DIRECTIONAL CHANGE IN A PRIORI ANTI-WINDUP COMPENSATORS VS. PREDICTION HORIZON

Dariusz Horla
Poznan University of Technology, Institute of Control and Information Engineering
Division of Control and Robotics, ul. Piotrowo 3a, 60-965, Poland
Dariusz.Horla@put.poznan.pl

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Abstract: The paper presents the correspondence in between directional change and anti-windup phenomenon with respect to a priori anti-windup compensator on the basis of MPC (simulation results include plants with not equal number of inputs and outputs). It shows what is the excess of directional change for consecutive predictions of control vectors for a given prediction horizons.

1 INTRODUCTION

Taking control limits into consideration is necessary to achieve high performance of the designed control systems (Horla, 2006b). There are two ways in which one can consider possible constraints at synthesis of controllers. In the first approach, imposing constraints during the design procedure of the controller usually leads to difficulties with obtaining explicit form of control laws, apart from very simple cases. The other way is to assume the system is fully linear and, subsequently, having designed the controller for unconstrained system (by means of optimisation, using Diophantine equations, etc) – impose constraints, what would require additional changes in control system due to presence of constraints (Horla, 2007b; Öhr, 2003; Peng et al., 1998).

The situation when because of constraints internal controller states do not correspond to the actual signals present in the control systems is referred in the literature as windup phenomenon (Öhr, 2003). One can expect inferior performance because of infeasibility of computed (unconstrained) control signals when control limits are not taken into account.

A few methods of compensating the windup phenomenon from SISO framework work well enough in the case of multivariable systems (Öhr, 2003; Walgama and Sternby, 1993). In such a case, apart from the windup phenomenon itself, one can also observe directional change in the control vector due to different implementations of constraints, what could affect direction of the unconstrained control vector (Horla, 2004; Horla, 2007a).

The other problem is, in general form, decoupling, with respect to not equal number of control signals and output signals, when control direction corresponds not only to input principal directions or maximal directional gain of the transfer function matrix, but also to the degree of decoupling (Albertos and Sala, 2004; Maciejowski, 1989).

The problem of directional change has been initially discussed in (Walgama and Sternby, 1993). The paper (Horla, 2007a) defined the connection of directional change problem with anti-windup compensation (AWC) for systems with equal numer of inputs and outputs.

The current paper has been given rise by research carried out in (Horla, 2004; Horla, 2007a; Horla, 2007b) and extends the understanding of anti-windup compensation to non-square systems with imposed constraints, comparing control performance with optimisation-based approach, related to MPC (Camacho and Bordons, 1999; Doná et al., 2000; Maciejowski, 2002) that is widely-spread and applied in the industry. In the paper, the problem of directional change has been studied with respect to optimal a priori anti-windup compensation and different prediction horizons.

2 A PRIORI AWC

One can perform anti-windup compensation by incorporating AWC implicitly into the controller. In order to use all the advantages of such an approach (as optimality of the solution, no need to design decoupling stages, etc.), let the optimal constrained control vector follow from

$$\underline{\underline{u}}_t^\star : J_t(\underline{\underline{u}}_t^\star) = \inf_{\underline{\underline{u}}_t \in \mathscr{D}(J_t)} \left\{ J_t(\underline{\underline{u}}_t) \right\}, \tag{1}$$

with constraints

$$|u_{j,i,t}| \quad \leq \quad \alpha_j\,, \tag{2}$$

where α_j is an amplitude constraint of the i–th element of control vector, and $\underline{u}_{j,t}$, $\mathscr{D}(J_t)$ is the set of all control vectors such that J_t has a finite value,

$$\underline{\underline{u}}_t = \begin{bmatrix} \underline{u}_{1,t} \\ \underline{u}_{2,t} \\ \vdots \\ \underline{u}_{m,t} \end{bmatrix}, \tag{3}$$

and $\underline{u}_{j,t}$ $(1 \leq j \leq m)$ comprises sequences of control actions applied to the j–th input with control horizon N_u (Horla, 2006a).

The controller is responsible for tracking given reference (implicit model) output vector $\underline{r}_{M,t}$ with $\underline{y}_t$ minimising

$$J_t \quad = \quad \sum_{k=1}^{p} \sum_{l=d}^{d+N_u-1} (r_{M,k,t+l} - y_{k,t+l})^2\,, \tag{4}$$

which can be presented in the sense of L_2 norm as

$$J_t \quad = \quad \left\| \underline{\underline{r}}_{M,t+d} - \underline{\underline{y}}_{t+d} - \hat{\underline{\underline{y}}}_{t+d} \right\|_2^2\,, \tag{5}$$

where

$$\underline{\underline{r}}_{M,t+d} = \begin{bmatrix} \underline{r}_{M,1,t+d} \\ \underline{r}_{M,2,t+d} \\ \vdots \\ \underline{r}_{M,p,t+d} \end{bmatrix} \tag{6}$$

comprises vectors including reference signals known for $d + N_u - 1$ steps in advance.

The vector of prediction of plant response forced by the sought control sequence is computed in an iterative manner

$$\hat{\underline{\underline{y}}}_{t+d} \quad = \quad \begin{bmatrix} \hat{\underline{y}}_{1,t+d} \\ \hat{\underline{y}}_{2,t+d} \\ \vdots \\ \hat{\underline{y}}_{p,t+d} \end{bmatrix}, \tag{7}$$

where $\hat{\underline{y}}_{i,t+d} = G\underline{\underline{u}}_t$, and G comprises matrices of plant impulse response samples.

The controller is to search for control vectors $\underline{u}_{j,t}$ with $1 \leq j \leq m$, each of them being control sequences applied to the j–th input $\{u_{j,t}, u_{j,t+1}, \ldots, u_{j,t+N_u-1}\}$ in horizon $N_u > 0$. Based on the superposition rule, the decay-response vector $\hat{\hat{\underline{\underline{y}}}}_{t+d}$ subject to initial conditions $\underline{u}_{t-k}$ $(k \geq 0)$ is computed iteratively alike.

Having expressed (4) as

$$J_t = \left(G\underline{\underline{u}}_t + \hat{\hat{\underline{\underline{y}}}}_{t+d} - \underline{\underline{r}}_{M,t+d}\right)^T \left(G\underline{\underline{u}}_t + \hat{\hat{\underline{\underline{y}}}}_{t+d} - \underline{\underline{r}}_{M,t+d}\right), \tag{8}$$

minimisation subject to constraints is equivalent to (Boyd et al., 1994; Boyd and Vandenberghe, 2004)

$$\begin{aligned} &\min \gamma \\ &\text{s.t.} \\ &\begin{bmatrix} I & \star \\ \underline{\underline{u}}_t^T (G^T G)^{1/2} & \begin{pmatrix} \gamma - (\underline{\underline{r}}_{M,t+d} - \hat{\hat{\underline{\underline{y}}}}_{t+d})^T \times \\ \times (\underline{\underline{r}}_{M,t+d} - \hat{\hat{\underline{\underline{y}}}}_{t+d}) + \\ +2(\underline{\underline{r}}_{M,t+d} - \hat{\hat{\underline{\underline{y}}}}_{t+d})^T G \underline{\underline{u}}_t \end{pmatrix} \end{bmatrix} \geq 0\,, \\ &\operatorname{diag}\left\{F^{(1,1)}, \ldots, F^{(1,m)}\right\} \geq 0 \geq 0\,, \\ &\operatorname{diag}\left\{F^{(2,1)}, \ldots, F^{(2,m)}\right\} \geq 0\,, \end{aligned} \tag{9}$$

where the last to LMIs define upper and lower bounds of $\underline{\underline{u}}_t$, and $\star$ is a symmetrical entry.

3 SIMULATION STUDIES

The following multivariable CARMA plant model will be of interest

$$A(q^{-1})\underline{y}_t = B(q^{-1})\underline{u}_{t-d}\,, \tag{10}$$

with left co-prime polynomial matrices $A(q^{-1})$, $B(q^{-1})$, delay $d = 1$, with $\underline{y}_t \in \mathscr{R}^p$ as the output vector, $\underline{u}_t \in \mathscr{R}^m$ is the constrained control vector ($\underline{v}_t \in \mathscr{R}^m$ will denote unconstrained control vector). The considered plants are assumed to be cross-coupled:

- P1 ($m = 2$, $p = 2$)

$$\begin{aligned} A(q^{-1}) \quad &= \quad I + \begin{bmatrix} 0.8 & -0.1 \\ 0.4 & -1.0 \end{bmatrix} q^{-1} + \\ &\quad + \begin{bmatrix} -0.49 & -0.10 \\ 0.10 & 0.25 \end{bmatrix} q^{-2}\,, \\ B(q^{-1}) \quad &= \quad \begin{bmatrix} 1.0 & 0.3 \\ 0.5 & 0.8 \end{bmatrix}, \end{aligned}$$

- P2 ($m = 3$, $p = 2$)

$$\begin{aligned} A(q^{-1}) \quad &= \quad I + \begin{bmatrix} 0.8 & -0.1 \\ 0.4 & -1.0 \end{bmatrix} q^{-1} + \\ &\quad + \begin{bmatrix} -0.49 & -0.10 \\ 0.10 & 0.25 \end{bmatrix} q^{-2}\,, \\ B(q^{-1}) \quad &= \quad \begin{bmatrix} 1.0 & 0.2 & 0.3 \\ 0.5 & 0.3 & 0.8 \end{bmatrix}, \end{aligned}$$

- P3 ($m = 2$, $p = 3$)

$$A(q^{-1}) = I + \begin{bmatrix} -0.7 & 0.0 & 0.1 \\ -0.1 & -0.8 & 0.2 \\ 0.1 & 0.0 & -0.8 \end{bmatrix} q^{-1} + \\ + \begin{bmatrix} -0.1 & 0.0 & 0.0 \\ 0.0 & 0.1 & 0.0 \\ 0.0 & 0.0 & 0.5 \end{bmatrix} q^{-2},$$

$$B(q^{-1}) = \begin{bmatrix} 1.0 & 0.1 \\ 0.2 & 1.0 \\ 0.5 & -0.1 \end{bmatrix}.$$

The reference vector is pre-filtered by implicit reference model with characteristic polynomial matrix

$$A_M(q^{-1}) = (1 - 0.5q^{-1})I^{p\times p},$$

what corresponds to closed-loop tracking with dynamics described by $A_M(q^{-1})$.

Evaluation of control performance connected with anti-windup compensation quality requires following performance indices to be introduced:

$$J_1 = \frac{1}{N}\sum_{i=1}^{p}\sum_{t=1}^{N}|r_{i,t} - y_{i,t}|, \qquad (11)$$

$$J_2 = \frac{1}{N}\sum_{i=1}^{p}\sum_{t=1}^{N}(r_{i,t} - y_{i,t})^2, \qquad (12)$$

$$\overline{\varphi}_i = \frac{1}{N}\sum_{t=1}^{N}|\varphi(\underline{v}_{t,i}) - \varphi(\underline{u}_{t,i})| \;\; [^\circ], \qquad (13)$$

$$\overline{\varphi}_i^2 = \frac{1}{N}\sum_{t=1}^{N}(\varphi(\underline{v}_{t,i}) - \varphi(\underline{u}_{t,i}))^2, \qquad (14)$$

where (11) corresponds to mean absolute tracking error of p outputs, (13) is a mean absolute direction change in between computed and constrained control vector, and $\varphi(i)$ denotes angle measure of control vector sequence in prediction horizon $N_u = i$.

4 SIMULATION RESULTS

For plants P1 and P2 the reference vectors comprise piecewise constant reference signals, whereas for P3 the third output is to be kept at zero at all times, what is difficult when there is a inferior number of control inputs in comparison with plant outputs.

Numerical results of performed simulations have been presented in Tables 1 and 2. The first set of simulations tested to what excess the directional change phenomenon will take place for plants P1–P3 and different prediction horizon.

As it can be seen from Table 1a and Figures 1 and 4, for P1, the greatest directional change (with respect to unconstrained control vector generated at the same time instant, but not applied) takes place in the current sample. The greater the prediction horizon, the smaller the directional change becomes. Since a mean angle deviations is approx. 1° then, one can say that constrained control vector is close to the computed unconstrained control vector. This might also take place because of equal number of inputs and outputs, what leads to easier decoupling.

In the case of P2 (Tab. 1b, Fig. 2, 5), mean angle deviation is near the right angle, what corresponds to to normal vectors with the third component unchanged, i.e. rotation with respect to a fixed axis. This might be be connected with plant principal directions and with the need to decouple outputs from inputs. Since the number of control inputs is greater than plant outputs, the excessive change in direction is needed, because one can obtain better tracking performance than for $m = p = 2$.

If the plant has insufficient number of control inputs (P3, Tab. 2b, Fig. 3, 6), it is impossible to assure high control performance and one has to cope with potential problem of uncontrollable modes. As it can be seen, the speed of transients has been reduced, what lead to better decoupling, aiding anti-windup compensation. In such a case, often directional change is a result of the need of decoupling.

For the case of no directional change requirement (Tab. 2), such a regime of work (present in some applications in robotics, or e.g. in tracking, (Öhr, 2003)), results in inferior control performance. For P1 and increasing N_u one obtains performance degradation, for P2 the closed-loop system becomes unstable (in order to decouple, the controller would have to alter control direction) the only improvement can be observed in the case of P3 because of $m < p$ (where some coupling is always present and results in proportions between control vector components that controller has to abide to).

5 SUMMARY

As it has been shown in the paper, the problem of directional change can be presented in a different way for plants with $m \neq p$ than in (Horla, 2007a; Walgama and Sternby, 1993). Not allowing directional change, may cause instability in the case of unstable plants (see P2), whereas for the other cases it degrades control performance.

Altering control direction is related to decoupling, thus one can expects problems with performance for $m > p$ and good control quality for $m < p$ when components of control vector must be kept in proportion (e.g., in a circular shape cutting task) at all times.

Table 1: a) $p = 2$, $m = 2$, b) $p = 2$, $m = 3$, c) $p = 3$, $m = 2$.

a)	$N_u = 1$	$N_u = 2$	$N_u = 3$	$N_u = 4$	$N_u = 5$
J_1	0.6354	0.6144	0.6034	0.5951	0.5911
J_2	1.9002	1.7126	1.6219	1.5683	1.5338
$\overline{\varphi}_1$	0.7171	0.8604	1.4556	1.5468	1.6631
$\overline{\varphi}_2$		0.7736	0.9795	1.1555	1.3119
$\overline{\varphi}_3$			0.7398	1.0107	1.1649
$\overline{\varphi}_4$				0.7299	0.9942
$\overline{\varphi}_5$					0.7207
$\overline{\varphi}_1^2$	11.8327	9.5201	111.7057	115.1060	124.3589
$\overline{\varphi}_2^2$		10.2881	13.1625	19.5663	25.4658
$\overline{\varphi}_3^2$			10.6466	15.1063	20.8268
$\overline{\varphi}_4^2$				10.6025	14.9204
$\overline{\varphi}_5^2$					10.5455

b)	$N_u = 1$	$N_u = 2$	$N_u = 3$	$N_u = 4$	$N_u = 5$
J_1	0.3535	0.3518	0.3560	0.3581	0.3585
J_2	0.7373	0.6985	0.6914	0.6923	0.6931
$\overline{\varphi}_1$	95.1171	89.5872	89.5007	96.3644	93.5828
$\overline{\varphi}_2$		88.0821	90.1826	88.2265	86.9010
$\overline{\varphi}_3$			88.9365	82.4423	91.6918
$\overline{\varphi}_4$				79.6011	87.4887
$\overline{\varphi}_5$					102.5099
$\overline{\varphi}_1^2$	9138.6	8218.2	8306.7	10250.0	9549.4
$\overline{\varphi}_2^2$		8341.1	8477.1	8938.1	8517.1
$\overline{\varphi}_3^2$			8429.2	7331.5	9513.0
$\overline{\varphi}_4^2$				7492.2	8101.1
$\overline{\varphi}_5^2$					11619.6

c)	$N_u = 1$	$N_u = 2$	$N_u = 3$	$N_u = 4$	$N_u = 5$
J_1	1.3293	1.2364	1.1721	1.1422	1.1407
J_2	1.8375	1.3703	1.2450	1.1629	1.1177
$\overline{\varphi}_1$	2.5244	3.6052	3.7189	3.8635	5.8012
$\overline{\varphi}_2$		1.8646	3.0450	3.2257	3.2991
$\overline{\varphi}_3$			2.1306	3.4264	3.6449
$\overline{\varphi}_4$				2.0898	3.4420
$\overline{\varphi}_5$					2.2384
$\overline{\varphi}_1^2$	37.0812	109.3184	95.4019	106.2623	687.9498
$\overline{\varphi}_2^2$		33.4335	116.5809	121.1664	106.5284
$\overline{\varphi}_3^2$			49.0232	135.6229	141.8349
$\overline{\varphi}_4^2$				46.1047	131.0049
$\overline{\varphi}_5^2$					47.5444

Table 2: no directional change, a) $p = 2$, $m = 2$, b) $p = 2$, $m = 3$, c) $p = 3$, $m = 2$ ($-$ denotes unstable closed-loop system).

a)	$N_u = 1$	$N_u = 2$	$N_u = 3$	$N_u = 4$	$N_u = 5$
J_1	0.8846	0.8994	0.8674	0.8914	0.8975
J_2	2.4978	2.6290	2.3793	2.3365	2.2729

b)	$N_u = 1$	$N_u = 2$	$N_u = 3$	$N_u = 4$	$N_u = 5$
J_1	9.5249	11.1171	10.6376	–	–
J_2	36.8874	77.8512	62.5581	–	–

c)	$N_u = 1$	$N_u = 2$	$N_u = 3$	$N_u = 4$	$N_u = 5$
J_1	1.4536	1.4110	1.3620	1.3450	1.3510
J_2	1.9418	1.6632	1.5240	1.4352	1.4242

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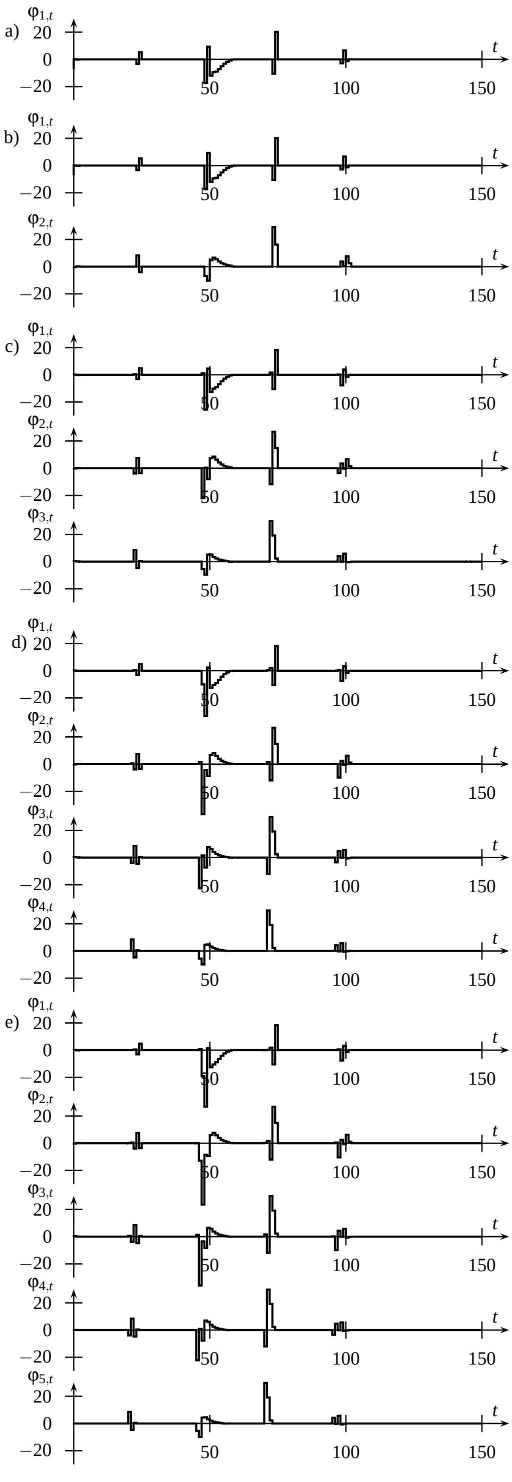

Figure 1: $p = 2$, $m = 2$, a) $N_u = 1$, b) $N_u = 2$, c) $N_u = 3$, d) $N_u = 4$, e) $N_u = 5$.

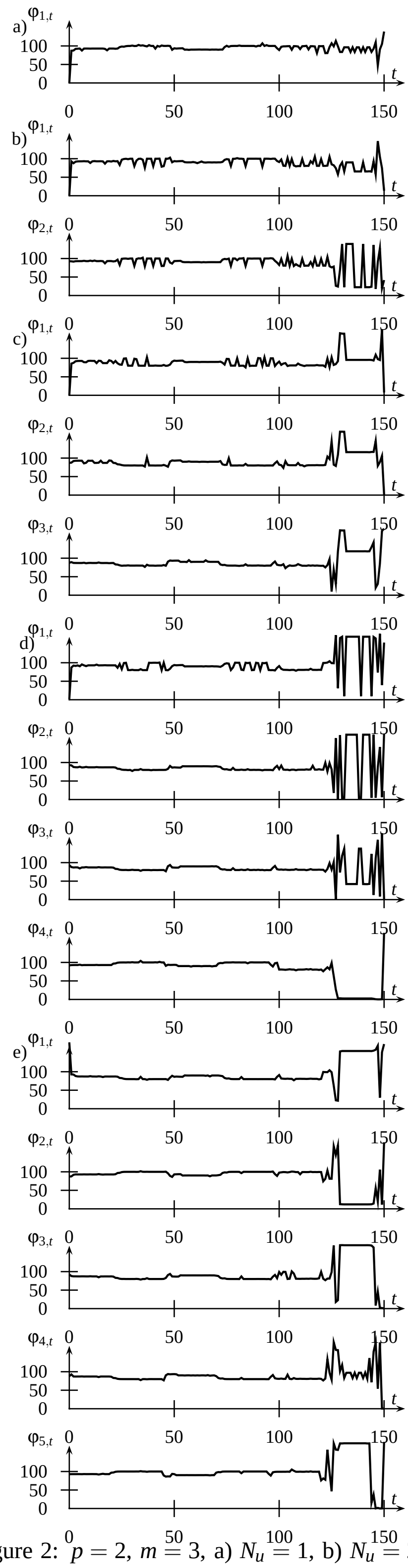

Figure 2: $p = 2$, $m = 3$, a) $N_u = 1$, b) $N_u = 2$, c) $N_u = 3$, d) $N_u = 4$, e) $N_u = 5$.

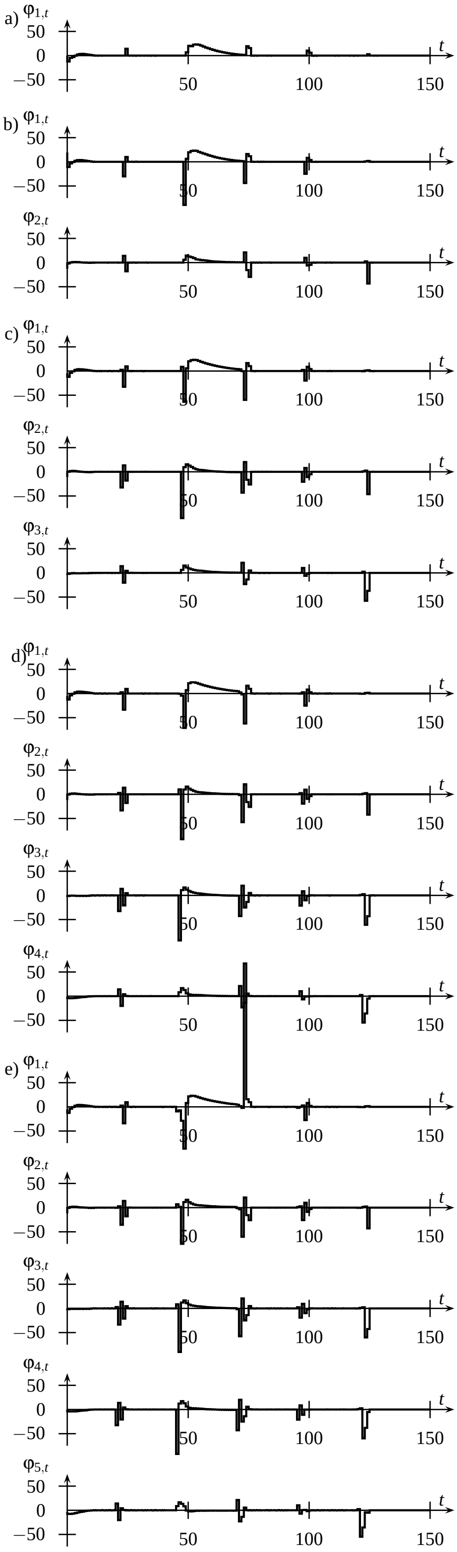

Figure 3: $p = 3$, $m = 2$, a) $N_u = 1$, b) $N_u = 2$, c) $N_u = 3$, d) $N_u = 4$, e) $N_u = 5$.

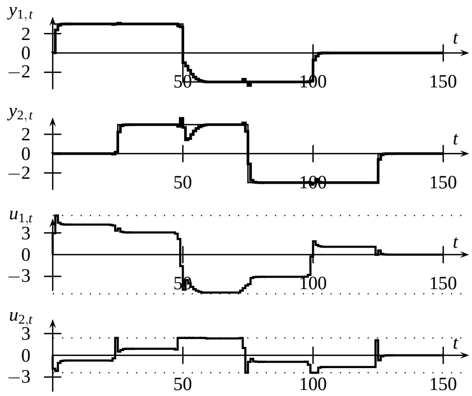

Figure 4: $p = 2$, $m = 2$, $N_u = 3$.

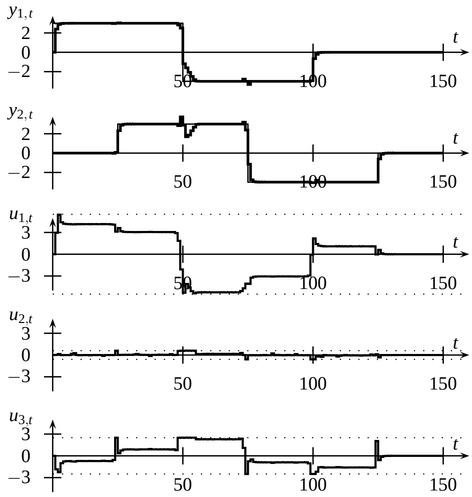

Figure 5: $p = 2$, $m = 3$.

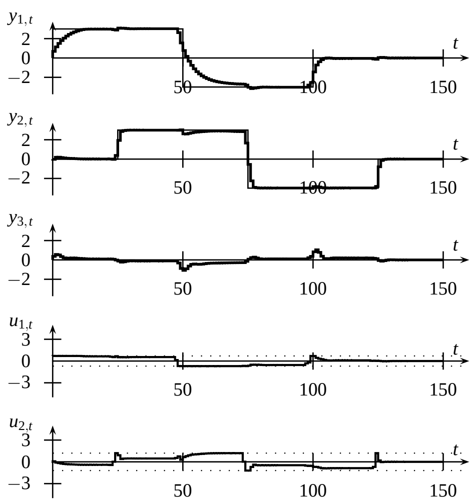

Figure 6: $p = 3$, $m = 2$.