A DISCRETE INTERFEROMETRIC MODEL FOR A LAYER OF RANDOM MEDIUM

S. S. Seker[1] R. H. Lang[2],
[1] Bogazici University, Department of Electrical and Electronic Engineering, Istanbul, 34342, Turkey, seker@boun.edu.tr
[2] George Washington University, Department of Electrical and Computer Engineering, Washington, DC 20052, USA, lang@gwu.edu

Abstract

Advancement in the field of radar interferometry has brought two additional quantities that contain target information. These quantities are the correlation coefficient and the interferogram phase. The interaction of waves with vegetation particles is essential to interpret these quantities in respect to parameter of interest in the target.

This work describes a discrete interferometric model of a layer of random medium to be used in the application of radar interferometry. The relation between radar interferometry and forest biophysical parameters can be established by the use of models.

The forest is modeled by characterizing tree trunks, branches and leaves with randomly oriented, lossy dielectric particles whose area and orientation in a layer of random medium are prescribed. The model results are compared to the theoretical results in the literature and good agreement are found.. The theory of discrete modeling and some applications are given by [1-3]. In references [2-3] formulates and demonstrates a method for extracting vegetation characteristic and underlying ground surface topography from interferometric synthetic aperture radar (ISAR) data.

Assume the far field from transmitter A, the layer of random medium is as shown in Figure 1.The mean field and Green's function for slab are obtained using the method developed by [1].

Following [1] and employing the distorted Born approximation, we find the scattered field at x due to a scatterer at s and the correlation of the fluctuating component of the scattered field as

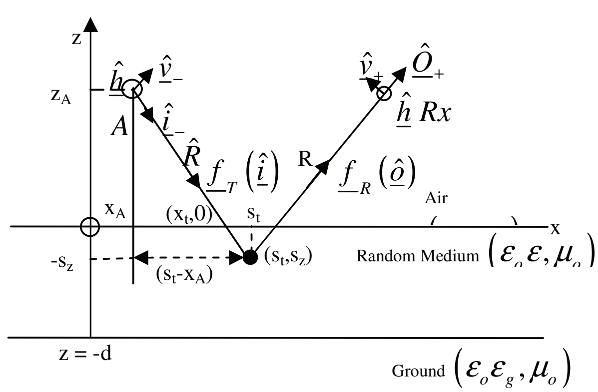

Figure 1: Layer of Random Medium

$$\underline{e}_{se}(\underline{x},\underline{s}) = \underline{e}_{se}^{(1)} + \underline{e}_{se}^{(2)} + \underline{e}_{se}^{(3)} + \underline{e}_{se}^{(4)} \tag{1}$$

$$C(x_1,x_2) = \left\langle \underline{E}(x_1)\underline{E}^*(x_2)\right\rangle = \rho_o \int d\underline{s}\,\underline{e}_{se}(\underline{x}_1,\underline{s})\underline{e}_{se}(\underline{x}_2,\underline{s}) \tag{2}$$

where the density of particles $\rho(s)=\rho_o$ is assumed. Eqs.(1),and(2) have four components. Namely, the direct, reflected-direct,direct-reflected,reflected-reflected fields as in [1].

Using the range resolution function Wr which is given by [2], in eq. (1) then substituted in eq. (2), and introduction of function W_η, which results from the process of synthesizing the aperture, we obtain for narrow-band signal with carrier frequency ω_o,

$$C_\alpha(x_1,x_2)=\sum_{\alpha=1}^{N}\left[C_\alpha^{(1)}+C_\alpha^{(2)}+C_\alpha^{(3)}+C_\alpha^{(4)}\right] \tag{3}$$

where $\alpha=1$ is for trunk, $\alpha_2=$ branch, $\alpha_3=$ leaves, with

$$C_\alpha^{(1)}=\gamma^{(1)}\int\left|W_r(\tau_1)\right|^2\left|W_\eta(\eta)\right|^2 e^{2\left(\operatorname{Im}\Delta K_s^+ + \operatorname{Im}\Delta K_i^-\right)s_z}ds_z \tag{4}$$

$$C_\alpha^{(2)}=\gamma^{(2)}\left|\underline{\Gamma}_{sq}\right|^2\int\left|W_r(\tau_2)\right|^2\left|W_\eta(\eta)\right|^2 e^{2\left(\operatorname{Im}\Delta K_s^+ - \operatorname{Im}\Delta K_i^-\right)s_z}ds_z \tag{5}$$

$$C_\alpha^{(3)}=\gamma^{(3)}\left|\underline{\Gamma}_{sp}\right|^2\int\left|W_r(\tau_3)\right|^2\left|W_\eta(\eta)\right|^2 e^{-2\left(\operatorname{Im}\Delta K_s^+ - \operatorname{Im}\Delta K_i^-\right)s_z}ds_z \tag{6}$$

$$C_\alpha^{(4)}=\gamma^{(4)}\left|\Gamma_{sp}\right|^2\left|\Gamma_{sq}\right|^2\int\left|W_r(\tau_4)\right|^2\left|W_\eta(\eta)\right|^2 e^{-2\left(\operatorname{Im}\Delta K_s^+ - \operatorname{Im}\Delta K_i^-\right)s_z}ds_z \tag{7}$$

where $W_\eta=e^{ik_o(2s_z-z_A)}\dfrac{\sin\left[(n+1)\right]k_o\upsilon T/2}{\sin\left(k_o\,\upsilon T/2\right)}$ $\quad\gamma^{(n)}=\rho_o\dfrac{\left|f_R(\hat{o})\right|^2\left|\underline{f}^{(1)}\cdot q\right|^2\left|f_T(\hat{i})\right|^2}{4\pi z_A^2 z_1 z_2}e^{ik_o(z_1-z_2)}A\quad,\quad n=1,4$

A is the area. , $\underline{f}_T(\hat{i})$ and $\underline{f}_R(\hat{o})$ are the antenna pattern of transmitter and receiver, respectively.

Special Case: If we assume $\theta_s=\theta_i$, then $\operatorname{Im}\overline{\underline{f}}(\underline{o})=\operatorname{Im}\overline{\underline{f}}(\underline{i})=\operatorname{Im}\overline{\underline{f}}$ hence

$$\operatorname{Im}\Delta K_s^+ + \operatorname{Im}K_i^- = \frac{4\pi\rho}{k_o\cos\theta}\operatorname{Im}\overline{\underline{f}}s_z \tag{9}$$

Thus eq. (4) takes the following form

$$C^{(1)}=\rho_o\frac{\left|f_R(\hat{o})\right|^2\left|\underline{f}\left(o^+,i^-\right)\cdot q^o\right|^2\left|f_T(\hat{i})\right|^2}{4\pi\hat{R}^2R_1R_2}\int\left|W_r\right|^2\left|W_\eta\right|^2 e^{ik_o(R_1-R_2)}e^{-\frac{8\pi\rho}{k_o\cos\theta}\operatorname{Im}\overline{f}s_Z}d\underline{s} \tag{10}$$

From ref. [2], we have eq. (22) as

$$\left\langle E(\underline{R}_1)E^*(\underline{R}_2)\right\rangle=\rho_o A^4\left\langle f^2\right\rangle\int\left|W_r\right|^2\left|W_\eta\right|^2 e^{ik(r_1-r_2)}e^{-\frac{8\pi\rho}{k_o\cos\theta}\operatorname{Im}\langle f\rangle s_z}dr_1^3 \tag{11}$$

Comparing eqs. (10) and (11), we conclude that both of them are the same. **Thus this work is the generalization of the work of ref. [2].**

References

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