

# Nonlinear Mixture Analysis for Hyperspectral Imagery

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Due to simplicity and effectiveness, linear mixture analysis is widely used. In some cases, nonlinear mixture analysis may improve unmixing accuracy and reduce residual errors. It was demonstrated that a nonlinear model can better explain the effects of multiple scattering in complex vegetated surfaces [1-2]. Neural network-based approaches are the major technique in nonlinear unmixing, but it does not have a clear physical meaning for the mixing structure. Also, training samples are needed.

A simple nonlinear mixture model was proposed in [3-5], where the nonlinear scattering effect between surfaces was approximated by the multiplication between endmembers; then the endmember products were used as new endmembers to participate in the ordinary linear mixture analysis. Let  $\mathbf{M} = [\mathbf{m}_1, \mathbf{m}_2, \dots, \mathbf{m}_p]$  be  $p$  endmembers for an image scene. If only the scatterings between two endmembers are considered, then the new signature matrix  $\mathbf{M}_{NL}$  for nonlinear mixture analysis becomes

$$\mathbf{M}_{NL} = [\mathbf{m}_1, \mathbf{m}_2, \dots, \mathbf{m}_p, \mathbf{m}_1\mathbf{m}_2, \mathbf{m}_1\mathbf{m}_3, \dots, \mathbf{m}_{p-1}\mathbf{m}_p]. \quad (1)$$

Hence, a pixel vector  $\mathbf{r}$  can be expressed as

$$\mathbf{r} = \mathbf{M}_{NL} \boldsymbol{\alpha}_{NL} + \mathbf{n} \quad (2)$$

where  $\mathbf{n}$  is the noise term. Then a least squares approach can be used to determine the abundances in  $\boldsymbol{\alpha}_{NL}$ .

We find out that multiple scatterings (i.e., products from three or more than three endmembers) are generally negligible because the corresponding abundances are very small. In this study, only the products between two endmember signatures are added. When the prior information about endmember types is known, the number of endmember products can be further reduced. For instance, when soil, grass, and tree are present in an image scene, it is obvious that the scattering between soil and tree is much more significant than the one between soil and grass, and the latter may be ignored. In some cases, the self-product of an endmember may need to be included as well. For instance, when tall trees are present, scattering between them occurs and including the self-product can increase the model fitness.

In [6], we investigated a new linear mixture model where the number of endmembers and their types was allowed to be changed from pixel to pixel, which yielded smaller pixel reconstruction error. Fast algorithms were proposed to determine the optimal endmember set from the global endmember set for each pixel. This model was referred to as endmember variable linear mixture model (EVLMM). We will adopt this approach for aforementioned nonlinear mixture analysis. It will automatically determine if nonlinear mixing is actually experienced for a given pixel.

The AVIRIS Lunar Lake used in the experiment is shown in Fig. 1. Six endmember signatures were obtained from the N-FINDR and their abundance maps from the original linear mixture model (LMM) with fully constraints were shown in Fig. 2. As shown in Table I, the

mean-squared-error (MSE) between the original and reconstructed pixels was 54.2645; when the EVLMM was used, MSE was reduced to 40.2811. If the 15 nonlinear endmembers (i.e., the products of the 15 pairs of 6 global endmembers) were added, using the original LMM the MSE was 50.5185; if the EVLMM was adopted, the MSE was further reduced to 39.6940. This preliminary result demonstrates the effectiveness of nonlinear mixture analysis in the improvement of hyperspectral image unmixing accuracy. The full paper will present the approaches to searching for major nonlinear pairs in order to reduce computational complexity, and provide more detailed performance discussion using both real and synthesized datasets.

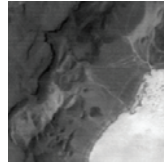


Figure 1. The original AVIRIS Lunar Lake image scene (200×200).

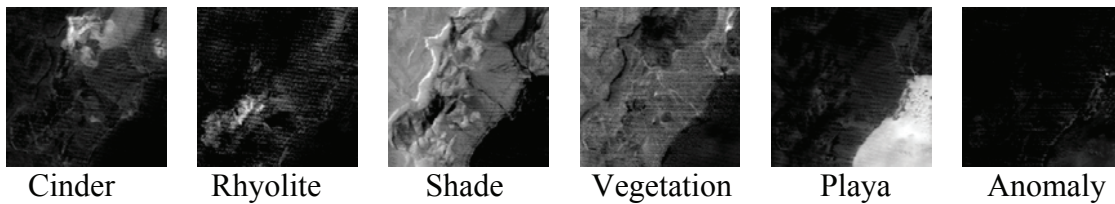


Figure 2. Six endmembers and their abundance maps using LMM.

TABLE I  
MSE USING DIFFERENT APPROACHES FOR THE AVIRIS LUNAR LAKE IMAGE SCENE

	MSE
LMM without nonlinear terms	54.2645
EVLMM without nonlinear terms	40.2811
LMM with nonlinear terms	50.5185
EVLMM with nonlinear terms	39.6940

## References

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