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# ANALYSIS OF NARROW RIDGE ESTIMATION FOR LINEAR CCD RPC MODEL AND ITS APPLICATION

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**Abstract:** Since IKONOS Company applied Rational Polynomial Camera (RPC) model to its high-resolution linear Charge Coupled Device (CCD) Push broom image, RPC model has been researched as a generic image model used for geometric rectification [1-3]. As to computation of the rational polynomial parameters, Hu Y. and Tao V [4-5], put forward a solution independent of terrain based on the least-squares mathematical method. However, matrix created by the parameters always shows morbid and unstable in process of computation. Then, narrow ridge estimation, a algorithm replacing the least-squares matrix  $A^T A$  by  $A^T A + kI$  where k is called the ridge estimator, was introduced to improve matrix nature. Q. Gui studied the variance indicators and proved the existence of a better ridge estimator [6]. G. Zhang [7] demonstrated that the RPC model's stability and accuracy would reach a suitable level for application if the ridge estimator equaled 0.0001. However, the problem about how to smooth the matrix with the ridge estimation has not yet been well resolved.

This paper first theoretically analyzes the process of the ridge estimation algorithm smoothing the morbid matrix. There are two inter-constraint conditions discovered in the process. One condition can be expressed as:

$$\left( \frac{k}{\lambda_{\max} + k} \right) \leq \frac{\|\bar{X} - X\|_2}{\|X\|_2} \leq \left( \frac{k}{\lambda_{\min} + k} \right). \text{ The relative error expression } \frac{\|\bar{X} - X\|_2}{\|X\|_2} \text{ is constrained by the ridge}$$

estimator k and the eigenvalue  $\lambda_{\max}$  and  $\lambda_{\min}$  of matrix. We discovered that the parameter estimation

accuracy would increase as the ridge estimator value decreased in a certain range. The other condition can be

$$\text{expressed as: } \text{cond}(A^T A + kI)_2 = \frac{\|A^T A + kI\|_2}{\|(A^T A + kI)^{-1}\|_2} = \frac{\lambda_{\max} + k}{\lambda_{\min} + k} < \frac{\lambda_{\max}}{\lambda_{\min}}, \lambda_{\max} > 0, \lambda_{\min} > 0, k > 0. \text{ This condition}$$

indicates that the ridge estimation, from view of numerical computing, can improve the stability of the matrix

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(measured by matrix condition number). Further study shows that the condition number of morbid matrix is strictly determined by its eigenvalue  $\lambda_{\max}$  and  $\lambda_{\min}$ . In light of the least eigenvalue which is close to zero, it can be concluded that the greater the ridge estimator, the better the stability. Such two mutual incompatible conditions constitute an interval of the ridge value in which engineering application is feasible.

Under the guidance of the theoretical analysis, different terrain multi-spectrum Spot5 images have been used for experiments. In those experiments error decay curves indicate that an optimal range  $[k_{\min}, k_{\max}]$  does exist for the ridge estimator. If  $k > k_{\max}$ , root of mean square (RMS) errors will decrease whereas if  $k < k_{\min}$ , the ridge estimator will not be improved. But in the interval  $[k_{\min}, k_{\max}]$ , plane RMS error is stable in  $[err_{\min} - 0.0005, err_{\min} + 0.0005]$ . Besides, with comparisons of experimental graphs from different grid densities, it can be concluded that the optimal interval for ridge value  $k$  moves to the direction of greater value along with the increment in grid density.

Based on the theoretical and experimental analysis, this paper finally recommends a ridge interval  $[10^{-10}, 10^{-5}]$  for engineering application.

**Key words:** RPC model, linear CCD, narrow ridge estimation, condition number, eigenvalue.

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