A QUALITATIVE TWO-STEP INVERSION APPROACH FOR THE RECONSTRUCTION OF SUBSURFACE DEFECTS

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1. INTRODUCTION

The detection of subsurface objects such as landmine or archaeological find, the location of sedimentary layer for geological inspections, or the identification of cracks and voids in host structures are some examples of applications where the reconstruction of the position and the shape of unknown targets embedded in inaccessible regions is required. In this framework, the imaging methods based on electromagnetic inverse scattering theory can play a key role [1]. As a matter of fact, the electromagnetic and geometric properties of the region under test can be quantitatively reconstructed starting from the observation of the scattered field. Unfortunately, the problem at hand presents several drawbacks such as non-linearity and ill-posedness that need to be taken into account especially when dealing with complex scenarios. However, a considerable amount of a-priori information is generally available in nondestructive testing and evaluation (NDT/NDE) applications, since the crack to be reconstructed is located in a known host medium [2]. Such a peculiarity can be profitably exploited in order to cope with the lack of information characterizing the inverse problem. In such a framework, microwave methodologies based on the use of heuristic optimizers and on the exploitation of the a-priori information have been effectively used for the detection of a crack in a known host structure [3] or when dealing with more complex and realistic scenarios characterized by multiple defects [4]. The proposed approaches demonstrate their feasibility and effectiveness in providing a coarse estimation of the targets in a qualitative fashion, but they are not suitable for the retrieval of complex shapes. In this work, a two-step procedure successfully adopted for NDT/NDE problems [5] has been employed for the qualitative reconstruction of complex subsurface targets. In particular, at the first step the target is localized and its shape is roughly estimated starting from the knowledge of the scattered field. Then, the second step is aimed at refining the contour of the target by means of a shape optimization technique characterized by the evolution of a level set function [6].

2. MATHEMATICAL FORMULATION

Let us consider a two-dimensional scenario where a homogeneous crack characterized by unknown position $\underline{r}_c$ and shape Ω lies in a cylindrical host region D characterized by known relative permittivity ε_D and conductivity σ_D. The defective host medium is probed by V electromagnetic TM plane waves with an incident field $\underline{E}^v_{inc}(\underline{r}) = E^v_{inc}(\underline{r})\hat{\underline{z}}$ and the total electromagnetic field is given by

$$E^v_{tot}(\underline{r}) = E^v_{inc(cf)}(\underline{r}) + \int\int_\Omega \tau_\Omega(\underline{r}')E^v_{tot(c)}(\underline{r}')G_1(\underline{r}'/\underline{r})d\underline{r}' \quad (1)$$

where $G_1(\underline{r}'/\underline{r})$ is the inhomogeneous Green's function [5] and $E^v_{inc(cf)}(\underline{r})$ is the total electric field of the scenario without defects. Furthermore, the electromagnetic properties of the region Ω are described by the differential object function $\tau_\Omega(\underline{r})$, defined as

$$\tau_{\Omega}(\underline{r}) = \begin{cases} (\varepsilon_C - \varepsilon_D) - j\frac{(\sigma_C - \sigma_D)}{2\pi f \varepsilon_0} & if\ \underline{r} \in \Omega \\ 0 & if\ \underline{r} \notin \Omega \end{cases} \quad (2)$$

where ε_C and σ_C are the known permittivity and conductivity of the crack. In order to properly reconstruct the shape Ω, the proposed strategy is based on the following steps:

1. At the first step, the region of interest (RoI) including the target is estimated by minimizing a suitable cost function by means of a genetic algorithm (GA) [3]. In particular, the RoI is localized and the occupation area is approximated with a rectangular region. The metric of the cost function is based on the values of the scattered field.
2. The second step is aimed at refining the estimation of the shape of the target, starting from the knowledge of the location and of the occupation area acquired at the first step. An initial trial shape is defined inside the RoI and then a level-set-based strategy is employed in order to estimate the shape of the unknown object [5][6].

3. PRELIMINARY NUMERICAL VALIDATION

An example of reconstruction of a complex shape (a double cross with permittivity $\varepsilon_C = 1.0$ and conductivity $\sigma_C = 0.0$) is reported in Fig. 1. As it can be noticed the object has been correctly localized and the occupation area was roughly estimated at the first step (dotted yellow line). Then, thanks to the level set approach, the shape of the object is retrieved at the second step with a satisfactory degree of accuracy.

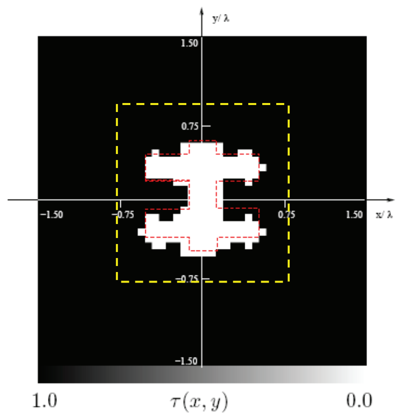

Figure 2 – Reconstruction of a double cross object included in an host medium characterized by $\varepsilon_D = 2.0$ and $\sigma_D = 0.0$ from noisy data. Actual profile (red dashed line). The region of interest defined at the end of the first step is depicted by the yellow dashed line.

4. REFERENCES

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