PASSIVITY OF A CLASS OF HOPFIELD NETWORKS
Application to Chaos Control

Adrian–Mihail Stoica
Faculty of Aerospace Engineering, University "Politehnica" of Bucharest, Str. Polizu, No. 1, Ro-011061, Bucharest, Romania
amstoica@rdslink.ro

Isaac Yaesh
Control Department, IMI Advanced Systems Div., P.O.B. 1044/77, Ramat–Hasharon, 47100, Israel
iyaesh@imi-israel.com

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Abstract: The paper presents passivity conditions for a class of stochastic Hopfield neural networks with state–dependent noise and with Markovian jumps. The contributions are mainly based on the stability analysis of the considered class of stochastic neural networks using infinitesimal generators of appropriate stochastic Lyapunov–type functions. The derived passivity conditions are expressed in terms of the solutions of some specific systems of linear matrix inequalities. The theoretical results are illustrated by a simplified adaptive control problem for a dynamic system with chaotic behavior.

1 INTRODUCTION

Hopfield networks are symmetric recurrent neural networks which exhibit motions in the state space which converge to minima of energy.

Symmetric Hopfield networks can be used to solve practical complex problems such as implement associative memory, linear programming solvers and optimal guidance problems. Recurrent networks which are non symmetric versions of Hopfield networks play an important role in understanding human motor tasks involving visual feedback (see (Cabrera and Milton, 2004) - (Cabrera et al., 2001) and the references therein). Such networks seem to be subject to effects of state-multiplicative noise, pure time delay (see (Hu et al., 2003), (X. Liao and Sanchez, 2002) and (Stoica and Yaesh, 2006)) and even multiple attractors which can be caused by Markov jumps. Even without Markov jumps, a non symmetric class of Hopfield networks is able to generate chaos (Kwok et al., 2003). Therefore, Hopfield networks can be used (Poznyak and Sanchez, 1999) as identifiers of unknown chaotic dynamic systems. The resulting identifier neural networks have been used in (Poznyak and Sanchez, 1999) to derive a locally optimal robust controller to remove the chaos in the system.

In this paper, we consider to replace the robust controller of (Poznyak and Sanchez, 1999) by a direct adaptive controller. More specifically we consider the so called Simplified Adaptive Control (SAC) method (Kaufman et al., 1998) which applies a simple proportional controller whose gain is adapted according the squared tracking error. Since such controllers' stability proof involves a passivity condition, we derive a passivity result for the Hopfield network. Our results are, in fact, developed for a generalized version of non symmetric Hopfield networks including Markov jumps of the parameters and state multiplicative noise thus allowing a wider stochastic class of chaos generating systems to be considered.

The paper is organized as follows. In Section 2, the problem is formulated and in Section 3 Linear Matrix Inequalities (LMIs) based conditions are derived for passivity analysis. In Section 4 a chaos control example is given and finally Section 5 includes concluding remarks.

Throughout the paper $\mathcal{R}^n$ denotes the n dimensional Euclidean space, $\mathcal{R}^{n\times m}$ is the set of all $n\times m$ real matrices, and the notation $P>0$, (respectively, $P\geq 0$) for $P\in\mathcal{R}^{n\times n}$ means that P is symmetric and positive definite (respectively, semi-definite). Throughout the paper $(\Omega,\mathcal{F},\mathcal{P})$ is a given probability space; the argument $\theta\in\Omega$ will be suppressed. Expectation is denoted by $E\{.\}$ and conditional expectation of x on the event $\theta(t)=i$ is denoted by $E[x|\theta(t)=i]$.

2 PROBLEM FORMULATION

The neural network proposed by Hopfield, can be described by an ordinary differential equation of the form

$$\dot{v}_i(t) = a_i v_i(t) + \sum_{j=1}^{n} b_{ij} g_j(v_j(t)) + \bar{c}_i = \kappa_i(v), 1 \le i \le n \quad (1)$$

where v_i represents the voltage on the input of the ith neuron, $a_i < 0, 1 \le i \le n$, $b_{ij} = b_{ji}$ and the activations $g_i(\cdot), i = 1, ..., n$ are C^1–bounded and strictly increasing functions.

This network is usually analyzed by defining the network energy functional:

$$\begin{aligned} E(v) = & -\sum_{i=1}^{n} a_i \int_0^{v_i} u \frac{dg_i(u)}{du} du \quad (2) \\ & -\frac{1}{2} \sum_{i,j=1}^{n} b_{ij} g_i(v_i) g_j(v_j) - \sum_{i=1}^{n} \bar{c}_i g_i(v_i) \end{aligned}$$

where it can be seen that $\frac{dE}{dt} = -\sum \frac{dg_i(v_i)}{dv_i} \kappa_i(v)^2 \le 0$ where the zero rate of the energy is obtained only in the equilibrium points, also referred to as attractors, where

$$\kappa_i(v^0) = 0, 1 \le i \le n \quad (3)$$

The network is then described in matrix form as:

$$\dot{v}(t) = Av(t) + Bg(v) + \bar{C}, 1 \le i \le n \quad (4)$$

where

$$A := diag(a_1, ..., a_n), B := [b_{ij}]_{i,j=1,...,n},$$
$$\bar{C} := \begin{bmatrix} \bar{c}_1 & \bar{c}_2 & \ldots & \bar{c}_n \end{bmatrix}^T, v := \begin{bmatrix} v_1 & v_2 & \ldots & v_n \end{bmatrix}^T$$

and where

$$g(v) := \begin{bmatrix} g_1(v_1) & g_2(v_2) & \ldots & g_n(v_n) \end{bmatrix}^T$$

The stochastic version of this network driven by white noise, has been considered in (Hu et al., 2003) where the stochastic stability of (1) has been analyzed and where it has been shown that the network is almost surely stable when the condition $\frac{dE}{dt} \le 0$ is replaced by $\mathcal{L}E \le 0$ where $\mathcal{L}$ is the infinitesimal generator associated with the Itô type stochastic differential equation (4). This condition has been shown in (Hu et al., 2003) to be satisfied only in cases where the driving noise in (1) is not persistent. This non persistent white noise can be interpreted as a white-noise type uncertainty in A and B, namely state-multiplicative noise. In (Stoica and Yaesh, 2005)-(Stoica and Yaesh, 2006) Hopfield networks with Markov jump parameters have been considered to represent also non zero mean uncertainties in these matrices. Encouraged by the insight gained in (Cabrera and Milton, 2004) and (Cabrera et al., 2001) regarding the role of state-multiplicative noise and time delay (see also (Mazenc and Niculescu, 2001)) in visuo-motor control loops, we generalize the results of (Stoica and Yaesh, 2005) to include this effect. The Lur'e - Postnikov systems approach ((Lure and Postnikov, 1944), (Boyd et al., 1994)) is invoked to analyze stability and disturbance attenuation (in the H_∞ norm sense) and the results are given in terms of Linear Matrix Inequalities (LMI).

To analyze input output properties we first define the error of the Hopfield network output with respect to its equilibrium points by

$$x(t) = v(t) - v^0. \quad (5)$$

and assume that the errors vector $x(t)$ satisfy

$$\begin{aligned} dx = & (A_0(\theta(t))x + B_0(\theta(t)) f(y) + D(\theta(t)) u(t))dt \\ & + A_1(\theta(t)) x d\eta + B_1(\theta(t)) f(y) d\xi \end{aligned} \quad (6)$$

where the system measured output is

$$z = L(\theta(t))x + N(\theta(t))u \quad (7)$$

and where

$$y = C(\theta(t))x \quad (8)$$

Note that (6) was obtained from (4) by replacing Adt by $A_0dt + A_1d\xi$, Bdt by $B_0dt + B_1gd\xi$ and $f(x) = g(x + v_0) - g(v_0)$. The control input $u(t)$ as introduced to provide a stochastic version of (Poznyak and Sanchez, 1999) allowing the considered Hopfield network to serve also a chaotic system identifier. We note that (Poznyak and Sanchez, 1999) the control signal is $u = \phi(r)u$ rather than just u where $\phi(r)$ is a diagonal matrix having $f_i(r_i)$ on its diagonal, where $r = H(\theta(t))x$. We have taken for simplicity $\phi = I$ which is also motivated by our example in Section IV.

Note also that the matrices $A_0(\theta(t)), A_1(\theta(t)), B_0(\theta(t)), B_1(\theta(t)), D(\theta(t))$, $C_1(\theta(t)), C_2(\theta(t))$ and $L(\theta(t))$ are piecewise constant matrices of appropriate dimensions whose entries are dependent upon the mode $\theta(t) \in \{1, ..., r\}$ where r is a positive integer denoting the number of possible models between which the Hopfield network parameters can jump. Namely, $A_0(\theta(t))$ attains the values of $A_{0,1}, A_{0,2}, ..., A_{0,r}$, etc. It is assumed that $\theta(t), t \ge 0$ is a right continuous homogeneous Markov chain on $\mathcal{D} = \{1, ..., r\}$ with a probability transition matrix

$$P(t) = e^{Qt}; Q = [q_{ij}]; q_{ii} < 0; \sum_{j=1}^{r} q_{ij} = 0; \quad i = 1, 2, .., r. \quad (9)$$

Given the initial condition $\theta(0) = i$, at each time instant t, the mode may maintain its current state or jump to another mode $i \ne j$. The transitions between the r possible states, $i \in \mathcal{D}$, may be the result of random fluctuations of the actual network components (*i.e.* resistors, capacitors) characteristics or can used to artificially model deliberate jumps which are the result of parameter changes in an optimization problem the network is used to solve. In visuo-motor tasks one may conjecture that proportional and derivative feedbacks are applied on the basis of time sharing, where transition probabilities define the statistics of switching between the two modes. Although there is no evidence for this conjecture, the model analyzed in the present paper can be used to check its stability and L_2 gain.

In the forthcoming analysis, we will assume that the components $f_i, i = 1,...,n$ of $f(\xi)$ are assumed to satisfy the sector conditions

$$0 \leq \zeta_i f_i(\zeta_i) \leq \zeta_i^2 \sigma_i \tag{10}$$

which are equivalent to

$$-F_i(\zeta_i, f_i) := f_i(\zeta_i)(f_i(\zeta_i) - \sigma_i \zeta_i) \leq 0 \tag{11}$$

We shall further assume that

$$\frac{\partial f_i}{\partial \zeta_i} \leq \sigma_i, \quad i = 1,...,n. \tag{12}$$

Although the latter assumption of (12) further restricts the sector–type one class of (11), the applicability of our results remains since it is fulfilled by the usual nonlinearities as saturation, sigmoid, etc., used in the neural networks.

Some additional notations are now in place. We define

$$S = diag\left(\sigma_1, \sigma_2, \quad ... \quad, \sigma_n \right)$$

where σ_i are the nonlinearity gains of (11).

As mentioned above we shall analyze passivity (in stochastic sense) conditions for the systems (6)-(8b) which is expressed as:

$$J = E\left\{ \int_0^\infty \left(z^T(t) u(t) \right) dt \right\} > 0, \; x(0) = 0. \tag{13}$$

3 PASSIVITY ANALYSIS

Introduce the Lyapunov–type function:

$$\begin{array}{c} V(x(t), \theta(t)) = x^T(t) P(\theta(t)) x(t) \\ +2\sum_{k=1}^n \lambda_k \int_0^{C_k x} f_k(s) ds. \end{array} \tag{14}$$

depending on the nonlinearities $f_i(y_i) = f_i(C_i x)$ via the constants λ_i where C_i is the i'th row in C. As it was mentioned in (Boyd et al., 1994), V of (14) defines a parameter-dependent Lypaunov function. To see this, consider the simple case of $f_i(x_i) = x_i \sigma_i$ and get $V(x, \sigma_1, \sigma_2, ..., \sigma_n) = x^T(P + S^{\frac{1}{2}} \Lambda S^{\frac{1}{2}})x$ which depends on the parameters $\sigma_i, i = 1,2,..,n$ and on the constants $\lambda_i, i = 1,2,..,n$ via (18) in the sequel. Applying the Itô–type formula (see (Dragan and Morozan, 1999), (Dragan and Morozan, 2004) and (Fen et al., 1992)) for $V(x,\theta)$ it follows that:

$$\begin{array}{c} E\{V(x, \theta(t)|\theta(0))\} - E\{V(0, \theta(0)|\theta(0))\} = \\ E\left\{ \int_0^t \mathcal{L}V(x, \theta(s)) ds \right\} \end{array}$$

where

$$\begin{array}{l} \mathcal{L}V(x,\theta) := (A_0(\theta)x + B_0(\theta) f(y) + D(\theta) u)^T \frac{\partial V}{\partial x} \\ +x^T A_1^T(\theta) \bar{P} A_1(\theta) x + f^T B_1^T(\theta) \bar{P} B_1(\theta) f \\ +\sum_{j=1}^r q_{ij} x^T P_j x. \end{array} \tag{15}$$

where

$$\bar{P}(\theta, \lambda_1, \lambda_2, ..., \lambda_n) := P(\theta) + diag\left(\lambda_1 \frac{\partial f_1}{\partial x_1}, ..., \lambda_n \frac{\partial f_n}{\partial x_n} \right),$$

with the dependence on its arguments being omitted and where for simplicity we have used the notation $f := f(y(t))$. Then the condition (13) is fulfilled if

$$\mathcal{L}V < 2z^T u \tag{16}$$

which becomes:

$$\begin{array}{l} \left(x^T A_{0i}^T + f^T B_{0i}^T + u^T D_i^T \right) \left(P_i x + C^T \Lambda f \right) \\ + \left(x^T P_i + f^T \Lambda C \right) \left(A_{0i} x + B_{0i} f + D_i u \right) \\ +x^T A_{1i}^T \bar{P}_i A_{1i} x + f^T B_{1i}^T \bar{P}_i B_{1i} f \\ +\sum_{j=1}^r q_{ij} x^T P_j x - u^T L_i x - x^T L_i^T u \\ -u^T (N_i + N_i^T) u < 0, \quad i = 1,...,r, \end{array} \tag{17}$$

where $\bar{P}_i$ denotes $\bar{P}(\theta = i, \lambda_1, \lambda_2, ..., \lambda_n)$ and where

$$\Lambda := diag\left(\lambda_1, \lambda_2, \quad ... \quad, \lambda_n \right). \tag{18}$$

In order to explicitly express (17), the assumption (12) will be used. Indeed, using the inequalities (12) it follows that conditions (17) are satisfied if the following inequalities are satisfied:

$$\begin{array}{l} -F_{i0}(x, f) := \\ x^T \left[A_{0i}^T P_i + P_i A_{0i} + A_{1i}^T \left(P_i + C^T S^{\frac{1}{2}} \Lambda S^{\frac{1}{2}} C \right) A_{1i} \right. \\ \left. +L_i^T L_i + \sum_{j=1}^r q_{ij} P_j \right] x + f^T \left[B_{0i}^T C^T \Lambda + \Lambda C B_{0i} \right. \\ \left. +B_{1i}^T \left(P_i + C^T S^{\frac{1}{2}} \Lambda S^{\frac{1}{2}} C \right) B_{1i} \right] f \\ + f^T \left(B_{0i}^T P_i + \Lambda C A_{0i} \right) x + x^T \left(P_i B_{0i} + A_{0i}^T C^T \Lambda \right) f \\ -u^T (L_i - D_i^T P_i) x - x^T (L_i^T - P_i D_i) u \\ -u^T (N_i + N_i^T) u < 0. \end{array} \tag{19}$$

Using the $\mathcal{S}$-procedure (*e.g.* (Boyd et al., 1994)) one, therefore, obtains that (16) subject to (11) is satisfied if there exist $\tau_i \geq 0, i = 1,2,...,n$ so that

$$F_{i0}(x, f) - \sum_{k=1}^n \tau_k F_k(x, f) \geq 0. \tag{20}$$

Denoting

$$T := diag\left(\ \tau_1, \tau_2, \quad \dots \quad, \tau_n\ \right) \tag{21}$$

and noticing that

$$\begin{aligned} -\sum_{k=1}^{n} \tau_k F_k(x,f) &= \sum_{k=1}^{n} \tau_k f_k^2 - \tau_k \sigma_k f_k y_k \\ &= f^T T f - \frac{1}{2} f^T T C S x \\ &\quad -\frac{1}{2} x^T S C^T T f, \end{aligned}$$

we get from (20) that:

$$\begin{aligned} & x^T \mathcal{Z}_{i11} x + f^T \mathcal{Z}_{i12}^T x + x^T \mathcal{Z}_{i12} f + f^T \mathcal{Z}_{i22} f \\ & -u^T (L_i - D_i^T P_i) x - x^T (L_i^T - P_i D_i) u \\ & -u^T (N_i + N_i^T) u < 0, \ \ i = 1, \dots, r. \end{aligned}$$

where

$$\begin{aligned} \mathcal{Z}_{i11} &:= A_{0i}^T P_i + P_i A_{0i} + A_{1i}^T \hat{P}_i A_{1i} + \sum_{j=1}^{r} q_{ij} P_j \\ \mathcal{Z}_{i12} &:= P_i B_{0i} + A_{0i}^T C^T \Lambda + \frac{1}{2} S C^T T \\ \mathcal{Z}_{i22} &:= B_{0i}^T C^T \Lambda + \Lambda C B_{0i} + B_{1i}^T \hat{P}_i B_{1i} - T \end{aligned} \tag{22}$$

where

$$\hat{P}_i = P_i + C^T S^{\frac{1}{2}} \Lambda S^{\frac{1}{2}} C \tag{23}$$

These conditions are fulfilled if:

$$\begin{bmatrix} \mathcal{Z}_{i11} & \mathcal{Z}_{i12} & P_i D_i - L_i^T \\ \mathcal{Z}_{i12}^T & \mathcal{Z}_{i22} & 0 \\ D_i^T P - L_i & 0 & -(N_i^T + N_i) \end{bmatrix} < 0, \tag{24}$$

$i = 1, \dots, r$, with the unknown variables P_i, Λ and T.

The above developments are concluded in the following result:

Theorem 1. *The system (6)–(7) is stochastically stable and strictly passive if there exist the symmetric matrices* $P_i > 0, i = 1, \dots, r$, *and the diagonal matrices* $\Lambda > 0$ *and* $T > 0$ *satisfying the system of LMIs (24) with the notations (22)-(23).* □

4 SIMPLIFIED ADAPTIVE CONTROL

In this section we show that the system of (6)-(8) should be regulated using a direct adaptive controller of the type:

$$u = -Kz \tag{25}$$

where

$$\dot{K} = zz^T. \tag{26}$$

Since this type of adaptive control is well–known in the deterministic case (see *e.g.* (Kaufman et al., 1998)), we shall just give a sketch of the proof, emphasizing the particularities arising in the stochastic framework (see also (Yaesh and Shaked, 2005)) analyzed in this paper. We first note that the system (6)-(8) is strictly passive when the passivity condition of (24) is satisfied with $N_i = \varepsilon I$ for ε that tends to zero. The latter is satisfied (see e.g. (Yaesh and Shaked, 2005) and (Kaufman et al., 1998)) if there exist the symmetric matrices $P_i > 0, i = 1, 2, \dots, r$ such that

$$\mathcal{Z}_i < 0 \ \text{ and } i = 1, 2, \dots, r, \tag{27}$$

and

$$P_i D_i = L_i^T, i = 1, 2, \dots, r, \tag{28}$$

where $\mathcal{Z}_i = \begin{bmatrix} \mathcal{Z}_{i11} & \mathcal{Z}_{i12} \\ \mathcal{Z}_{i12}^T & \mathcal{Z}_{i22} \end{bmatrix}$. The stochastic closed-loop system obtained from (6), (8) with $u = Kz$ can be written as:

$$\begin{aligned} dx &= [(A_0(\theta(t)) - D(\theta(t)) K_e L(\theta) x) \\ &\quad + B_0(\theta(t)) f(y) + D(\theta(t)) \bar{u}] dt \\ &\quad + A_1(\theta(t)) x d\eta + B_1(\theta(t)) f(y) d\xi \\ z &= L(\theta(t)) x. \end{aligned} \tag{29}$$

where $\bar{u} = -(K - K_e) z$. The above equations hold for any K_e of appropriate dimensions but in the following it will be assumed that K_e is a constant gain for which the system (29) is stochastically passive (some authors call in this case the open-loop system *almost passive*-AP). Note that K_e's existence is needed just for stability analysis and but its value is not utilized in the implementation. In our stochastic context the stochastic stability of this direct adaptive controller (25), (26) (which usually referred to as *simplified adaptive control*-SAC) will be guaranteed by the stochastic version of the AP property. To this end, as in (Kaufman et al., 1998) we will choose the following generalization of the Lyapunov function of (14) to prove the closed-loop stability:

$$\begin{aligned} \mathcal{V}(x(t), K(t), \theta(t)) &= V(x(t), \theta(t)) \\ &+ \ tr(K(t) - K_e)^T (K(t) - K_e) \end{aligned} \tag{30}$$

where tr denotes the trace and V has the expression (14) with $P(i),\ i = 1, \dots, r$ satisfying the conditions of form (27) and (28) written for the passive system (29) relating $\bar{u}$ and z. Then, direct computations show that the infinitesimal generator of $\mathcal{V}$ of the form (30) along the trajectory (29) and subject to the conditions (28) has the expression:

$$\mathcal{L}\mathcal{V}(x(t), K(t), \theta(t) = i) = \bar{x}^T \mathcal{Z}_i \bar{x} + 2tr\left(\bar{K}^T \dot{\bar{K}} - \bar{K}^T z z^T\right) \tag{31}$$

where $\bar{K} := K - K_e$ and $\bar{x} = \begin{bmatrix} x \\ f \end{bmatrix}$. Since the system (29) was assumed passive (i.e. (27) is satisfied with

$A_{0i} - D_i K_e L_i$ replacing A_{0i}) it follows that $\mathcal{L}V < 0$ and then, choosing $\bar{K} = zz^T$ it results that $\mathcal{L}\mathcal{V} < 0$ which proves the stochastic stability of the resulting closed-loop system. □

We next apply this result in a chaos control problem.

5 EXAMPLE - CHAOS CONTROL

Consider a slightly modified version of the third order chaos generator model of (Kwok et al., 2003) described by (6)-(8), where

$$A_0 = \begin{bmatrix} -\varepsilon & 1 & 0 \\ 0 & -\varepsilon & 1 \\ a_1 & a_2 & a_3 \end{bmatrix}, B_0 = D = L^T = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma \end{bmatrix}, B_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$C^T = \begin{bmatrix} \beta \\ 0 \\ 0 \end{bmatrix}, \tag{32}$$

where $a_1 = -2, a_2 = -1.48, a_3 = -1, \sigma = 0.1, \varepsilon = 0.01$ and $\beta = 10$. The nonlinearity is $f(y) = \alpha tanh(y)$ where $\alpha = 1$.

To establish stability we verify (27)-(28) with $K_e = 10^5$ and find using YALMIP (Löfberg, 2004a)-(Löfberg, 2004b) where A_0 is replaced by $A_0 - DK_eL$. Therefore, by the results of Section 4 above, the closed-loop system with the controller (25), (26) is expected to be stochastically stable.

Next we simulate the above system for $500sec$ with an integration step of $0.001sec$ with $u = 0$ for $t \leq 250sec$ and with the SAC controller $u = -Kz$ where $\dot{K} = z^2$ in the rest of the time. The results are given in Fig. 1 - 3 : the phase-plane (i.e. x_1 versus x_2) trajectories are depicted in Fig. 1, the components $x_i, i = 1,2,3$ of the state-vector and the control input are depicted in Fig. 2, and the adaptive gain K is depicted in Fig. 3. It is seen from these figures that the chaotic behavior characterizing the system in open-loop, is replaced by a stable trajectory at $t \geq 250sec$ where the SAC is applied.

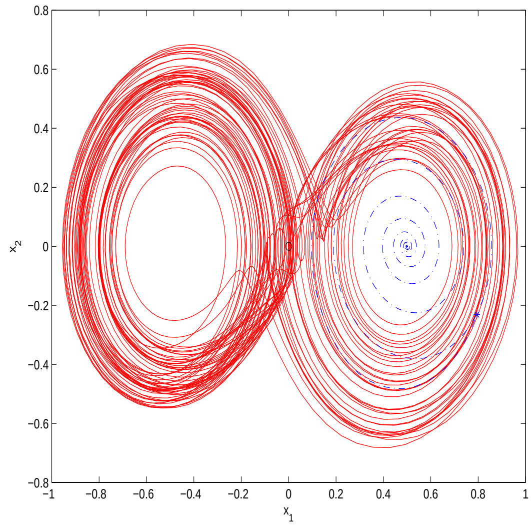

Figure 1: Simulation Results.

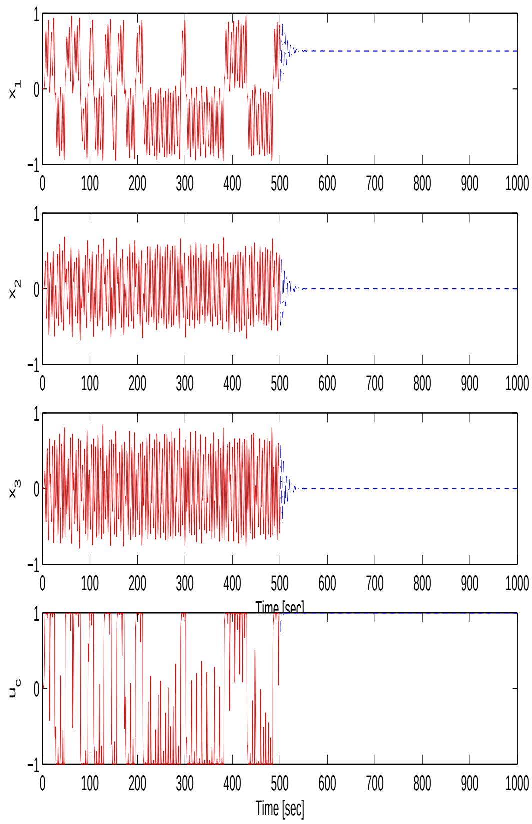

Figure 2: Simulation Results.

6 CONCLUSIONS

A class of stochastic Hopfield networks subject to state-multiplicative noise where the network weights jump according a Markov chain process have been considered. Stochastic passivity conditions for such systems have been derived in terms of Linear Matrix Inequalities. The results have been illustrated via simplified adaptive control of a dynamic system which exhibits a chaotic behavior when its is not controlled.

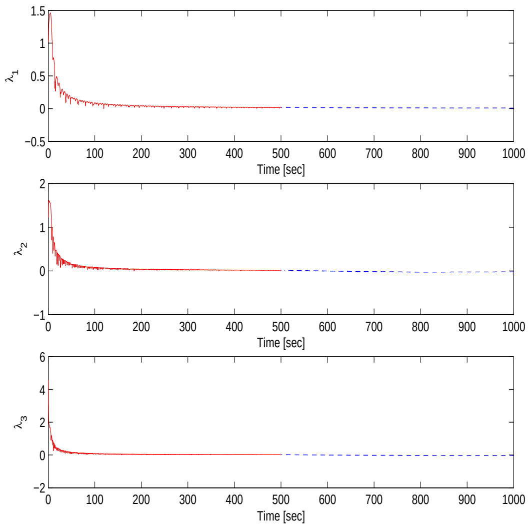

Figure 3: Simulation Results.

The control efficiency in stabilizing the chaotic process has been demonstrated with simulations. The results of this paper should encourage further study of attempts to control chaotic systems with simplified adaptive controllers.

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