MODEL-BASED STATISTICAL ANALYSIS OF POLSAR DATA.

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Polarimetric SAR (POLSAR) data are complex multidimensional image data, which can be analyzed along several processing schemes. In the literature, we find that much emphasis has been put on analysis based on target decomposition theorems. Through this approach, information about scattering mechanisms can be gained. The knowledge of the exact statistical properties of POLSAR data founds the basis for another strategy of multidimensional image analysis, which in some cases are complementary to the target decomposition approach. Statistical properties can be used to discriminate different types of land cover, and to develop specialized filters for speckle noise reduction, to mention a few applications. Many studies have been conducted in order to relate physical features and statistical properties of SAR and POLSAR data.

The multiplicative model has been widely used in the modeling, processing, and analysis of single polarimetric synthetic aperture radar images. This model states that, under certain conditions, the backscattered signal results from the product between a Gaussian speckle noise component and the terrain backscatter. Several distributions could be used for the terrain backscatter, in order to model different types of surface classes with their characteristic spatial correlation properties and degrees of homogeneity.

A multidimensional extension of the multiplicative model is straight forward. This can be done by letting $\mathbf{X}$ be a d-dimensional, zero mean Gaussian variable with covariance matrix equal to the identity matrix. Let furthermore, $\mathbf{\Gamma} \in \mathcal{R}^{d\times d}$ be a positive definite, symmetric, matrix with determinant $\det \mathbf{\Gamma} = 1$, and let Z be a scalar random variable with pdf $p_Z(z)$, which can attain only positive values. A new variable $\mathbf{Y}$ is now generated as

$$\boldsymbol{Y} = \sqrt{Z}\mathbf{\Gamma}^{\frac{1}{2}}\boldsymbol{X}. \tag{1}$$

The matrix $\mathbf{\Gamma}$ defines the internal covariance structure of the component variables of $\mathbf{Y}$. For this reason we will refer to this matrix as the covariance structure matrix. The model in (**??**) may be further generalized by

$$\boldsymbol{Y} = \boldsymbol{\mu} + Z\mathbf{\Gamma}\boldsymbol{\beta} + \sqrt{Z}\mathbf{\Gamma}^{\frac{1}{2}}\boldsymbol{X}, \tag{2}$$

where $\boldsymbol{\mu}$ is a location vector, $\boldsymbol{\beta}$ is a vector parameter accounting for the linear scaling of the mean of $\boldsymbol{Y}$ as function of Z. The model in (**??**) is referred to as *a multivariate variance-mean mixture variate*, and may be regarded as a generalized *scale mixture of Gaussian model* (SMoG).

To obtain the marginal pdf of $\boldsymbol{Y}$, an integration over the prior distribution $p_Z(z)$ must be performed, which accordingly gives us

$$p_{\boldsymbol{Y}}(\boldsymbol{y}) = \int_0^\infty p_Z(z)\frac{1}{(2\pi\, z)^{\frac{d}{2}}} \times \exp\left(-\frac{(\boldsymbol{y}-\boldsymbol{\mu}-\mathbf{\Gamma}\boldsymbol{\beta}z)^t\mathbf{\Gamma}^{-1}(\boldsymbol{y}-\boldsymbol{\mu}-\mathbf{\Gamma}\boldsymbol{\beta}z)}{2z}\right)dz. \tag{3}$$

The superindex t denotes transpose of the matrix.

The above modeling schemes constitutes flexible models, which have the capabilities to model data which ranges in Gaussianity from highly non-Gaussian to Gaussian data. These models can also easily be used to develop non-Gaussian distributions to many signal quantities. The multivariate K-distribution can be formalized in this framework. If the data is K-distributed, the distribution for the corresponding sample covariance matrix is found to be K-Wishart distributed. The rich parameter space associated with the *a multivariate variance-mean mixture models* allows for the development of image segmentation algorithms, both in data space and in the parametric feature space.

In this paper some characteristics related to SMoG models are briefly discussed, as well as certain approaches to the analysis of POLSAR data in this framework. Experimental results from various application areas will be presented.