WALKING PLANNING AND CONTROL FOR A BIPED ROBOT UPSTAIRS

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Keywords: Humanoid robot, stability control, gait planning, stability margin, ZMP, FZMP.

Abstract: The focus of this paper is the problem of walking stability control in humanoid robot going upstairs. Walking stability is a very important problem in the field of robotics. Lots of researches have been done to get stable walking on plane. But it is very limited on going upstairs. We first plan the gate of ankle and hip when going upstairs as well as the calculation of stable region and stability margin. Then the emergency-coping strategy of enlarging the support polygon is provided. At last, a control system which is proved to be effective by simulation is presented. If the ZMP is in the support polygon, this control system makes fine setting to gait to get higher stability. If the ZMP is out of the support polygon, the control system adjusts the location of ZMP through the emergency coping strategy.

1 INTRODUCTION

Biped humanoid robots have better mobility than wheeled robots, especially for moving on rough terrain, steep stairs and obstacle environments (Huang, 1999). Research on humanoid robots has become one of the most exciting topics in the robotics field and there are many ongoing projects (Kaneko, 2002; Konno, 2002; Pfeiffer, 2002). Many researches are made on the walking stability of biped robots (Kajita, 2003; Stojic, 2000).

In order to realize stably walking, many different models are proposed. Such as the zero-moment point (ZMP), Vukobratovic (Vukobratovic, 1990; Vukobratovic, 2004) first proposed; the criterion of “Tumble Stability Criterion” for integrated locomotion and manipulation systems, proposed by Yoneda etc (Yoneda, 1996); the foot-rotation indicator (FRI), introduced by Goswami (Goswami, 1999) and so on. This paper uses the Fictitious Zero-Moment Point (FZMP) (Yin, 2005) criterion to calculate stability.

There are different walking patterns in different environment. Tatsuo Narikiyo etc (Narikiyo, 2006) researched walking control of robot when walking in space. Shuji Kajita etc (Kajita, 2004) researched a biped which can jump. But as humanoid robot which will be used widely in our daily life, it will be more welcomed if the robot can walk up and down stairs. So in this paper, we discuss the gait planning and present an effective control strategy of robot going upstairs.

2 STABILITY CALCULATION

In order to evaluate dynamic stability, we use the ZMP principle. The ZMP is the point where the influence of all forces acting on the mechanism can be replaced by one single force. If the ZMP is inside
the support polygon, the biped robot can be stable. If the ZMP is on the boundary of the support polygon, the robot will fall down or have a trend of falling down. If the computed ZMP is outside the support polygon, then the robot will fall down and in this case, the computed ZMP is called fictitious ZMP. The link model of the humanoid robot is shown in Figure 1.

The projection of position vector of computed ZMP can be computed by the following equations:

\[
x_{\text{zmp}} = \sum_{i=1}^{5} \frac{m_i(z_i + g)}{\sum_{i=1}^{5} m_i} x_i - \sum_{i=1}^{5} \frac{m_i(z_i + g)}{\sum_{i=1}^{5} m_i} y_i + \sum_{i=1}^{5} \frac{m_i(z_i + g)}{\sum_{i=1}^{5} m_i} z_i + \sum_{i=1}^{5} M_i \]  

\[
y_{\text{zmp}} = \sum_{i=1}^{5} \frac{m_i(z_i + g)}{\sum_{i=1}^{5} m_i} y_i - \sum_{i=1}^{5} \frac{m_i(z_i + g)}{\sum_{i=1}^{5} m_i} x_i + \sum_{i=1}^{5} \frac{m_i(z_i + g)}{\sum_{i=1}^{5} m_i} y_i + \sum_{i=1}^{5} M_i \]  

where \(m_i\) is mass of every links, \((x_i, y_i, z_i)\) is the coordinate of the mass center of the links, \((M_i, M_i)^T\) is the moment vector.

If the ZMP is inside the support polygon and the minimum distance between the ZMP and the boundaries of support polygon is large, then the biped will be in high stable, and this distance is called the stability margin. We can know the situation of walking stability from the stability margin.

As shown in Figure 2, if the ZMP is outside the support polygon, i.e. FZMP, the norm of vector \(s\) represents the shortest distance between FZMP and the edges of the support polygon. This edge is called rotation edge. The direction of vector \(s\) is the rotation direction of the robot.

### 3 GAIT PLANNING OF ANKLE WHEN GOING UPSTAIRS

In order to simplify research process we first discuss how to get ankle trajectory and hip trajectory. Then the knee trajectory could be got by kinematics. Here we take the left foot for example and the right foot is similar only with a delay of half cycle. The link model we used is shown in Figure 3.

#### 3.1 Gait Planning of Ankle

According to the walking procedure of human, we suppose that the walking cycle is \(T_c\), \(t = kT_c\) is the \(k\) th cycle begins with the moment when the left foot is just apart from the ground and ends with the left foot gets into contact with the ground; \(kT_c < t \leq kT_c + T_s\) is double support phase, during which the sole is rotated about toes; the swing foot reaches the highest point when \(t = kT_c + T_s\).

![Figure 3: The model of the humanoid robot going upstairs.](image)

We get the key point \((x_i, z_i, \theta_i)\) of ankle in plane \(XOZ\) as follows:
\[ x_f = \begin{cases} 
0 & t = kT_e \\
 \frac{x_{\mu} + 2Lk}{t} & t = kT_e + T_d \\
 \frac{x_{\mu} + 2Lk + L}{t} & t = kT_e + T_a \\
 \frac{x_{\mu} + 2L(k + 1) - l_s(1 - \cos \theta_f) - h_f}{t} & t = (k + 1)T_e \\
 \frac{x_{\mu} + 2L(k + 1)}{t} & t = (k + 1)T_e + T_d \\
 \frac{x_{\mu} + 2L(k + 2)}{t} & t = (k + 2)T_e 
\end{cases} \]

\[ z_f = \begin{cases} 
0 & t = kT_e \\
 \frac{z_{\mu} + 2Hk}{t} & t = kT_e + T_d \\
 \frac{z_{\mu} + 2Hk + h_f}{t} & t = kT_e + T_a \\
 \frac{z_{\mu} + 2H(k + 1) + h_f}{t} & t = (k + 1)T_e \\
 \frac{z_{\mu} + 2H(k + 1) + h_f}{t} & t = (k + 1)T_e + T_d \\
 \frac{z_{\mu} + 2H(k + 2) + h_f}{t} & t = (k + 2)T_e 
\end{cases} \]

where \( l_s \) is the distance between tiptoe and the centre of gravity of sole; \( l_p \) is the distance between heel and the centre of gravity of sole; \( h_f \) is the height of heel and \( T_a \) is the time when the robot just walks through a step.

The key point of the angle between sole and ground can be denoted as follows:

\[ \Theta_f = \begin{cases} 
0 & t = kT_e \\
 \frac{\theta_f}{t} & t = kT_e + T_d \\
 \frac{\theta_f}{t} & t = (k + 1)T_e \\
 \frac{\theta_f}{t} & t = (k + 1)T_e + T_d 
\end{cases} \]

Since the whole sole of the right foot is in contact with the ground at \( t = kT_e \) and \( t = (k + 1)T_e + T_d \), the following derivative constraints must be satisfied:

\[ \begin{align*}
\dot{x}_f (kT_e) &= 0 \\
\dot{x}_f ((k + 1)T_e + T_d) &= 0 \\
\dot{z}_f (kT_e) &= 0 \\
\dot{z}_f ((k + 1)T_e + T_d) &= 0 \end{align*} \]

\[ \begin{align*}
\dot{\theta}_f (kT_e) &= 0 \\
\dot{\theta}_f ((k + 1)T_e + T_d) &= 0 \end{align*} \]

### 3.2 Gait Planning of Hip

We assume that the robot is decelerated in double support phase and accelerated in single support phase and the acceleration in direction of \( x \)-axis and \( z \)-axis are \( a_{x_h} \) and \( a_{z_h} \) respectively. The distance between the hip and the ankle of supporting leg is \( x_s \) at the beginning of the double support phase and \( x_e \) at the end of the double support phase.

The changes in the direction of \( z \)-axis are \( z_s \) and \( z_e \) at the beginning and end of the double support phase respectively. Then the trajectory of hip can be expressed like this:

It must satisfy the following constraints:

- The derivative constraints \( \dot{x}_h (kT_e) = \dot{x}_h (k + 1)T_e \) \( \dot{z}_h (kT_e) = \dot{z}_h (k + 1)T_e \) and \( \dot{z}_h (kT_e) = \dot{z}_h (k + 1)T_e \) must be satisfied.
- \( z_h (t) \leq h_{max} \), \( h_{max} \) is the maximum height of hip; \( h_{max} = l_i + l_2 + h_f \), \( l_i, l_2 \) are the length of thigh and shin respectively, \( h_f \) is the height of ankle.
- \( z_h (t) \geq h_{min} \), \( h_{min} \) is the minimum height of hip and it’s value can be set according to the process of human walking.

\[ \left[ (x_s(t) - x_a(t))^2 + (z_s(t) - z_a(t))^2 \right]^{1/2} \leq l_i + l_2 \]
4 STABILITY CALCULATION

The maximum region enclosed by two soles’ projection on the ground is called the stable region, as shown in Figure 4.

\[ x_h(t) = \begin{cases} x_h + 2Lk + x_s, & t = kT_e \\ x_h + 2Lk + x_s + a_{sh}t, & t = kT_e + T_d \\ x_h + 2Lk + L, & t = kT_e + T_s \\ x_h + 2L(k + 1) - x_s - a_{sh}t, & t = (k + 1)T_e \\ x_h + 2L(k + 1) - x_s, & t = (k + 1)T_e + T_d \\ x_h + 2L(k + 1) + x_s, & t = (k + 2)T_e \\ \end{cases} \] (9)

\[ z_h(t) = \begin{cases} z_h + 2Hk + z_s, & t = kT_e \\ z_h + 2Hk + z_s + a_{zh}t, & t = kT_e + T_d \\ z_h + 2Hk + H, & t = kT_e + T_s \\ z_h + 2H(k + 1) - z_s - a_{zh}t, & t = (k + 1)T_e \\ z_h + 2H(k + 1) - z_s, & t = (k + 1)T_e + T_d \\ z_h + 2H(k + 1) + z_s, & t = (k + 2)T_e \\ \end{cases} \] (10)

\[ \begin{align*} x_{\text{max}} &= \max(x_{Q1}, x_{Q2}) + l_a \\ x_{\text{min}} &= \min(x_{Q1}, x_{Q2}) - l_b \\ y_{\text{max}} &= \max(y_{Q1}, y_{Q2}) + l_w \\ y_{\text{min}} &= \min(y_{Q1}, y_{Q2}) - l_w \\ l_{12} \\ l_{34} \end{align*} \] (13)

We can get the distances between ZMP and every boundary of stable region easily and the stability margin can be expressed as:

\[ \gamma = \min(d_{x_{\text{max}}}, d_{x_{\text{min}}}, d_{y_{\text{max}}}, d_{y_{\text{min}}}, dl_{12}, dl_{34}) \] (14)

5 MAINTAIN STABILITY BY ENLARGING SUPPORT POLYGON

As we said above, if the computed ZMP is outside the support polygon, the robot cannot be in dynamic stable and has the trend of falling down. In this case we can enlarge the support polygon to maintain stability.

As shown in Figure 5, the changed angle \( \alpha^*_f \) of moving direction of the foot is determined by the following equation:

\[ \alpha^*_f = \cos^{-1} \frac{e \cdot s}{\|e\| \|s\|} \] (15)
where \( e \) is the normal vector of planned moving direction and \( s \) is the changed vector. If the gradient of step is \( \theta \) and the distance between the feet is \( d \), then the foot moving distance \( f' \) relative to planed landing position is determined by the following formula:

\[
\begin{align*}
\tan(\alpha' - \beta) &= \left[ \tan(\theta + (d + 30)^\circ \sin^2 \theta \cos(\gamma - \alpha') - b \right] \left[ \tan(\theta + (d + 30)^\circ \sin^2 \theta \cos(\gamma - \alpha') - a \right] \\
\end{align*}
\]

where \( \beta \) is the original angle of foot and \( \gamma \) is the changed angle of foot centre. \( a, b \) express half of the foot’s long and width respectively.

6 CONTROL STRATEGY

If the computed ZMP is inside the support polygon then the robot can be in stable. But, the error between the computed ZMP and the designed ZMP is unavoidable. It means that the stability condition is not the best and we can make the robot more stable. Our control system (the left part) shown in Figure 7 can optimize the position of actual ZMP to enlarge the stable margin.

As shown in Figure 6, the inertial force \( F_R \) and ground reaction force \( R \) are not in the same line. In this case \( F_R, R \) and the error of ZMP form a moment which makes the robot roll. So it is necessary to reduce the error to diminish the moment. The rolling moment can be depicted like this

\[
\Delta M = (DZMP - AZMP) \times R
\]

where \( DZMP \) means desired ZMP; \( AZMP \) means actual ZMP.

We can obtain the error of ZMP (\( \Delta ZMP \)) according to the desired ZMP and the actual ZMP. The gait adjustment parameter \( \Delta \theta \) can be got using inverse kinematics.

\[
\Delta \theta = [\Delta \alpha_1, \Delta \alpha_2, \Delta \alpha_3, \Delta \beta_1, \Delta \beta_2]^T
\]

\[
= K \Delta ZMP = [K_{a1}, K_{a2}, K_{a3}, K_{b1}, K_{b2}] \Delta ZMP
\]

where \( K \) is the adjustment coefficient matrix by experience.

If there is an external disturbance, the computed ZMP may be out of the support polygon and the robot may tip over, at this moment the control system (the right part) will take action.

Two important parameters can be got according to the definition of FZMP, i.e. the distance between FZMP and the rotation edge and the rotation direction. We can also get the change of the angle of link \( \Delta \theta \) by the sensor fitted at the link. The emergency-coping strategies such as enlarging the support polygon, moving the upper body and contacting with surrounding by hands can be used to make the FZMP located in the support polygon and maintain the stability of the robot. Whereas, in some cases one method along may be unrealistic, two or three methods can be combined.

7 SIMULATION

We have constructed a simulator of a humanoid robot by using dynamic analysis software package ADMAS and the control system is built in Matlab. This allows us to analyze the joint torque, the change of ZMP, etc. The simulator built in ADMAS is shown in Figure 8.

Figure 5: The determination of foot landing position.

Figure 6: Error of ZMP.
Figure 7: The control strategy.

We simulate the procedure of walking up and down stairs with a height of 0.2m and a width of 0.6m. The pictures of series of walking upstairs are shown in Figure 10 at the time of 0.0s, 0.2s, 0.4s, 0.6s, 0.8s and 1.0s.

Figure 8: The ADMS model of humanoid robot.

The structure of control system based on Matlab/simulink is shown as follows:

Figure 9: The structure of control system.

The velocity and acceleration of ankle and hip are given in Figure 11 and 12 respectively. The x-axis represents time and the y-axis represents the velocity of ankle and hip. The red real line is velocity and the blue dashed is acceleration.
Figure 11: Velocity and acceleration of ankle.

Figure 12: Velocity and acceleration of hip.

Figure 11 shows that the maximum velocity happens when the swing foot is descending and it is consistent with the real walking process of human. It also shows that the velocity and acceleration are close to zero at the time of 1.0s and the slope is getting smaller when the time is getting nearer to 1.0s. We all know that the landing acceleration of ankle is very important for the stability of robot. If the acceleration is too big, the impact force between robot and ground may be very large and the robot may become unstable. So in our simulation, the impact force is very small and the robot can walk stably. It shows that the strategy described above works.

8 CONCLUSIONS

Biped robots have better mobility than wheeled robots but tip over easily, so the walking stability is even more important. When going upstairs the stability problem is especially crucial and the research is far from enough. In order to make the robot go upstairs stably, it is necessary to have an efficient control strategy to scout and make adjustment on time. In this paper, we propose a method to plan a walking pattern and the way of calculating the stable region and stability margin is also presented. The stability maintenance method of enlarging support polygon is given out. The optimization control strategy which is proved to be useful by numerical simulation is proposed.

REFERENCES


