# COMPARISON OF GAUSSIAN AND RAYLEIGH NOISE MODELS IN INVERSION OF SUBSURFACE PARAMETERS OF LAYERED ROUGH SURFACES USING SIMULATED ANNEALING

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## 1. INTRODUCTION

The problem of determining the subsurface properties of layered rough surface structures—representative models for soil, rivers, and lakes—from scattering data arises in many areas of science and engineering. It has been shown that Simulated Annealing is a powerful tool for inversion of the model parameters of these structures [1]. The sensitivity of the Simulated Annealing method to measurement noise has also been investigated assuming Gaussian noise contaminates the measured scattering coefficient, or measured *scattered power* [1]. While the analysis based on this noise model offers insight into the sensitivity of the inversion algorithm to measured data that deviate from what commonly-used forward models predict, it is more appropriate to consider that Gaussian noise perturbs the measured *scattered field*. This assumption is shown to be equivalent to assuming the scattered power being contaminated by noise with a Rayleigh distribution. The results of the new noise analysis, presented in this work, are expected to be more consistent with inversion results when real data are used.

## 2. NOISE MODELS

We can consider two different noise models in investigating the performance of an inverse scattering problem associated with rough surfaces. One choice is to assume the measured bistatic scattering coefficient is directly contaminated by Gaussian noise as in

$$\gamma_n^o = \gamma_{syn}^o + \mathcal{N}(0, \sigma^2) \tag{1}$$

where $\gamma_n^o$ denotes the measured bistatic scattering coefficient and $\gamma_{syn}^o$ denotes the synthesized noise-free bistatic scattering coefficient. In this work, we use Small Perturbation Method (SPM) to synthesize data [2]. The quantity $\mathcal{N}(0, \sigma^2)$ represents the measurement noise, which is assumed to have a Gaussian distribution with a zero mean and a standard deviation of $\sigma$. We assume that the standard deviation of the measurement noise is directly proportional to the strength of the signal, i.e., $\sigma = \Delta\gamma_{syn}^o$. Since $\mathcal{N}(0, \sigma^2) = \sigma\mathcal{N}(0, 1) = \Delta\gamma_{syn}^o\mathcal{N}(0, 1)$,

$$\gamma_n^o = \gamma_{syn}^o + \Delta\gamma_{syn}^o\mathcal{N}(0, 1) \tag{2}$$

We may use another noise model in which the Gaussian noise contaminates the received electric field. Assume

$$v_n = v + \mathcal{N}(0, \sigma^2) \tag{3}$$

where $v$ denotes the noise free voltage and $v_n$ denotes the measured voltage received at the antenna. Assuming $\sigma = \Delta v$, we have

$$v_n^2 = v^2 + 2v\Delta\mathcal{N}(0, 1) + \Delta^2 v^2\mathcal{N}(0, 1)^2 \tag{4}$$

Averaging (4) over realizations of the rough surface with the assumption that $\mathcal{N}(0, 1)$ and $v$ are independent random variables,

$$\langle v_n^2\rangle = \langle v^2\rangle + \Delta^2\langle v^2\rangle\mathcal{N}(0, 1)^2 \tag{5}$$

Since the measured voltage is proportional to the measured electric field,

$$\gamma_n^o = \gamma_{syn}^o + \Delta^2\gamma_{syn}^o\mathcal{N}(0, 1)^2 \tag{6}$$

where $\mathcal{N}(0, 1)^2$ has a Rayleigh distribution.

## 3. INVERSION AND NOISE ANALYSIS

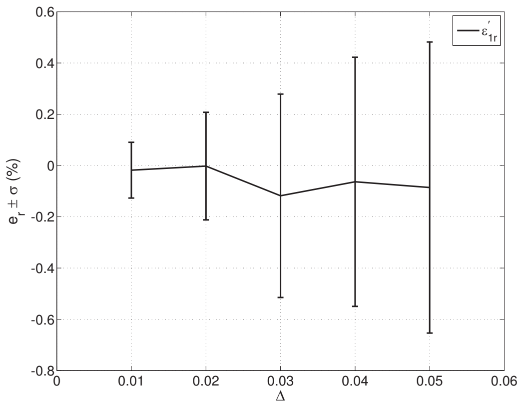
To estimate the model parameters of a two-layer rough surface structure, we minimize

$$L(\mathbf{X}) = \sum_{i=1}^{N_f} \sum_{j=1}^{N_\theta} \sum_{pq} \left( \frac{\gamma_{pq}^o (\mathbf{X}; f_i, \theta_j) - d_{pq}(f_i, \theta_j)}{d_{pq}(f_i, \theta_j)} \right)^2 \quad (7)$$

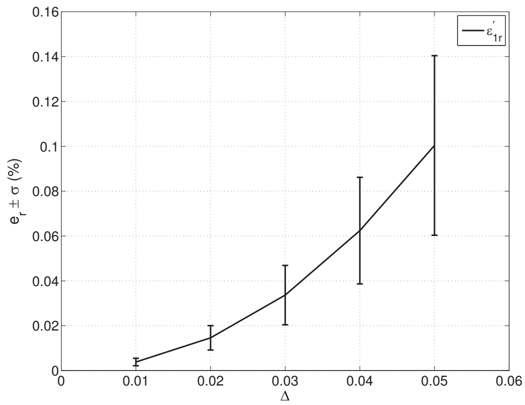
where $\gamma_{pq}^o(\mathbf{X}; f_i, \theta_j)$ and $d_{pq}(f_i, \theta_j)$ are, respectively, the calculated and measured scattering coefficients of the layered structure at a certain frequency and observation angle for $pq$ polarization. The vector $\mathbf{X}$ represents the vector of model parameters—layers permittivities ($\epsilon_{1r}^{'}$ and $\epsilon_{2r}^{'}$), conductivities of the layers ($\sigma_1$ and $\sigma_2$), the standard deviation and correlation length of each interface ($\sigma_{f_i}$ and $l_{f_i}$), and the mean separation between the two rough interfaces ($d$). The values of $N_f$ and $N_\theta$ are, respectively, the number of frequency points and measurement angles used in data collection. For each realization of the noise, the inversion is carried out using the Simulated Annealing method. Using several realizations of the noise, the average and standard deviation of the error in the estimated model parameters are calculated.

## 4. RESULTS

Consider a case where only $\epsilon_{1r}'$ is unknown and where $\Delta$ =0.01, 0.02, 0.03, 0.04, 0.05.[1] Fig. 1 shows, in the form of error bars, the average and standard deviation of the relative output error ($e_r \pm \sigma$) in the estimated $\epsilon_{1r}'$ versus $\Delta$. As expected, as $\Delta$ increases, both the average and standard deviation of the output error increase in the Rayleigh model. In the Gaussian model, however, the average error may not increase when $\Delta$ increases. In this work, we evaluate the sensitivity of Simulated Annealing to measurement noise for different noise strengths when *all of the nine model parameters* are considered unknown. We use Rayleigh model and compare the results with those obtained when Gaussian model is used. As the Rayleigh model is more representative of measurement noise, the new results will be more consistent with those obtained using real data.




(a) Gaussian




(b) Rayleigh

**Fig. 1**. Average and standard deviation of the error in the estimated $\epsilon_{1r}'$ versus $\Delta$ for (a) Gaussian and (b) Rayleigh noise models. Number of noise realizations is 100.

## 5. REFERENCES


[1] A. Tabatabaeenejad and M. Moghaddam, "Inversion of subsurface properties of layered dielectric structures with random slightly-rough interfaces using the method of Simulated Annealing," *IEEE Trans. Geosci. Remote Sens.*, in press.

[2] A. Tabatabaeenejad and M. Moghaddam, "Bistatic scattering from dielectric structures with two rough boundaries using the Small Perturbation Method," *IEEE Trans. Geosci. Remote Sens.*, vol. 44, no. 8, pp. 2102–2124, Aug. 2006.


[1] Noting that $\langle \mathcal{N}(0,1)^2 \rangle = 1$, these noise strengths are, respectively, equivalent to SNR $\sim$ 40, 34, 30, 28, 26 dB in the Rayleigh noise model.