A Fast Numerical Method for Scattering from Dielectric Rough Surfaces

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The need for numerical methods in studying electromagnetic scattering from rough surfaces is well established. The most commonly used numerical method is the surface integral equation method and its solution by the method of moments (MoM). Conventional implementation of MoM requires $O(N^3)$ operations, where N is the number of surface unknowns. When N is large, as frequently encountered in studying electromagnetic scattering from rough surfaces, in particular at near grazing incidence, given the high computational complexity of conventional implementation of MoM, fast numerical methods are called for and several have been proposed in the literature.

Among the methods some of the most popular ones are the banded matrix iterative approach /canonical grid (BMIA/CAG) for PEC rough surface, the physics-based two-grid method (PBTG) for dielectric rough surface, and the iterative forward-backward method (FBM) for both PEC and dielectric rough surfaces. In BMIA/CAG, the impedance matrix from Green's function is decomposed into a sum of a banded matrix, which recognizes the coherent mutual interaction for two points in the neighborhood of each other, and a Taylor expanded flat surface matrix, which is based on observation that outside the neighborhood the Green's function connecting the two points on the rough surface is close to that of a flat surface, the considered canonical grid, and the Green's function is approximated by a Taylor expansion. The advantage of this approach is that, for the banded matrix part, its product with a surface current column vector can be more efficiently computed than that of a full matrix version, while for the Taylor expanded flat surface matrix part, its product with the vector can be speedily computed by the fast Fourier transform (FFT). The computational complexity of this method is $O(N \log N)$. The PBTG method uses two grids: a dense grid and a coarse grid. It is combined with BMIA/CAG to attain the appealing computational property of the latter. In FBM, electric and magnetic equivalent surface current densities are split into forward and backward components, a treatment naturally gives up to an iterative solution scheme. Its computational complexity is $O(N^2)$. Its iterative process converges very fast, except that for extremely rough surfaces.

However, the computational efficiency achieved in each of the above fast methods can not be taken for granted to be applicable to universal roughness conditions. Rather, it has been observed that each method can break down or become computationally intensive. For instance, in studying ocean-radar scattering, for typical microwave radar frequencies and sea-surface roughness, BMIA was found frequently to either diverge or converge to the incorrect solution. The FBM, in its conventional form with a computational complexity of $O(N^2)$, may not be suitable when N is large as in the study of rough surface scattering at near grazing incidence. Moreover, it also suffers from convergence issues. For typical value of dielectric constant for sea water at microwave frequencies, it is reported that FBM shows significantly slow convergence, and it often fails to converge for very and extremely rough surfaces, in particular for HH polarization. The fact that the relative residual error defined by was fixed to 10^{-2} indicates that the convergence issue can be even more severe if a more restrictive value for the relative residual error is used, say 10^{-4} as commonly used in the literature. In dealing with computational complexity of FBM, the spectral acceleration (SA) technique was combined to reduce the computational complexity to $O(N)$. However, there is a price to pay. The FBM-SA scheme is found sensitive to the approximation error induced by SA by extensive numerical simulations. It often fails to converge for cases where FBM converges.

In this paper we propose a fast numerical technique to improve over the aforementioned methods. After discretization of the integral equation, a new matrix splitting scheme is employed for the associated iterative system. In the splitting of the impedance matrix Z, each of the two subblock matrices with a dominant diagonal is split as the sum of a diagonal matrix and the remainder. This treatment is motivated by the observation that for PEC rough surfaces the MFIE-induced impedance matrix is diagonally dominated by the value one half where favorable performance is obtained in a similar treatment in our previous work. For each of the remaining two subblock matrices, a three-part partition is carried out as in the BMIA/CG approach, and the strong part $Z^{(s)}$ and canonical grid weak part $Z^{(FS)}$ are remained in the left side of the system. When updating the right side of the system, production of the strong part with a vector can be more readily calculated than the full matrix version, and the production of the weak part with the vector is performed using the spectral acceleration technique.

To validate the proposed method, in the numerical simulation we consider Gaussian rough surfaces with both Gaussian and exponential spectra. Several relative dielectric constants of the medium below the rough surface are considered, including 15+4i for comparison with the literature, and 80+66i which is typical of ocean at microwave frequency. The roughness runs from smooth (around 0.1 wavelength) to extremely rough (10 wavelengths), and the ratio of the rms height with the correlation length, representative of the rms slope, ranges from 0.15 to 1. The incidence angle ranges from normal to near grazing (85 degrees). The number of surface unknowns can be as large as 32768.

Through extensive numerical simulation, several appealing features have been observed for our proposed method and are worth listing as follows:

1) It is at least several times faster than the FBM method;

2) It is comparable to FBM-SA in terms of run time but is considerably robust against approximation error induced by SA.

3) At large roughness, while convergence behavior of FBM for VV is much better than HH, that of our proposed method curiously shows the reverse. Although cause of such behavior is an ongoing research subject, from an operational perspective it suggests that our proposed method complements FBM for polarization considerations at large roughness.

In summary, our proposed method has demonstrated much improved speed over FBM, and considerably improved convergence behavior over FBM-SA. Moreover, its polarizational behavior at large roughness satisfactorily complements that of FBM. It thus holds the potential for statistical characterization of scattering from natural surface or ocean surface at near grazing angles.