. A straight forward approach is to convert MOO to a single objective optimization problem. One simple implementation of the conversion is the so-called weighted aggregation approach which sums all the objectives to form a weighted combination (Shi, 2004) (Mendes, 2004). Weights can be either fixed or adapted dynamically during the optimization.

Other approaches combine Pareto dominance with PSO in order to identify Pareto fronts. Most of the research studies developed in this field used two dimensional objectives. It may seem that using only two objectives oversimplifies the problem (Mendes, 2004). In this paper, an application of the particle swarm optimization algorithm to the flexible job shop scheduling with three objectives is reported. The main goal of our research is to design mechanisms to extend PSO such that it can generate solutions of "good quality" either for the individual optimization of criteria or for the compromise between the different objectives.

## 2 MATHEMATICAL FORMULATION

The flexible job shop scheduling problem was studied in (Chetouane, 1995), (Mesghouni, 1999), (Kacem, 2002), (Dupas, 2004), (Xia, 2005), (Abraham, 2006), (Liu, 2006), (Liu, 2007). FJSP belongs to the  $NP$ -hard family (Sakarovitch, 1984). It presents two difficulties. The first one is the assignment of each operation to a machine, and the second is the scheduling of this set of operations in order to optimize our criteria. The result of a scheduling algorithm must be a schedule that contains a start time and a resource assignment to each operation.

The data, constraints and objectives of our problem are as follows:

### 2.1 Data

- $M$  represents a set of  $m$  machines. A machine is called  $M_k$  ( $k = 1, \dots, m$ ), each  $M_k$  has a load called  $W_k$ .
- $N$  represents a set of  $n$  jobs. A job is called  $J_i$  ( $i = 1, \dots, n$ ), each job has a linear sequence of  $n_i$  operations.
- $O_{i,j}$  represents the operation number  $j$  of the job number  $i$ . The realization of each operation  $O_{i,j}$  requires a machine  $M_k$  and a processing time

$p_{i,j,k}$ . The starting time of  $O_{i,j}$  is  $t_{i,j}$  and the ending time is  $t_{f_{i,j}}$ .

## 2.2 Constraints

- Machines are independent of one another.
- Each machine can perform operations one after another.
- Each machine is available during the scheduling.
- A started operation runs to completion.
- Jobs are independent of each another.

In our work, we suppose that:

- Machines are available since the date  $t = 0$ .
- Each job  $j_i$  can start at the date  $t = 0$ .
- The total number of operations to perform is greater than the number of machines.

## 2.3 Criteria

We have to minimize  $Cr1$ ,  $Cr2$  and  $Cr3$ :

- The makespan:

$$Cr1 = \max_{1 \leq i \leq n} (\max_{1 \leq j \leq n_i} (t_{f_{i,j}}))$$

- The total workload of machines:

$$Cr2 = \sum_{1 \leq k \leq m} (W_k)$$

- The workload of the most loaded machine:

$$Cr3 = \max_{1 \leq k \leq m} (W_k)$$

These criteria are often conflicting. In fact, balance resource usage by minimizing the utilization of bottleneck equipment can be antagonistic with the minimization of the total time of production.

## 2.4 Lower Bounds

Lower bounds are usually used to measure the quality of solutions found. For our work, we use lower bounds proposed in (Dupas, 2004):

- $BCr1$ : (lower bound for  $Cr1$ )

$$BCr1 = \max_i (\sum_j \min_k (p_{i,j,k}))$$

- $BCr2$ : (lower bound for  $Cr2$ )

$$BCr2 = \sum_{i,j} \min_k (p_{i,j,k})$$

- $BCr3$ : (lower bound for  $Cr3$ )

$$BCr3 = \lceil BCr2/m \rceil$$

## 3 PSO FOR FJSP

### 3.1 Particle Representation and Initial Swarm Generation

One of the key issues when designing the PSO algorithm lies on its solution representation which directly affects its feasibility and performance. In this paper, an operation-based representation is used. For the ( $m$  machines,  $n$  jobs,  $O$  operations) FJSP, each particle contains  $O$  number of dimensions corresponding to  $O$  operations and has a continuous set of values for its dimensions which represents particle's positions. The Smallest Position Value (Tasgetiren, 2004) (Tasgetiren, 2006), the SPV rule is used to find the permutation of operations and a randomly generated number is used to find the machine to which a task is assigned to during the course of PSO. Figure 3 illustrates the solution representation of a particle corresponding to FJSP described in table 1 and table 2. The smallest component of the particle's position is  $-2,25$  which corresponds to the operation number 6 of job number 2. Thus, job 2 is scheduled first. The second smallest component of the particle's position is  $-0,99$  which corresponds to the operation number 2 of job number 1. Therefore, job 1 is the second job in the ordering, etc.

Table 1: Processing time of operations of a (3 Jobs, 5 Machines) problem.

	M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>	M <sub>4</sub>	M <sub>5</sub>
O <sub>1,1</sub>	1	9	3	7	5
O <sub>1,2</sub>	3	5	2	6	4
O <sub>1,3</sub>	6	7	1	4	3
O <sub>2,1</sub>	1	4	5	3	8
O <sub>2,2</sub>	2	8	4	9	3
O <sub>2,3</sub>	9	5	1	2	4
O <sub>3,1</sub>	1	8	9	3	2
O <sub>3,2</sub>	5	9	2	4	3

Table 2: The operating sequences of jobs of a (3 Jobs, 5 Machines) problem.

J <sub>1</sub>	O <sub>1,1</sub>	O <sub>1,2</sub>	O <sub>1,3</sub>
J <sub>2</sub>	O <sub>2,1</sub>	O <sub>2,2</sub>	O <sub>2,3</sub>
J <sub>3</sub>	O <sub>3,1</sub>	O <sub>3,2</sub>	

The PSO randomly generates an initial swarm of  $S$  particles, where  $S$  is the swarm size. These particle vectors will be iteratively updated based on collective experiences in order to enhance their solution quality.

Jobs	$J_1$			$J_2$			$J_3$	
Operations	$O_{1,1}$	$O_{1,2}$	$O_{1,3}$	$O_{2,1}$	$O_{2,2}$	$O_{2,3}$	$O_{3,1}$	$O_{3,2}$
Particle position	1,8	-0,99	3,01	0,72	-0,45	-2,25	5,3	4,8
Sorted positions	6	2	5	4	1	3	8	7
Sequence of jobs	2	1	2	2	1	1	3	3
Sorted operations	$O_{2,1}$	$O_{1,1}$	$O_{2,2}$	$O_{2,3}$	$O_{1,2}$	$O_{1,3}$	$O_{3,1}$	$O_{3,2}$
Random numbers	2	3,7	5,3	1,4	5,1	5	4	4,6
Machines	$(M_{2,t_1})$	$(M_{3,t_2})$	$(M_{5,t_3})$	$(M_{1,t_4})$	$(M_{5,t_5})$	$(M_{5,t_6})$	$(M_{4,t_7})$	$(M_{4,t_8})$

Figure 3: The mapping between particle and FJSP.

### 3.2 Our Approach

Our approach is a novel proposal to solve multiobjective optimization problems using PSO. It is inspired by The Vector Evaluated Particle Swarm Optimization (VEPSO)(Parsopoulos, 2002b) algorithm wich incorporates ideas from the Vector Evaluated Genetic Algorithm (VEGA) (Shaffer, 1985).

Our approach is based on the use of Weighted Average Ranking (WAR) (Collette, 2002) and a subdivision of decision variable space into  $(k + 1)$  sub-swarms ( $k$ : is the number of criteria). Each sub-swarm  $i$  ( $i$  between 1 and  $k$ ) is exclusively evaluated with the objective function number  $i$ , but, information coming from other sub-swarm(s) especially from the sub-swarm number  $(k + 1)$  is used to influence its motion in the search space. The execution of the flight of each sub-swarm can be seen as an entire PSO process (with the difference that it will optimize only a part of the search space and not the entire search space). The sub-swarm  $(k + 1)$  looks for the solutions of compromise between the  $k$  studied criteria. It generates the leaders set among the particle swarm set by using the Weighted Average Ranking. Leaders of other sub-swarms can migrate to the sub-swarm  $(k + 1)$  until a number of iterations is reached in order to variate the selection pressure. The procedure of exchanging information among sub-swarms can lead to Pareto optimal points.

Stages of the algorithm described in figure 1 are repeated until a certain prefixed number of iterations is reached.

## 4 PERFORMANCE MEASURES

Different instances of the present problem have been chosen to test our approach, in order to ensure a certain diversity. These instances present a number of operations between 8 and 56 (the number of jobs is

between 3 and 15) and a number of resources between 4 and 10 machines. The studied problem nature is varied enough according to the performance of resources, their flexibility and the number of the precedence constraints. So, cases of parallel machine problems, where all the machines have the same performance, have been also tested. We also studied total and partial flexibility cases when machines presented variable performances. As results to the simulations, some findings can be pulled:

- Most particular problems have been solved in an optimal manner (case of problems having parallel machines).
- The problems with parallel machines are easier to solve than the problems having machines with variable performance.
- The found solutions are generally of a good quality. Is is noted while comparing them with the existing approaches in the literature and also while comparing obtained values of the criteria with the computed lower bounds. As an illustration, we choose to present the following instance: we consider the problem described in table 3 (10 jobs, 30 operations, 10 machines). The computation of the different lower bounds gives the following values:  $BCr1 = 7$ ,  $BCr2 = 41$ ,  $BCr3 = 4$ . This example has been already processed in the literature by many methods: temporal decomposition (Chetouane, 1995), classic GAs (Mesghouni, 1999), approach by localization and approach by localization and controlled EAs and approach by hybridizing particle swarm optimization and simulated annealing (Xia, 2005). The schedule obtained in these cases is characterized by the following values presented in figure 4.

	$C1$	$C2$	$C3$
Temporal decomposition	16	59	16
Classic GA	7	53	7
Approach by localization	8	46	6
AL+CGA	7	45	5
PSO+SA	7	44	6
Our approach	7	44	6

Figure 4: Solutions in the literature of (10J, 10M).

Table 3: Matrix of processing times of FJSP (10J,10M).

		M <sub>1</sub>	M <sub>2</sub>	M <sub>3</sub>	M <sub>4</sub>	M <sub>5</sub>	M <sub>6</sub>	M <sub>7</sub>	M <sub>8</sub>	M <sub>9</sub>	M <sub>10</sub>
J <sub>1</sub>	O <sub>1,1</sub>	1	4	6	9	3	5	2	8	9	5
	O <sub>1,2</sub>	4	1	1	3	4	8	10	4	11	4
	O <sub>1,3</sub>	3	2	5	1	5	6	9	5	10	3
J <sub>2</sub>	O <sub>2,1</sub>	2	10	4	5	9	8	4	15	8	4
	O <sub>2,2</sub>	4	8	7	1	9	6	1	10	7	1
	O <sub>2,3</sub>	6	11	2	7	5	3	5	14	9	2
J <sub>3</sub>	O <sub>3,1</sub>	8	5	8	9	4	3	5	3	8	1
	O <sub>3,2</sub>	9	3	6	1	2	6	4	1	7	2
	O <sub>3,3</sub>	7	1	8	5	4	9	1	2	3	4
J <sub>4</sub>	O <sub>4,1</sub>	5	10	6	4	9	5	1	7	1	6
	O <sub>4,2</sub>	4	2	3	8	7	4	6	9	8	4
	O <sub>4,3</sub>	7	3	12	1	6	5	8	3	5	2
J <sub>5</sub>	O <sub>5,1</sub>	7	10	4	5	6	3	5	15	2	6
	O <sub>5,2</sub>	5	6	3	9	8	2	8	6	1	7
	O <sub>5,3</sub>	6	1	4	1	10	4	3	11	13	9
J <sub>6</sub>	O <sub>6,1</sub>	8	9	10	8	4	2	7	8	3	10
	O <sub>6,2</sub>	7	3	12	5	4	3	6	9	2	15
	O <sub>6,3</sub>	4	7	3	6	3	4	1	5	1	11
J <sub>7</sub>	O <sub>7,1</sub>	1	7	8	3	4	9	4	13	10	7
	O <sub>7,2</sub>	3	8	1	2	3	6	11	2	13	3
	O <sub>7,3</sub>	5	4	2	1	2	1	8	14	5	7
J <sub>8</sub>	O <sub>8,1</sub>	5	7	11	3	2	9	8	5	12	8
	O <sub>8,2</sub>	8	3	10	7	5	13	4	6	8	4
	O <sub>8,3</sub>	6	2	13	5	4	3	5	7	9	5
J <sub>9</sub>	O <sub>9,1</sub>	3	9	1	3	8	1	6	7	5	4
	O <sub>9,2</sub>	4	6	2	5	7	3	1	9	6	7
	O <sub>9,3</sub>	8	5	4	8	6	1	2	3	10	12
J <sub>10</sub>	O <sub>10,1</sub>	4	3	1	6	7	1	2	6	20	6
	O <sub>10,2</sub>	3	1	8	1	9	4	1	4	17	15
	O <sub>10,3</sub>	9	2	4	2	3	5	2	4	10	23

## 5 CONCLUSIONS

This paper presents a novel approach using particle swarm optimization to solve the multicriteria flexible job shop scheduling with total or partial flexibility. It is based on the vector evaluated particle swarm optimization and the weighted average ranking.

Our work, resulted in to the development of a generic method to resolve multiobjective optimization. It provides relevant solutions for the individual optimization of criteria or for the compromise between the different objectives. Future research will cover an investigation on the effects of diversity control in the search performances of multiobjective particle swarm optimization.

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