CONTRIBUTION CONCERNING ROBOT ACCURACY USING NUMERICAL MODELING

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Abstract: The kinematical accuracy of robot is very important. It is induced by the rigidity of each mechanism of the robot. The paper presents a numerical method to evaluate the rigidity of worm-gearing teeth. The software, including setting-up and graphic display, could be adopted of any kind of cylindrical worm-gear drive or for spur gear drives and bevel gear drives, mechanisms which are in the robot structure. Besides, we can determine geometrical parameters of the gear drives which influence the increase of accuracy of robot linkages.

1 INTRODUCTION

Into the kinematical chain there are worm-gear drives, screw-nut mechanisms and pinion-rack drives. During the working, these gear drives and mechanisms of the robot deform under the load, leading to the motion errors. The errors can not be entirely eliminated, but their maximum values must be limited. The theoretical advantage of the conjugate action in involute gears is lost due to the deflection of the teeth under load and due to the manufacturing and assembling errors. These factors produce instantaneous variations in the gear ratio.

As it is well-known, the rigidity of the meshing teeth changes as the contact point moves from the initial point of contact to the final point of contact. During the meshing the normal force is mobile on the tooth flank, it changes continuously the position with respect to the fixing zone of the teeth. The load is unevenly distributed, depending on the contact ratio. Consequently, all these factors causes rotative speed variations of the driven shaft, vibrations, shocks, noise, power loss, low durability of gears. The purpose of the present work is to develop a methodology to evaluate the rigidity of the worm-gearing tooth. By means of this methodology the performances of the robot mechanisms may be improved.

2 GEOMETRY OF THE WORM-GEARING TOOTH

In order to analyze the rigidity of the worm-gearing tooth we assume that the spatial gearing consists of more plane-gearings (pinion-rack drives), that in fact are cross sections perpendicular to worm-gear axis (Figure 1). The analytic solving of the problem, even for a ruled worm-gearing, is very difficult due to the complexity of the equations of the plane-gearing profiles that are involved in the enveloping. Consequently, we use the "minimum distance method" applied in the case of the "discrete representation" of the enveloping profiles. Thus, the enveloping profile of the elementary worm-gear (plane-gear) can be determined numerically by knowing "discretely" a matrix having as elements the coordinates of the worm axial section and by using the theorem of the "minimum distance method".

The minimum distance theorem in "discrete way" states (Ghelase, D., Daschievici, L., 2006):

The envelope to the family of curves, represented in "discrete way" as massive of the coordinates of the points belonging to the family curves, consists of the all points there are on these curves, for which, at a certain size of the increment φ_1, the distance at the meshing pole is minimum.

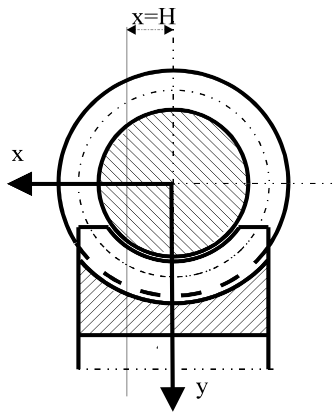

Figure1: Worm-gear drive.

2.1 Worm-Geometry

In order to determine the coordinates of the worm axial section, we focus on the case of a worm-gearing with modified profile could ensure, as well as possible, the generalization of the model from the geometrical viewpoint. Hence, let the axial section (x=0) of the worm (Figure 2) with constant pitch, having a circular arch profile with the centre in O_1 for the right flank and in O_2 for the left flank. The coordinates of the centre O_1, respectively O_2, are given by the following relations:

$$\begin{cases} Y_{O1}=R_e-u\cdot\cos\alpha-a\cdot\sin\alpha \\ Z_{O1}=b+u\cdot\sin\alpha-a\cdot\cos\alpha \end{cases}$$
$$\begin{cases} Y_{O2}=R_e-u\cdot\cos\alpha-a\cdot\sin\alpha \\ Z_{O2}=-b-u\cdot\sin\alpha-a\cdot\cos\alpha \end{cases} \quad (1)$$

where: *a* is a constant parameter;

$b=\pi\cdot m/4-1.25\cdot m\cdot tg\,\alpha$;

$p=m/2$;

$u=1.25\cdot m/\cos\alpha$;

$R=\sqrt{a^2+u^2}$ is the radius of the circular arc profile;

R_e is the tip radius of the worm tooth, all measured in mm (see Figure 2).

2.1.1 Equations of the Worm Flanks

In accordance with Figure 2, a point of the worm flank has the following coordinates:

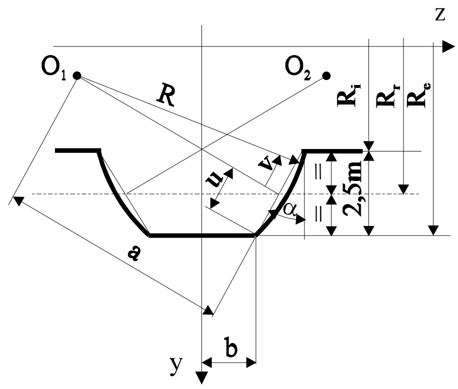

Figure 2: Worm flank geometry.

- For the right flank:

$$\begin{cases} X=0; \\ Y=Y_{O1}+R\cdot\cos(\frac{\pi}{2}-\alpha+v_1); \\ Z=Z_{O1}+R\cdot\sin(\frac{\pi}{2}-\alpha+v_1); \end{cases} \quad (2)$$

- For the left flank:

$$\begin{cases} X=0; \\ Y=Y_{O2}+R\cdot\cos(\frac{\pi}{2}-\alpha+v_2); \\ Z=Z_{O2}-R\cdot\sin(\frac{\pi}{2}-\alpha+v_2). \end{cases} \quad (3)$$

In the above relations, ν_1 and ν_2 are variable parameters of the right flank and left flank, respectively. Generally, the helical motion can be written by means of two coordinate transformations corresponding to simple motions, components of the helical motion: rotation about Oz axis, having parameter φ, and translation on the same axis, proportional to the rotation angle $p\cdot\varphi$, p being helical parameter. In this way, the helical motion of the movable coordinate system XYZ is described by the matrix equation:

$$x=\omega_3^T(\varphi)\cdot X+a \quad (4)$$

or

$$\begin{Vmatrix} x \\ y \\ z \end{Vmatrix} = \begin{Vmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} X \\ Y \\ Z \end{Vmatrix} + \begin{Vmatrix} 0 \\ 0 \\ p\cdot\varphi \end{Vmatrix} \quad (5)$$

where x is the matrix of a point coordinates with respect to the coordinate system xyz fixed to the frame, X is the matrix of the same point coordinates with respect to the movable coordinate system, a is the matrix of the point O coordinates (the origin of the movable coordinate system) with respect to the point O_1 (see Figure 3), and $\omega_3(\varphi)$ is the matrix of the rotation transformation.

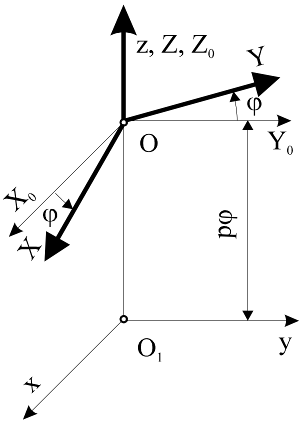

Figure 3: Coordinate system applied for the helical motion.

Substituting (1), (2) and (3) in (4), we obtain the parametric equations of the right flank surface and left flank surface.

Then, crossing these surfaces with the plane x=H, the curve representing the worm profile corresponding to the sectional plane takes the form (for example, the right flank):

$$\Sigma_{DH}\begin{cases}\sin\varphi_1 = \dfrac{H}{-[Y_{O1} + R\cdot\sin(\alpha - v_1)]};\\ y = [Y_{O1} + R\cdot(\alpha - v_1)]\cdot\cos\varphi_1;\\ z = Z_{O1} + R\cdot\cos(\alpha - v_1) + p\cdot\varphi_1;\end{cases} \tag{6}$$

2.2 Determination of the Worm-Gear Flank Profile

The worm-gear tooth surface is generated by the rolling.

We apply the "minimum distance method" on the algorithm of the discretization in the case of generation with the rack-bar tool.

First of all, we get the discretization of the generating curve C_Σ, which in this case is the worm profile, represented by the vector (7), where:

y_i and z_i are the coordinates of the profile from the "H" plane, which were determined by (5).

The gear flank generation of the elementary gear drive is made with the rack-bar tool (see Figure 4).

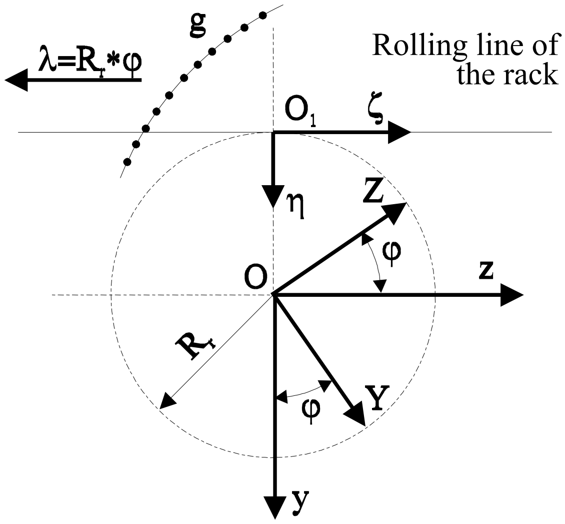

Figure 4: Worm-gear flank generation.

$$g = \begin{Vmatrix} y_1 & z_1 \\ y_2 & z_2 \\ . & . \\ . & . \\ . & . \\ y_n & z_n \end{Vmatrix} \tag{7}$$

The rolling condition interpreted in "discrete" way is the following:

$$K\cdot\Delta\lambda = R_r\cdot\Delta\varphi\cdot j \tag{8}$$

Where $\Delta\varphi$ is the angular increment of the rolling. It is then obvious that for generating a profile with high accuracy from the technical viewpoint, this increment has to be enough small.

2.2.1 Generation Motion

The generation motions of the worm-gear flank are:
1) Rotation of centroid, associated to the gear of the elementary gear drive, with respect to the fixed coordinate system xyz, described by the matrix equation

$$x = \omega_1^T(j\cdot\Delta\varphi)\,X. \tag{9}$$

In this relation, x is the matrix of the point coordinates with respect to the fixed coordinate system, X is the coordinates matrix of the same point with respect to movable coordinate system XYZ and $\omega_1(\varphi)$ is matrix of the rotation transformation about O_x axis;
2) Translation of the movable coordinate system ξηζ associated to the rack, with respect to the fixed

coordinate system, described by the equation (a is the coordinates matrix of the point O_1, the origin of the movable coordinate system, with respect to the point O):

$$x=\xi+a \tag{10}$$

with

$$a = \begin{Vmatrix} 0 \\ -R_r \\ -R_r \cdot (j \cdot \Delta\varphi) \end{Vmatrix}. \tag{11}$$

3) Relative motions

Combining (9) and (10) we obtain that the motion equation of a point on the generating curve “g” (Figure 4) from the coordinate system XYZ with respect to the coordinate system $\xi\eta\zeta$ is as follows:

$$\xi=\omega_1^T(j\cdot\Delta\varphi)\ X\text{-}a \tag{12}$$

$$X=\omega_1^T(j\cdot\Delta\varphi)[\xi+a]. \tag{13}$$

From the last equation, we infer that

$$\begin{cases} X = \xi; \\ Y = (\eta - R_r)\cdot\cos(j\cdot\Delta\varphi) + [\zeta - R_r\cdot(j\cdot\Delta\varphi)]\cdot \\ \sin(j\cdot\Delta\varphi); \\ Z = -(\eta - R_r)\cdot\sin(j\cdot\Delta\varphi) + [\zeta - R_r\cdot(j\cdot\Delta\varphi)]\cdot \\ \cos(j\cdot\Delta\varphi). \end{cases} \tag{14}$$

The system of equations (14) represents the family of generating curves “g” with respect to the coordinate system of the worm-gear, η and ζ being the coordinates of the points that are on the generating curve (Figure 5).

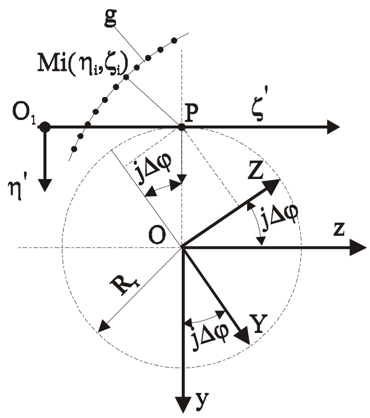

Figure 5: Coordinates of the meshing pole P.

The envelope to the family (14) is what we have to determine, more precisely, the gear profile (see Figure 6, flank of gear).

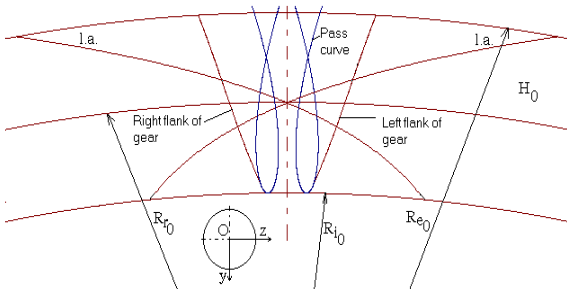

Figure 6: Line of contact (l.a.) in the plane H_0.

The enveloping condition is given by the minimum of distance

$$d = \sqrt{(Y - Y_P)^2 + (Z - Z_P)^2}\ , \tag{15}$$

where Y_P and Z_P (coordinates of the meshing pole) are:

$$Y_P= -R_r\cdot\cos(j\cdot\Delta\varphi);\ Z_P= R_r\cdot\sin(j\cdot\Delta\varphi). \tag{16}$$

2.3 Surface of Contact

The surface of contact is defined as locus of the contact points of the two conjugated surfaces (which are in enveloping) in the fixed coordinate system xyz (Figure 4). The parametric equations of the surface of contact are obtained associating the enveloping condition to the absolute motion equation of the worm-gear flank profile. In the sectional plane x=H, the line of contact is given by:

$$\begin{cases} y = Y\cdot\cos(j\cdot\Delta\varphi) - Z\cdot\sin(j\cdot\Delta\varphi); \\ z = Y\cdot\sin(j\cdot\Delta\varphi) + Z\cdot\cos(j\cdot\Delta\varphi). \end{cases} \tag{17}$$

3 WORM-GEARING TOOTH RIGIDITY

Once the algorithm for the determination of the contact points, both on the flank height and along the line of contact, is performed, then it is possible to evaluate the rigidity of the worm-gearing tooth.

3.1 Bases of Design

The mathematical model is based on the following assumptions:

- The worm-gearing is errors free and the gears are rigid except the teeth;
- Taken into consideration only the bending produced by the meshing normal force;

- Consider that the worm-gearing consists of more plane-gear drives (pinion-rack drives), named “elementary gear drives”, that in fact are cross sections perpendicular to axis of rotation of the worm-gear (Figure 1);
- The tooth of the elementary gear drive is supposed to be a beam fixed at one end in the body of gear;
- The assembly of the plane-gear drives into the worm-gear drive was made provided that the teeth of the elementary gear drives to deform together and not separately under the same load.

3.2 Computer Program

Our algorithm to evaluate the rigidity of the worm-gearing tooth is the following (Ghelase, D., 2005):

1) Computation of the rigidity for an elementary tooth;
2) Computation of the rigidity for a pair of elementary teeth;
3) Computation of the rigidity for an elementary gearing tooth (pinion-rack drive);
4) Computation of the rigidity for the worm-gearing tooth.

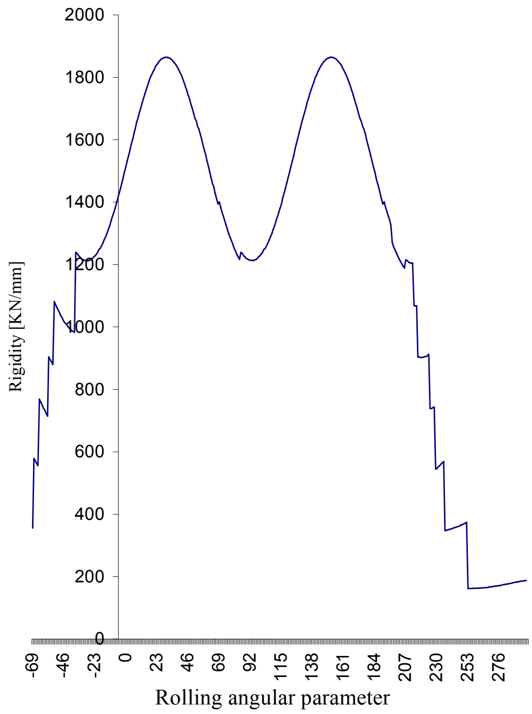

Figure 7: Rigidity of the worm-gearing tooth.

By means of the numerical modelling, these steps will be added to the computer program used for the study of the worm-gearing tooth geometry, finally providing the instrument for the determination of the worm-gearing tooth rigidity. The computation diagram of rigidity of worm-gearing tooth can be seen in Figure 7.

The cvasisinusoidal zone of the curve from Figure 7 repeats periodically, because it represents the rigidity during the meshing when the all plane-gear drives are involved in the meshing. Thus, if the input and output rigidities are eliminated, being less importing for our study, we get the elasticity characteristic of the worm-gearing tooth.

3.3 Elasticity Characteristic

The elasticity characteristic represents the variation of rigidity of the worm-gearing tooth depending on the rolling angle ($j \cdot \Delta\varphi$), where “*j*” is the rolling angular parameter (Ghelase, D., Tomulescu, L., 2003). It is cvasisinusoidal curve with the high jumps when a tooth binds or recesses (Figure 8).

The investigation of the elasticity characteristic is very important for the study of an elastic system, such as: gearing, linkage. Hence, the introduction of this concept contributes to the completion of the used gearing study and it leads to increase of the gearing tooth rigidity.

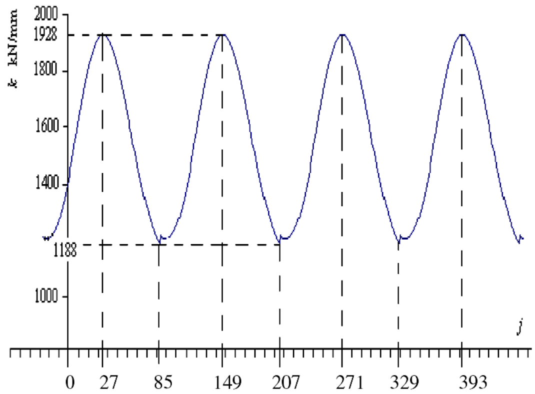

Figure 8: Elasticity characteristic of the worm-gearing tooth.

3.4 Influence of Geometrical Parameters

The influence of the geometrical parameters on the rigidity was obtained by means of the computerized simulation (Ghelase, D., 2003). It was applied to 150 worm-gear drives and we can present the following conclusions:

1. The rigidity of worm-gearing tooth increases if diametral quotient q increases and radius of profile curvature R increases (Ghelase, D., 2003).

2. The rigidity of worm-gearing tooth reduces if profile angle α increases (Ghelase, D., 2003) and number of the gear teeth z_2 increases (Figure 9, Table 1).

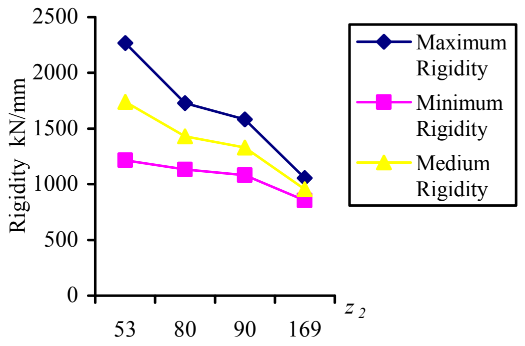

Figure 9: Rigidity depending on number of gear teeth z_2.

Table 1: Influence of number of gear teeth on rigidity.

z_2	Maximum Rigidity [kN/mm]	Minimum Rigidity [kN/mm]	Medium Rigidity [kN/mm]
53	2267.385	1215.140	1741.262
80	1727.633	1132.201	1429.917
90	1581.896	1079.696	1330.796
169	1055.990	853.826	954.908

4 CONCLUSIONS

Finally, we can draw the following conclusions:

1) A method to evaluate the rigidity of worm-gearing tooth was developed;

2) The proposed approach may be applied for any types of cylindrical worm-gearing and for spur gearing and bevel gearing. These mechanisms are in the structure of robot and by them rigidity depends the kinematical accuracy of robot;

3) The introduction of "elasticity characteristic" concept contributes to the completion of study for the used mechanisms;

4) The developed computer program enables to obtain numerical solutions and graphic illustration;

5) The numerical method, proposed and analyzed in this paper, affords the geometry optimization and the study of the meshing for various geometrical parameters of the worm-gearing, being in fact a simulation of meshing;

6) Moreover, we can determine the parameters which influence the improvement of rigidity for worm-gearing tooth and the increase of accuracy of robot linkages.

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