EXPERIMENTAL OPEN-LOOP AND CLOSED-LOOP IDENTIFICATION OF A MULTI-MASS ELECTROMECHANICAL SERVO SYSTEM

Usama Abou-Zayed, Mahmoud Ashry and Tim Breikin
Control Systems Centre, The University of Manchester, PO BOX 88, M60 1QD, U.K.
usama.abou-zayed@postgrad.manchester.ac.uk

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Abstract: The procedure of system identification of multi-mass servo system using different methods is described in this paper. Different black-box models are identified. Previous experimental results show that a model consisting of three-masses connected by springs and dampers gives an acceptable description of the dynamics of the servo system. However, this work shows that a lower order black-box model, identified using off-line or on-line experiments, gives better fit. The purpose of this contribution is to present experimental identification of a multi-mass servo system using different algorithms.

1 INTRODUCTION

An important step in designing a control system is proper modeling of the system to be controlled. An exact system model should produce output responses similar to those of the actual system. The complexity of most physical systems makes the development of exact models infeasible. Therefore, in order to design a controller that is reliable and easy to understand in practice, simplified system models should be obtained around operating points and/or model order reduction (Ziaei, 2000).

System identification is an established modeling tool in engineering and numerous successful applications have been reported. The theory is well developed (Ljung, 1999; Soderstrom, 1989), and there are powerful software tools available, e.g., the System Identification Toolbox (SIT) (Ljung, 1997).

Different physical models of electromechanical servo systems based on multi-mass representation were discussed in (Abou-Zayed, 2008). Using grey-box off-line identification, inertial parameters and parameters describing flexibilities were identified. The physical parameters estimates showed no variations in the mechanical parameters, and acceptable variations in the electrical parameters. Experimental results in (Abou-Zayed, 2008) show that a model consisting of three masses connected by springs and dampers gives an acceptable description of the dynamics of the servo system. However, this model is a six-order state-space model.

The objective of this paper is to present our recent experimental studies on black-box open-loop and closed-loop identification of a three-mass electromechanical system. The closed-loop tests are performed using a local-optimal controller.

The paper is organized as follows. In section 2, the servo system is described briefly. In Section 3, the results of black-box off-line identification are presented. On-line open-loop and closed-loop identification of the studied system is discussed in section 4. Finally, Section 5 contains some conclusions.

2 EXPERIMENTAL SETUP

A view from the experimental setup is shown in Fig.1. The DC servo mechanism setup to be studied operates at ±10V input voltage with a permissible output motor shaft speed of 2200 r.p.m. The shaft is connected to an inertial load through a coupling gear with ratio (r=1/30). The load shaft carries an absolute position sensor with linear range ±10V. A personal computer PC (Pentium III, 700 MHz, 256 MB RAM), running the MATLAB software, is
connected to the servo system setup through a data acquisition card. This PC is used as a signal generator for the servo system input. It also used as a data logger to store the relevant system parameters at fixed sample time. The third function is a digital controller for closed-loop identification purposes.

The DC servo system setup, shown in Fig.1 can be viewed as single input single output (SISO) for the present case, where the motor armature voltage \( v_a \) is the input, while the output is the angular position of the load \( \theta_L \). Since the measurement noise is fairly small, a reasonable estimate of the load angular speed \( \omega_L \) is obtained for the identification purpose. Therefore, the load angular speed will be used as the output signal.

3 OFF-LINE IDENTIFICATION OF SERVO SYSTEM

This section presents the results of the study and realization of the off-line identification of servo systems for different types of models with different excitations. First, some dynamical properties of the system are obtained using the process reaction curve method. Then, black-box models, describing the system, are identified.

3.1 Process Reaction Curve Identification

It is one of the widely used approaches to predetermine the dynamic behaviour of a system before performing the data collection for system identification. An input step signal change is applied to the system, and the output response is measured. Rise time, settling time, bandwidth, time constant, time delay, and type of response can be determined using the Process reaction curve (Ziegler, 1942).

System step response is shown in Fig.2. The sample time chosen for this step test is 0.01sec to observe the system dynamical behaviour. An 8V input voltage (dashed line) is applied to the system. The output response (solid line) acts like a first order plus time delay system with average steady state output 7.18V, rise time about 1.4sec, and bandwidth around 1.6rad/s. Using the process reaction curve method the system can be modeled as in the classical case:

\[
g(s) = \frac{Ke^{-Tds}}{\tau s + 1}
\]

where \( K \) is the steady state gain (\( K = 0.898 \)), \( T_d \) is the time delay (\( T_d = 10 \) ms), and \( \tau \) is the time constant (\( \tau = 0.73 \) sec).

3.2 Experiment Design

The results of the identification experiments reported here are based on two data sets where the excitation signal has different character:

Set 1: A sum of 16 sinusoids with amplitude 1.8 and equidistant spacing in frequency, between 0.1 and 6.1 rad/sec. The resulting crest factor (Ljung, 1999) is 1.8 due to the Schroeder phase choice (Schroeder, 1970). The time response and power spectrum of the set are shown in Fig.3.

Set 2: A linear swept-frequency sinusoidal signal with amplitude 9 and time-varying frequency over a certain band ranges from 0.1 to 6.1rad/sec over a certain time period 100sec. The resulting input signal has a crest factor 1.42, and shown in Fig.4.
3.3 Black-box Transfer Function Model Identification

The starting point is the general linear model structure (Ljung, 1999),
\[ y(t) = G(q, \theta)u(t) + H(q, \theta)e(t) \]  

where \( q \) denotes the shift operator.

Two different model structures will be studied, and these are the ARX structure, defined by:
\[ G(q, \theta) = B(q, \theta)A(q, \theta), \]
\[ H(q, \theta) = \frac{1}{A(q, \theta)} \]  

and the OE structure, where:
\[ G(q, \theta) = B(q, \theta)/F(q, \theta), \]
\[ H(q, \theta) = 1 \]

For the two model structures mentioned above, the estimation of the model parameters will be carried out generally using prediction error method (PEM). The identification experiments are carried out using the SIT (Ljung, 1997).

Tables 1, and 2 show the results of the estimated models using data set1 for ARX and OE model structures respectively. Both data sets show nearly similar estimates. The notation (ModelStructure pzid) denotes the p\textsuperscript{th} order model with ‘z’ zeroes and delay ‘d’. The comparison is carried out using two different quantities. The first is MSE as:
\[ MSE = \frac{1}{N} \sum_{t=1}^{N} (y(t) - \hat{y}(t))^2 \]  

The second is the FIT:
\[ FIT = \left(1 - \left[ \frac{\sum_{t=1}^{N} (y(t) - \hat{y}(t))^2}{\sum_{t=1}^{N} (y(t) - \bar{y})^2} \right] \right) \times 100\% \]

\( \bar{y} \) is the mean value of the measured output.

Using k-step ahead predictors \( \hat{y}_k = \hat{y}(t - k; \theta) \). The two extreme predictors is defined as:
\[ \hat{y}_1(t) = H^{-1}(q)G(q)u(t) \]
\[ \hat{y}_\infty(t) = G(q)u(t) \]

Table 1: Comparison of black-box ARX models.

<table>
<thead>
<tr>
<th>Model</th>
<th>MSE×10(^{-3})</th>
<th>fit (cross validation ) %</th>
</tr>
</thead>
<tbody>
<tr>
<td>k=1</td>
<td>k=\infty</td>
<td></td>
</tr>
<tr>
<td>ARX 211</td>
<td>5.95</td>
<td>96.01</td>
</tr>
<tr>
<td>ARX 311</td>
<td>2.29</td>
<td>97.72</td>
</tr>
<tr>
<td>ARX 411</td>
<td>0.76</td>
<td>98.44</td>
</tr>
<tr>
<td>ARX 511</td>
<td>0.58</td>
<td>98.76</td>
</tr>
<tr>
<td>ARX 611</td>
<td>0.35</td>
<td>99.03</td>
</tr>
</tbody>
</table>

Table 2: Comparison of black-box OE models.

<table>
<thead>
<tr>
<th>Model</th>
<th>MSE×10(^{-3})</th>
<th>fit %</th>
</tr>
</thead>
<tbody>
<tr>
<td>OE 211</td>
<td>39.90</td>
<td>85.53</td>
</tr>
<tr>
<td>OE 311</td>
<td>38.20</td>
<td>84.99</td>
</tr>
<tr>
<td>OE 321</td>
<td>38.30</td>
<td>84.98</td>
</tr>
<tr>
<td>OE 421</td>
<td>37.60</td>
<td>85.66</td>
</tr>
<tr>
<td>OE 611</td>
<td>37.70</td>
<td>85.72</td>
</tr>
<tr>
<td>OE 621</td>
<td>34.70</td>
<td>86.17</td>
</tr>
</tbody>
</table>

It is clear that for OE models, there is no difference between both predictors. Otherwise, there is considerable difference between them. The one-step ahead predictor can give fits that “look good,” even though the model may be bad. Therefore, the simulation fit can be used for invalidating the bad models.

Figure 3: Multi-sine signal time response and power spectrum.

Figure 4: Chirp sine signal time response and power spectrum.
4 ON-LINE IDENTIFICATION OF SERVO SYSTEM

4.1 Open-loop System Identification

Experiments are performed to find the discrete-time model that can best represent the system using RLS method. Let the system model is given in the form:

\[ A(z^{-1})y(t) = B(z^{-1})u(t-1) \]  

(9)

where \( z^{-1} \) is the back shift operator, and

\[ A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_{n_a} z^{-n_a} \]

\[ B(z^{-1}) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_{n_b} z^{-n_b} \]  

(10)

A model of the system in (9) can be presented in the form of

\[ y(t) = \varphi^T(t)\theta \]  

(11)

where \( \theta \) is a vector of unknown parameters defined by:

\[ \theta = [a_1, \ldots, a_{n_a}, b_0, \ldots, b_{n_b}]^T \]  

(12)

and \( \varphi \) is a vector of regression which consists of measured values of inputs and outputs

\[ \varphi^T(t) = [-y(t-1), \ldots, -y(t-n_a), u(t-1), \ldots, u(t-n_b-1)] \]  

(13)

The model given in (11) presents an accurate description of the system. However, in this expression the vector of system parameters \( \theta \) is unknown. It is important to determine it by using available data in signal samples at system output and input. For that purpose a model of the system is supposed

\[ \hat{y}(t) = \varphi^T(t) \hat{\theta}(t-1) \]  

(14)

For the RLS algorithm to be able to update the parameters at each sample time, it is necessary to define an error. The model prediction error, \( e(t) \) is a key variable in RLS algorithm and is defined as

\[ e(t) = y(t) - \hat{y}(t) = y(t) - \varphi^T(t) \hat{\theta}(t-1) \]  

(15)

The error \( e(t) \) is the difference between the system output and the estimated model output. This model prediction error is used to update the parameter estimates as

\[ \hat{\theta}(t) = \hat{\theta}(t-1) + P(t)\varphi(t)e(t) \]  

(16)

where the estimator covariance matrix \( P(t) \) is updated using

\[ P(t) = \frac{1}{2} P(t-1) \left[ I_p + \varphi(t)\varphi^T(t)P(t-1) \right] \frac{1}{\lambda + \varphi(t)\varphi^T(t)P(t-1)\varphi(t)} \]  

(17)

where the subscript ‘\( p \)’ is the dimension of the identity matrix, \( p = n_a + n_b + 1 \), \( \lambda \) is the forgetting factor, 0 < \( \lambda \) < 1. The property of the forgetting factor, \( \lambda \), is that \( \lambda \) controls the speed of parameter convergence: \( \lambda = 1 \) yields the slowest speed, but provides the best robustness towards noise, and decreasing values of \( \lambda \) result in increasing speed of parameter convergence. In general, choosing 0.98 < \( \lambda \) < 0.995 gives a good balance between convergence speed and noise susceptibility (Alexander, 2001).

Application of RLS method demands supposition of the initial values of \( P(0) \) and \( \hat{\theta}(0) \). The technique which is chosen as estimates and then allowed to settle to their final values as the program goes through several iterations. There is no unique way to initialize the algorithm. One suggestion is using a supposition that the system is an integrator of the first order with unit gain to set \( \hat{\theta}(0) \). While, a standard choice of \( P(0) \) is the unit matrix scaled by a positive scalar \( \alpha \), (i.e. \( P(0) = \alpha I_p \)), where \( \alpha \) is recommended to be chosen 1 < \( \alpha \) < 100 depending on the existence of prior knowledge about the system parameters (Wellstead, 1991).

A square wave perturbation signal with a frequency of approximately 0.2 of the system bandwidth ensures that most of the square wave power, associated with the first three harmonic components, is inside the system bandwidth. A square wave perturbation signal with a frequency of \( f = 0.05 \) Hz that is superimposed on a step signal was applied to the system input. The RLS algorithm is implemented for experimental tests using SIMULINK and real-time windows target.

<table>
<thead>
<tr>
<th>Order</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSE</td>
<td>0.00617</td>
<td>0.00368</td>
<td>0.00357</td>
<td>0.00355</td>
<td>0.00392</td>
</tr>
</tbody>
</table>

For comparison purposes, Table 3 shows the MSE calculated for different model orders. Third order
model appears to be suitable for describing the system. Further increase in the model order brought no significant improvement.

The performance of the estimated parameters and the model output error for a third-order model are shown in Fig. 5. Estimated parameters converge after a certain time. The speed of parameter convergence depends on the forgetting factor used. Faster parameter convergence can be obtained if the value of the forgetting factor is reduced, but noise amplification. The measured system output and predicted model output is shown in Fig. 6. It can be seen that both output signals are in good agreement.

4.2 Closed-loop System Identification

Closed-loop identification using direct method is considered in this section. Knowledge of the controller or the nature of the feedback is not a certain requirement. A local-optimal controller (Abou-Zayed, 2008) that provides stable closed-loop servo operation is implemented, using SIMULINK and real-time windows target.

The estimated parameters for the third-order model are shown in Fig. 7. The parameters converge faster than open-loop identification. Further, the variations in the estimated parameters are smaller than that obtained from the open-loop identification. This phenomenon is due to the closed-loop feedback control since the local-optimal controller filters high frequency signal components and limits the bandwidth.

Table 4: Estimated parameters of third-order model for open-loop and closed-loop experiments.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Open-loop</th>
<th>Closed-loop</th>
</tr>
</thead>
<tbody>
<tr>
<td>a_1</td>
<td>-1.821</td>
<td>-1.855</td>
</tr>
<tr>
<td></td>
<td>0.204</td>
<td>0.082</td>
</tr>
<tr>
<td>a_2</td>
<td>1.065</td>
<td>0.942</td>
</tr>
<tr>
<td></td>
<td>0.172</td>
<td>0.136</td>
</tr>
<tr>
<td>a_3</td>
<td>-0.122</td>
<td>-0.131</td>
</tr>
<tr>
<td></td>
<td>0.144</td>
<td>0.080</td>
</tr>
<tr>
<td>b_1</td>
<td>0.132</td>
<td>0.125</td>
</tr>
<tr>
<td></td>
<td>0.025</td>
<td>0.008</td>
</tr>
<tr>
<td>b_2</td>
<td>-0.105</td>
<td>-0.113</td>
</tr>
<tr>
<td></td>
<td>0.029</td>
<td>0.009</td>
</tr>
</tbody>
</table>

Table 4 shows the third-order parameters estimates for both open-loop and closed-loop experiments. It shows smaller variations for all parameters estimated using closed-loop experiment. That comes due to the closed-loop local-optimal control which filters high frequency signal components and limits the bandwidth

5 CONCLUSIONS

This paper presents theoretical and experimental identification of a three-mass electromechanical servo system using different algorithms. The aim of
this research is also to highlight some of the more practical implications of plant identification and to describe the well-established algorithm, recursive least squares, used to perform system identification.

On-line open-loop and closed-loop identification of the studied system is discussed. A real-time implementation of the RLS estimator is presented using SIMULINK and real-time windows target. The application of the RLS method is also demonstrated on a real-time experimental set-up such that it is practical and easy to use. A third-order discrete-time linear model is shown to be flexible enough to fit the observations well. It also became apparent that the order of the suitable linear model was lower than the theoretical one. Closed-loop identification gives faster parameters convergence than open-loop identification. Further, the variations in the estimated parameters are smaller than that obtained from the open-loop identification. This phenomenon is due to the closed-loop local-optimal control which filters high frequency signal components and limits the bandwidth.

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**REFERENCES**


