IMPROVING THE RECONSTRUCTION ACCURACY OF INVERSION STRATEGIES THROUGH ENERGETIC CONSTRAINTS

Davide Franceschini, Manuel Benedetti, Federico Viani, and Andrea Massa

ELEDIA Group – Department of Information Engineering and Computer Science,
University of Trento, Via Sommarive 14, I-38050 Trento, Italy.
E-mail: andrea.massa@ing.unitn.it, Web-page: http://www.eledia.ing.unitn.it

1. INTRODUCTION

Microwave imaging techniques are widely investigated in many applied sciences such as the subsurface prospecting [1] and the medical diagnosis [2]. Unfortunately, when inverse scattering approaches are considered, the drawbacks of non-linearity and ill-posedness [3] have unavoidably to be dealt with. Therefore, the problem at hand is usually recast as the minimization of a suitable cost function to be properly minimized in order to find the optimal solution. Such a cost function is usually defined by two discrepancy terms based on the knowledge of the scattered field measured in a suitable observation region and of the incident field in the investigation region. Moreover, further regularization terms have been studied and successfully applied in order to enhance the quality of the reconstructions (e.g., [4][5]). In such a framework, this contribution presents a new constraint based on the conservation of energy to improve the reconstruction accuracy of inverse scattering procedures.

2. PROBLEM FORMULATION

Let us refer to a two-dimensional scenario characterized by the contrast function $\tau(\mathbf{r}) = \varepsilon_r(\mathbf{r}) - 1$, $\varepsilon_r(\mathbf{r})$ being the dielectric permittivity of the medium and $\mathbf{r}$ indicates the position vector, and illuminated by a set of known incident electric fields at fixed angular frequency ω. Such fields impinge from v=1,...,V different directions and the information available from the scattered field is non-invasively collected in a set of measurement points for each illumination. Because of the non-linearity and the ill-posedness, the inverse scattering problem is reformulated in terms of the minimization of a cost function, which is defined by the discrepancy between measured and estimated data according to an integral equation model [3]. In the proposed approach, we aim at investigating the effectiveness of a new regularization term based on the conservation of energy. Accordingly, the arising cost function is obtained as follows

$$\Phi\left(\tau, \mathbf{E}_v^t\right) = \alpha\phi_d\left(\tau, \mathbf{E}_v^t\right) + \beta\phi_s\left(\tau, \mathbf{E}_v^t\right) + \gamma\phi_e\left(\tau, \mathbf{E}_v^t\right) \tag{1}$$

where ϕ_d and ϕ_s indicate the "Data" and "State" terms [3], respectively, while ϕ_e denotes the energetic term and $\mathbf{E}_v^t$ is the internal field. Furthermore, α , β and γ are weighting factors. As a consequence, the minimization of (1) leads to a configuration of τ and $\mathbf{E}_v^t$ that provides a satisfactory matching with the available scattering equations (i.e., "Data" and "State" terms) and fits the following relationship in the area under investigation

$$\nabla \cdot \mathbf{S}_v(\mathbf{r}) + j\frac{\omega}{2}\left\{\mu_0 \mathbf{H}_v^t(\mathbf{r}) \cdot \mathbf{H}_v^{t*}(\mathbf{r}) - \varepsilon_0 \left\{\mathrm{Re}\left[\tau(\mathbf{r}) + 1\right]\right\}\mathbf{E}_v^t(\mathbf{r}) \cdot \mathbf{E}_v^{t*}(\mathbf{r})\right\} = 0 \tag{2}$$

where $\mathbf{S}_v(\mathbf{r})$ is the Poynting's vector, $\mathbf{H}_v^t(\mathbf{r})$ the internal magnetic field and * denotes the complex conjugate, under the assumption of an isotropic lossless medium. Moreover, μ_0 and ε_0 indicate the vacuum permeability and permittivity, respectively. The energetic constraint (2) can be estimated for each direction of illumination and for each position $\mathbf{r}$ inside the investigation domain.

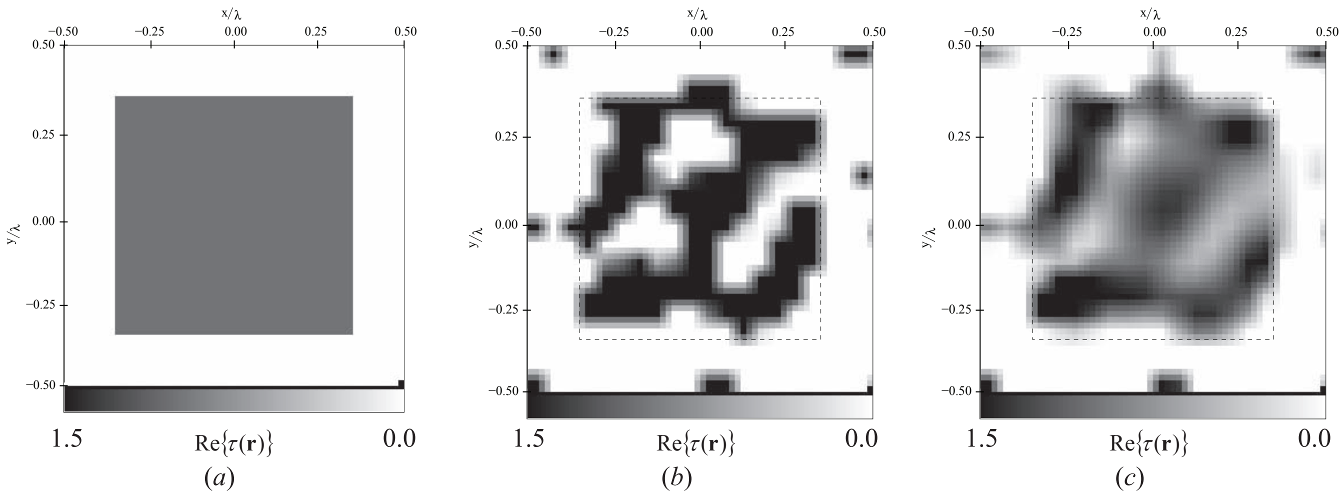

Fig. 1. (*a*) Actual contrast function. Reconstructions achieved (*b*) without and (*c*) with energetic constraints (SNR=10dB).

3. NUMERICAL RESULTS

As a representative test case, Fig. 1(*a*) shows the reference profile of a homogeneous cylinder ($\tau = 1.0$) of square cross section ($L_{obj} = 0.7\lambda$). λ indicates the free-space wavelength of the probing plane waves, which are supposed to illuminate the investigation domain impinging from *V*=8 different directions. For each incidence, the scattered field is acquired in 35 measurements points and the data are blurred adding a Gaussian noise characterized by a signal-to-noise ratio (SNR) of 10 dB. Moreover, the reconstructions have been carried out discretizing the investigation area in 400 subdomains. As far as the obtained results are concerned, the contrast function of Fig. 1(*b*) shows the reconstruction accuracy achieved through a standard conjugate gradient-based algorithm that minimizes the cost functional composed only by the “Data” and “State” terms. Even though the scatterer is localized, the homogeneity of the structure is not well retrieved. On the other hand, the exploitation of energetic constraints [Fig. 1(*c*)] may increase the accuracy in the reconstruction of the unknown object function. To quantitatively compare Fig. 1(*b*) and Fig. 1(*c*), let us consider the average reconstruction error, which decreases from 21% [Fig. 1(*b*)] to 13% [Fig. 1(*c*)] when the energetic constraints are used. Moreover, if we restrict the estimation of the reconstruction error on the area actually occupied by the scatterer [dashed lines in Figs 1(*b*)(*c*)], the value deceases from 33% to 18% when the minimization process is forced to search for a solution that satisfies the conservation of energy. Further experiments are currently under development to analyze the potentialities and the limitations of the proposed approach, considering both simulated and real data.

4. REFERENCES

[1] Y. Yu, T. Yu, and L. Carin, “Three-dimensional inverse scattering of a dielectric target embedded in a lossy half-space,” *IEEE Trans. Geosci. Remote Sensing*, vol. 42, pp. 957-973, May 2004

[2] E. C. Fear and M. A. Stuchly, “Microwave detection of breast cancer,” *IEEE Trans. Microwave Theory Tech.*, vol. 48, pp. 1854-1863, Nov. 2000.

[3] D. Colton and R. Krees, *Inverse acoustics and electromagnetic scattering theory*. Berlin, Germany: Springer-Verlag, 1992.

[4] L. Crocco and T. Isernia, “Inverse scattering with real data: detecting and imaging homogeneous dielectric objects,” *Inverse Problems*, vol. 17, pp. 1573-1583, Dec. 2001.

[5] A. Abubakar, P. M. van den Berg, and S. Y. Semenov, ”Two- and three-dimensional algorithms for microwave imaging and inverse scattering,” *J. Electromag. Waves Appl.*, vol. 17, pp. 209-231, Feb. 2003.