AN IMPROVED FRACTAL CONSTRUCTION ON 3D DEM TERRAIN PROFILE

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1. INTRODUCTION

The fractal interpolation functions(FIFs) introduced by Barnsley[1] provide us a new scheme to model rough surfaces such as terrain profile. The construction of fractal interpolation surfaces(FISs) over triangular domains is described in literatures [2,3,4], which requires more iterations than rectangular domains. Although FISs over rectangle were constructed successfully in literatures[5,6] , they need much more iterations when the grids are non-uniform. Furthermore, it is very tedious for the users to set many contraction factors in the methods mentioned above. The motivation of this work is to remove these drawbacks by some improved algorithms.

2. THE IMPROVED FRACTAL CONSTRUCTION ALGORITHMS 3D DEM TERRAIN PROFILE

Since the aforementioned algorithms have certain limitations, this paper proposes a new algorithm in which a map is selected according to the probability varied directly with the area of the corresponding subdomain and contractor factors are set automatically according to the change rate of slopes.

A 3D space (K) in this paper refers to a 2D space (F) plus an one-dimensional height (H), i.e. $K = F \times H, I_{nm} = I_n \times J_m (n \in \{1,2,..., N\}, m \in \{1,2,..., M\})$, wherein N and M represent respectively the total original data of axle x and axle y, and continuous Affine function: $\phi_n(x) : I \to I_n$, $\varphi_m(y) : J \to J_m$. FIFs in 3D space can be expressed as follows:

$$W_{nm} = \begin{cases} [(1-(-1)^n)\cdot x_{n-1} + (1-(-1)^{n-1})\cdot x_n]/2 + (-1)^{n-1}\cdot(x_n - x_{n-1})(x - x_0)/(x_N - x_0) \\ [(1-(-1)^m)\cdot y_{m-1} + (1-(-1)^{m-1})\cdot y_m]/2 + (-1)^{m-1}\cdot(y_m - y_{m-1})(y - y_0)/(y_M - y_0) \\ F_{nm}(x,y,z) = g_{nm}(\phi_n(x), \varphi_m(y)) + d_{nm}(z - h(x,y)) \\ n \in \{1,2,...,N\}, m \in \{1,2,...,M\} \end{cases} \quad (1)$$

It is easy to verify that equations (1) satisfy “join-up” conditions[5], which is essential to construct continuous surfaces.

In regular fractal interpolation process, the selection of affine transformation function $W_{ij}(i \in \{1,2,..., N\}, j \in \{1,2,..., M\})$ is stochastic with equal probability , which means there are equal data points generated on each subdomain. Due to the equal probability the interpolated population appears dense somewhere and sparse elsewhere. It becomes a tedious task to generate the required amount of interpolated points to reconstruct the profile. Therefore, we propose a relatively speedy method based on unequal probability to overcome the long time-consuming drawback. We take the area ratio between the subdomain and the total domain as the probability. Define a labelling map $J : \{1,2,...,N\} \times \{1,2,...,M\} \to \{1,2,...,N\cdot M\}$ with $J(i,j) = (i-1)\times M + j$ （2）

Let $\phi:\{1,2,...,N\}\times\{1,2,...,M\}\rightarrow\{1,2,...,N\cdot M\}$ be a 1-1 function. Then the iterative fractal interpolation process becomes:

$$p_{\phi(i,j)}=\sum_{1}^{i}\sum_{1}^{j}area(D_{i,j})/area(D)=\frac{y_{i-1}}{y_M}+\frac{x_i}{x_N}\cdot\frac{(y_i-y_{i-1})}{y_M} \qquad (3)$$

r=rand

if $p_k \le r < p_{k+1}$ then for k=1,2,…,M×N-1

l=k+1

end

$$p''=W_{\phi^{-1}(l)}(p').$$

Wherein p' point in 3D space denotes the last iteration result. This procedure is repeated until the iteration points are generated to meet the user demands.

It is very tedious to set factors respectively for if the number of the interpolation points is great. To solve the problem, we take production between the global contraction factor and the change rate of slopes that have been mapped to interval [0,1]. In this case, d_{nm} in equations(1) will be replaced by $D_g\times S''_{nm}$, wherein D_g denotes the global contraction factor and S''_{nm} represents the change rate of slopes corresponding to every interpolation point .In the improved algorithm, the user need only one time to set global contraction factor other than many parameters in the regular methods, then the continuous profiles with different roughness on various position can be achieved.

3. EXPERIMENTAL RESULTS ANALYSIS

The tested data of this study is 30m×30m DEM terrain with 121×121 points. Some rows and columns are randomly selected from the original data to reconstruct surfaces respectively by means of the regular scheme[5] and the proposed method in this paper. The primary fourth-order moment statistics and iterations are used to analyze and compare the differences between the two methods so as to evaluate the new algorithm. In the same condition, the cost of time is reduced over 50% and the statistics errors are also decreased markedly.

4. CONCLUSION

The experiment results indicate that the improved algorithm is superior to regular methods in both computational efficiency and features preserving.

5. REFERENCES

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