# ADAPTIVE DECOMPOSITION OF POLARIMETRIC SAR COVARIANCE MATRICES

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## 1. INTRODUCTION

One of the most complex problems in polarimetric radar imaging is how to interpret the measured covariance matrix in terms of known scattering mechanisms. Two main approaches are commonly used for this purpose. The first, introduced by Cloude [1] involves calculating the eigenvectors of the measured covariance matrix. This method provides a unique result. However, as shown by van Zyl [2], when these results are interpreted in terms of scattering mechanisms, some approximations have to be made.

A second method, introduced by Freeman and Durden [3] and extended by Yamaguchi *et al.* [4] uses a linear combination of covariance matrices from known scattering models to approximate the observed covariance matrix. This method has the advantage that the results are directly interpreted through the component covariance matrices. For this type of decomposition to be meaningful, one must require that each component covariance matrix indeed represent a physical scattering process. This means that all covariance matrices resulting from the decomposition *must* individually have non-negative eigenvalues. In fact, we have shown previously that the decomposition introduced by Freeman and Durden [3] result in a large percentage of pixels with negative eigenvalues. This fact was recognized by Yamaguchi and his team [4], and they introduced a fourth step in the decomposition to reduce the number of pixels with negative eigenvalues.

## 2. THE PROPOSED DECOMPOSITION FRAMEWORK

Consider a generic decomposition where we want to express the scattering as follows

$$\left\langle [\mathbf{C}] \right\rangle = a[\mathbf{C}_{model}] + [\mathbf{C}_{remainder}] \tag{1}$$

Here, the first term on the right represents the covariance matrix predicted by some model such as randomly oriented branches in the case of vegetated terrain. Recognizing that the form of this covariance matrix may be different from the measured matrix, we add the second term, which will contain whatever is in the measured matrix that is not consistent with the model matrix. The question now is what value of $a$ to use in equation (1). Can we at least provide some boundaries for the range of values for $a$? To answer this question, we need to recognize that all matrices in equation (1) must represent physically realizable covariance matrices. That is, if we look at each matrix by itself, it should satisfy all the restrictions that we expect for a measured covariance matrix. In particular, we need to insist that the eigenvalues for each matrix are real and greater than or equal to zero. Let us rewrite equation (1) in the form

$$[\mathbf{C}_{remainder}] = \left\langle [\mathbf{C}] \right\rangle - a[\mathbf{C}_{model}] \tag{1}$$

The matrix on the left must have eigenvalues that are real and are larger than or equal to zero. This requirement allows us to derive a limit on the values of $a$. The largest value of $a$ that still ensures that all three eigenvalues of the matrix on the left would be greater than or equal to zero, is the maximum value of $a$ that we could use in equation (1). We note that the only unknown on the right-hand side of equation (1) is $a$. Therefore, solving for the eigenvalues of the matrix on the right-hand side gives three equations, leading to three values of $a$ for which each eigenvalue will go negative. We then choose the smallest of these values of $a$, which ensures that one eigenvalue will be zero, while the remaining ones will either be zero or positive.

For vegetated terrain, we perform the decomposition in two steps. First, we assume some model for the canopy. In the case of the Freeman and Durden decomposition, the orientation of the scatterers is assumed to be uniformly random. We then perform the decomposition as described above, picking the largest value of $a$ that still ensures that the remainder matrix has non-negative eigenvalues. The second step in the decomposition uses an eigenvalue decomposition to further separate the remainder matrix into a single and double scattering components plus a remainder term. This follows the same procedure as described by van Zyl [2] when applying the Cloude eigenvalue decomposition. The only difference is that we apply this process only to the matrix that remains after we subtracted the canopy model component from the measure matrix.

## 3. ADAPTIVE DECOMPOSITION

Within the framework described in the previous Section, we can easily extend the decomposition to find the "best" model for the canopy scattering. We assume scatterers that are randomly oriented about a mean angle, with varying amounts of variance in the orientation. If the variance is zero, the canopy is represented by cylinders all with the exact same orientation. If the variance is a maximum, they are uniformly randomly oriented. We then perform the decomposition as described above for all mean orientation angles, and all variances. The decomposition that results in the smallest "remainder" term provides the best fit to the observed data. We perform this adaptive decomposition for all pixels in the SAR image. In our presentation, we will show examples of this decomposition for different vegetation types and compare the results to previous decompositions.

## 4. REFERENCES

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