STATISTICAL CHARACTERIZATION OF THE SINCLAIR MATRIX: APPLICATION TO POLARIMETRIC IMAGE SEGMENTATION

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1. INTRODUCTION

Radar polarimetric data have shown their potential for land use estimation. A widely used classification method is the one based on the theoretical Wishart distribution that radar data are *a priori* supposed to verify over natural surfaces. Such assumption is based on Goodman hypothesis, assuming homogeneous areas. However, this hypothesis may be invalid when the resolution cell size is quite small with respect to the wavelength, leading to the presence of texture for example. Such a situation may occur not only for airborne data but can also be expected with new spaceborne sensors such as Radarasat-2 and TerraSAR-X, and their associated metric resolution, or ALOS/PALSAR operating at a larger wavelenth at L band.

This paper focuses on the flexibility of a multidimensional model of probability density function (pdf) to describe distribution of complex data in polarimetric SAR images.

This model is based on Copulas Theory for characterizing the dependence between the polarimetric channels (HH, VV, HV, VH). This corresponds to finding a model based on multidimensional copulas to describe the behavior of the covariance matrix. The advantage in using copulas theory is to extend correlation concept to a wider dependence one, which may be non-linear. So, from this point of view, the model is more flexible than the classical Wishart distribution. This multidimensional characterisation may be linked to pdf which are not necessary issued from a circular Gaussian process. It yields a flexible model, for characterizing statistical behavior of the polarimetric SAR data, that may be derived to produce a segmentation algorithm without requiring a de-speckling pre-processing step.

2. MULTICOMPONENT STATISTICAL MODEL

The Copula theory [1] is used to model the dependence between the components of a multidimensional random variable (RV) $\boldsymbol{X} = (X_1, X_2, \dots, X_n)^t$, t standing for transpose. The model states that any joint cumulative distribution function (cdf) $F_{\boldsymbol{X}}(\boldsymbol{x})$ may be written in the form:

$$F_{\boldsymbol{X}}(\boldsymbol{x}) = C\Big(F_{X_1}(x_1), F_{X_2}(x_2), \dots, F_{X_n}(x_n)\Big) \tag{1}$$

where $F_{X_i}(\cdot)$ stands for the marginal cdf of scalar RV X_i that can be defined by any 1D distribution function. The copula $C(\cdot)$ is a unique nD cdf on $[0,1]^n$ with uniform marginals that models eq. (1), if the marginals $F_{X_i}(\cdot)$ are continuous, according to the Sklar theorem [1]. The main difficulty is to find an appropriated model for such a nD copula. Many techniques exist in the construction of nD copula with $n > 2$. Unfortunately, most of them are not capable to model any kind of dependencies. A general technique have been implemented here by using the general D-vine construction scheme [2, 3]. This construction principle is very flexible and yields a consistent multidimensional copula whatever the combination of the $n(n-1)/2$ parameters. Furthermore, there is no restrictions on the bivariate copulas. Nevertheless, the Ali-Mikhail-Haq copula has been found to be the more representative (unconditional as well as the conditional ones) for radar data [4].

Since the copula model splits the dependence characterization and the marginal distribution, it gives a flexible tool for characterizing each marginal pdf $f_{X_i}(x_i)$ of the multicomponent VA $\boldsymbol{X}$ to be processed. When considering coefficients of the Sinclair matrix [5], components are complex and may be defined from two ways:

1) The components may be defined from their real and imaginary parts. Gaussian [6], Generalized Gaussian [7], or more appropriate distributions may be considered to model those pdf.

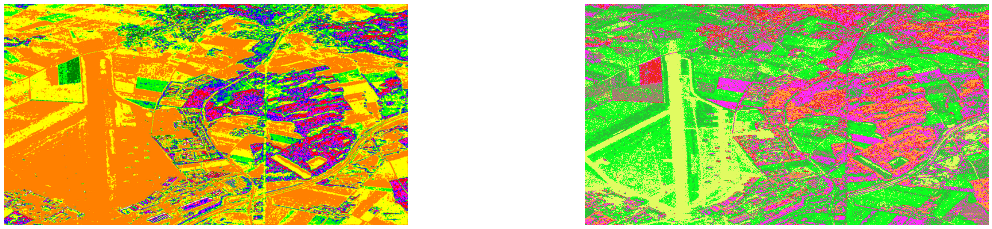

Fig. 1. Segmentation results by using the (H, α) wishart classification, performed by the polSarPro software, on the left; and a randomly initialized copula-based SEM algorithm on the right.

2) The components may be defined from their modulus and phase. Then, many distributions may be found from the literature to describe the amplitude (or better the intensity) of radar scattering. The statistical characterization of the phase did not find a similar interest. In fact, the absolute phase has to be removed to better characterize co-polarized and cross-polarized phase angle distribution [8]. Then, a Von-Mises distribution is used for such a model [9].

3. APPLICATION TO POLARIZATION SEGMENTATION

The scattering matrix can be written in the form:

$$\boldsymbol{\mathcal{S}} = e^{j\phi_{\mathrm{VV}}} \begin{pmatrix} S_{\mathrm{HH}} e^{j\Phi_c} & S_{\mathrm{HV}} e^{j\Phi_x} \\ S_{\mathrm{HV}} e^{j\Phi_x} & S_{\mathrm{VV}} \end{pmatrix}, \tag{2}$$

where $\Phi_x = \Phi_{\mathrm{HV}} - \Phi_{\mathrm{VV}} = \Phi_{\mathrm{VH}} - \Phi_{\mathrm{VV}}$ is the cross-polarized phase angle and $\Phi_c = \Phi_{\mathrm{HH}} - \Phi_{\mathrm{VV}}$ the co-polarized phase angle. S_{ij} stands for the modulus of the complex scattering coefficients $\boldsymbol{S}_{\mathrm{ij}}$ of polarization $\mathrm{ij} =$ HH, VV of HV (assuming HV=VH). Then, the statistical characterization can be written in the form (ignoring the ϕ_{VV} term):

$$F_{\boldsymbol{\mathcal{S}}}(\boldsymbol{\mathcal{S}}) = C\Big(F_{S_{\mathrm{HH}}}(S_{\mathrm{HH}}), F_{S_{\mathrm{VV}}}(S_{\mathrm{VV}}), F_{S_{\mathrm{HV}}}(S_{\mathrm{HV}}), F_{\Phi_c}(\Phi_c), F_{\Phi_x}(\Phi_x)\Big). \tag{3}$$

The Stochastic Estimation Maximization (SEM) segmentation procedure is a stochastic version of the classical EM algorithm where estimation of parameters from given samples (E stage) and estimation of the probability of each class (M stage) are iteratively processed. In this application, the parameters to be estimated come from the ones of the marginal distributions as well as the parameters of the 5D copula. Peliminary results are given on Fig. 1.

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