AN ALGEBRAIC APPROACH TO GROUND-VOLUME DECOMPOSITION FROM MULTI-BASELINE POLINSAR DATA

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1. ABSTRACT

This paper focuses on the analysis of multiple mechanisms of distributed scattering, as occurring in forested and vegetated areas, on the basis of multi-polarimetric and multi-baseline (MPMB) Synthetic Aperture Radar (SAR) data, for various hypotheses on the nature of the illuminated scene. Two basic assumptions will be retained: i) statistical independence of the different scattering mechanisms (i.e.; ground, volume, ground-trunk scattering); ii) independence of volumetric and temporal coherence losses on the choice of the polarimetric channel. Then, the contribution of each scattering mechanism to the covariance matrix of the MPMB data becomes the Kronecker product of two matrices, the first accounting for the polarimetric signature and the second for the coherence losses. In formula:

$$\mathbf{W}_K = \sum_{k=1}^{K} \mathbf{C}_k \otimes \mathbf{R}_k \tag{1}$$

where K is the number of scattering mechanisms that are supposed to contribute to the received signal (typically $K = 2$, representing ground and volume scattering).

Although still unexploited in SAR literature, the Sum of Kronecker Products (SKP) structure offers the possibility to discuss the problem of mechanism separation and characterization from a very general point of view, including not only model based approaches, commonly retained in the analysis of forested scenarios, but also model free, and hybrid approaches, as it will be shown in this paper. In general, mechanism identification is performed by looking for the best approximation of the sample covariance matrix of the MPMB data, $\widehat{\mathbf{W}}$, with K Kronecker products, namely:

$$\widehat{\mathbf{W}}_K = \arg\min \left\{ \left\| \widehat{\mathbf{W}} - \mathbf{W}_K \right\| \right\} \tag{2}$$

In model based approaches $\mathbf{W}_K$ is parameterized basing on explicit electromagnetic models that describe the coherence loss undergone by each scattering mechanism across different baselines [1], [2], and/or the polarimetric behavior associated to each scattering mechanism [3], [4]. Model inversion is then performed by minimizing (2) with respect to the parameterization of $\mathbf{W}_K$, without explicitly exploiting the SKP structure. In the framework of a model based approach, the key point is that the number of observations is sufficient to ensure a non-ambiguous model inversion. If the number of observations is insufficient, then a more simple model has to be adopted. Conversely, a very high number of observations offers the possibility to improve the estimate accuracy and mitigate the impact of model mismatches. Nevertheless, model based approaches are affected by an intrinsic limitation, in that data interpretation can be carried out only on the basis of the model that has been adopted.

As it will be shown in this paper, the explicit exploitation of the SKP structure allows to proceed to mechanism separation even in absence of a physical model of the imaged scene. The key to obtain a model free separation of the scattering mechanisms is the noticeable result, due to *Van Loan and Pitsianis* in [5], that if $\widehat{\mathbf{W}}_K = \sum_{k=1}^{K} \mathbf{C}_k \otimes \mathbf{R}_k$ is the best approximation of the sample covariance matrix with K Kronecker products, then:

$$\mathbf{C}_k = \sum_{n=1}^{K} \left\{ \alpha^{-T} \right\}_{kn} \widetilde{\mathbf{C}}_n; \quad \mathbf{R}_k = \sum_{n=1}^{K} \left\{ \alpha \right\}_{kn} \widetilde{\mathbf{R}}_n \tag{3}$$

where $\left\{ \widetilde{\mathbf{C}}_k \right\}, \left\{ \widetilde{\mathbf{R}}_k \right\}$ are two sets of matrices that can be easily obtained in closed form from the sample covariance matrix, α is a $K \times K$ non singular matrix with real entries, and $\{\alpha\}_{kn}$ denotes the $kn - th$ element of α. Notice that the choice of α does affect only the matrices $\mathbf{C}_k$ and $\mathbf{R}_k$, whereas the sum of the Kronecker Products between $\mathbf{C}_k$ and $\mathbf{R}_k$ results invariant by construction. It follows that the problem of identifying the scattering mechanisms is equivalent to the problem of finding a real valued square matrix $\widehat{\alpha}$ such that:

1. both $\mathbf{C}_k$ and $\mathbf{R}_k$ are non-negative definite for all k: this condition ensures the physical validity of the solutions;
2. some criterion is optimized: this condition allows to select a unique solution among the set of all the physically valid solutions.

As for the choice of the criterion to be optimized, two different approaches are possible. One is to determine a solution $\widehat{\alpha}$ such that the corresponding $\mathbf{C}_k$ and $\mathbf{R}_k$ are as close as possible to some physically based model. This is a hybrid approach, in that SKP structure is exploited jointly with the chosen model. Another choice is to select the solution according to some mathematical criterion, for example by maximizing the diversity among all the matrices $\mathbf{C}_k$ or $\mathbf{R}_k$, thus obtaining a model free separation of the scattering mechanisms. An experiment on real data has been performed basing on a data set of 9 P-Band fully polarimetric SAR images acquired by DLR's E-SAR over

the forested site of Remningstorp, Sweden, in the framework of the ESA experiment BioSAR. In this case it has been observed that the data covariance matrix may be well approximated as the sum of just two Kronecker products ($K = 2$). Mechanism separation has been performed in a model free fashion, by maximizing the diversity between the matrices $\mathbf{R}_1$ and $\mathbf{R}_2$, associated to the first and the second scattering mechanisms. In order to give a physical interpretation of the solution, a MUSIC based tomographic analysis of the matrices $\mathbf{R}_1$ and $\mathbf{R}_2$ has been performed. As a result, both spectra have given rise to very well defined spatial structures, see Fig. (1), bottom left, which have shown a very good accordance to LIDAR based terrain and canopy elevation estimates, provided by the Swedish Defence Research Agency (FOI). It is worth noticing that no clear separation has been observed by evaluating the tomograms relative to the HH and the HV channels, presumably due to the sparsity of the vegetation, see Fig. (1), middle left. Furthermore, the co-polar phase has been observed be about 100° for the first scattering mechanism and about 0° for the second scattering mechanism, see Fig. (1), bottom right. In the case of Remningstorp, the two independent scattering mechanisms identified by the model free SKP can therefore be reasonably associated to double bounce scattering from ground-trunk interactions and volume scattering, thus synthesizing (and validating) that model from the data.

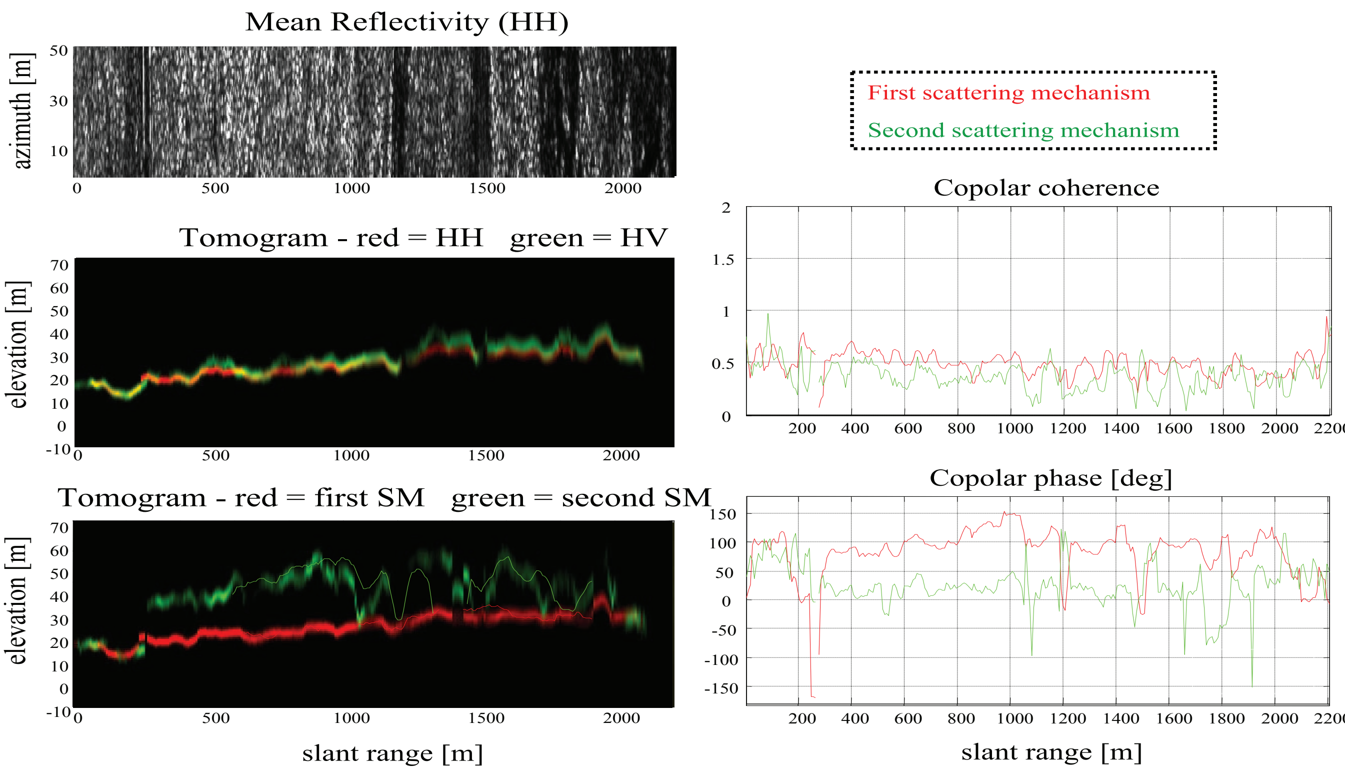

Fig. 1. Model free separation of two scattering mechanisms. Top left: mean reflectivity of the data (HH channel) within a stripe as wide as 50 m in the azimuth direction. Middle left: MUSIC based Tomograms for the HH (red) and the HV (green) channels. Bottom panel: MUSIC based Tomograms for the first (red) and the second (green) scattering mechanism obtained by maximizing the diversity between $\mathbf{R}_1$ and $\mathbf{R}_2$. The red and green lines are relative to terrain and canopy elevation estimates obtained from LIDAR measurements. Top right: HHVV coherence of the two scattering mechanisms. Bottom right: HHVV phase of the two scattering mechanisms.

2. REFERENCES

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