# OBTAIN LONGEST AXIS OF COHERENCE REGION AND ITS APPLICATIONS TO ESTIMATE TOPOGRAPHIC PHASE

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This paper presents a new method to identify the coherences samples in the coherence region that give the maximum distance. This method directly shifts the coherence region without stretching or rotation so that the shirted coherence regions preserve the original shape. The coherence region is shifted by large length in upper, right directions so that shifted regions locate within the upper, right half plane. The projection vectors giving the phase diversity of the shifted coherence regions provides the nearly estimation for the longest axis. Utilizing the phase diversity method proposed by Tabb et. al, the proposed method estimates four projection vectors' candidates for the maximum distance from the two shifted coherence regions. Selected From the four candidates, the two candidates giving the maximum distances are the final estimations. Since the longest axis coincides with the orientation of the elliptical coherence region, the longest axis is used to estimate topographic phase. Experiments are applied on the ESAR polarimetric interferometric data to validate the proposed method and assess its potentials to estimate topographic phase.

# 1. INTRODUCTION

Polarimetric SAR interferometry introduces Polarimetric techniques into interferometric applications. It investigates the dependency of complex interferometric coherence on polarization states so as to separate scattering mechanisms existing in the same resolution cell and to obtain the targets' vertical structure [1]. The coherence region, within which all possible copolar complex coherences locate in, is of great interest [2]. It is because coherence region is necessary to analyze the variations of complex coherences with polarization states. Lots of efforts have been done to extract the shape of coherence region [3] and to estimate the phase diversity of coherence region [2]. However, they need to rotate the coherence region and calculate the eigenvectors of every rotated polarimetric interferometric data matrix. Therefore, it consumes lots of time to compute the information about coherence regions.

#### 2. PROBLEM DESCRIPTION

According to RVoG model, coherences in different polarization states distribute along a straight line in complex plane. The topographic phase can be estimated from the intersections of the line and the unit circle. In other words, any two different coherence samples are feasible to fit the line and to obtain the topographic estimation [4]. However, the coherences disperse over an elliptical or triangle region other than a straight line [2]. Therefore, different choices of coherence samples lead to distinct estimations for the topographic phase. To obtain the stable estimation for topographic phase, we propose to use the longest axis of coherence region to estimate the topographic phase.

One method to calculate longest axis is to calculate the shape of the coherence region and to search for the maximum distance. However, the method needs extensive computation and consumes lots of time. It computation efficiency needs to improve.

## 3. OUR SOLUTION

We propose directly shifts the coherence region without stretching or rotation. As shown the Fig. 1 and Fig. 2, the coherence regions marked by black solid ellipses are shifted in the upper, right directions respectively. The shifted distance should be large. The longer the shifted distance is, the closer the projection vectors estimated from phase diversity method are to those

giving the longest axis. As indicated by phase diversity method [2], the projection vectors  $\vec{\omega}_A$  and  $\vec{\omega}_B$ , which give the phase diversity of shifted coherence region in upper half plane, are estimated from:

$$\begin{cases} \vec{\omega}_{A} = \mathbf{B}^{-1/2} \vec{u}_{1} \\ \vec{\omega}_{B} = \mathbf{B}^{-1/2} \vec{u}_{2} \end{cases} \text{ while } \mathbf{B}^{-1/2} \mathbf{A} \mathbf{B}^{-1/2} = \sum_{i=1}^{3} \lambda_{i} \vec{u}_{i} \vec{u}_{i}^{H}, \mathbf{A} = \left( \mathbf{\Omega}_{12} + \mathbf{\Omega}_{12}^{H} \right) / 2, \mathbf{B} = -j/2 \cdot \left( \mathbf{\Omega}_{12} - \mathbf{\Omega}_{12}^{H} \right) + L \cdot \mathbf{I}_{3} \text{ and } \lambda_{1} > \lambda_{2} > \lambda_{3} . \tag{1}$$

where the matrix  $\Omega_{12}$  is defined as  $\langle \vec{k}_1 \vec{k}_2^H \rangle$ , symbol *L* indicates the shifted length and I<sub>3</sub> means the unitary matrix.

Similar to equation (1), the phase diversity of the shifted coherence in right half plane are given by the projection vectors:

$$\begin{cases} \vec{\omega}_{C} = \mathbf{B}^{-1/2} \vec{u}_{1} \\ \vec{\omega}_{D} = \mathbf{B}^{-1/2} \vec{u}_{2} \end{cases} \text{ while } \mathbf{B}^{-1/2} \mathbf{A} \mathbf{B}^{-1/2} = \sum_{i=1}^{3} \lambda_{i} \vec{u}_{i} \vec{u}_{i}^{H}, \mathbf{A} = -j/2 \cdot \left( \mathbf{\Omega}_{12} - \mathbf{\Omega}_{12}^{H} \right), \mathbf{B} = \left( \mathbf{\Omega}_{12} + \mathbf{\Omega}_{12}^{H} \right)/2 + L \cdot \mathbf{I}_{3} \text{ and } \lambda_{1} > \lambda_{2} > \lambda_{3} . \tag{2}$$

Selected From the candidates  $\bar{\omega}_A$ ,  $\bar{\omega}_B$ ,  $\bar{\omega}_C$  and  $\bar{\omega}_D$ , the two candidates giving the maximum distances between their original coherences are used to estimate the longest axis of coherence region. The distance between candidates is calculated from:

$$D_{MN} = \left| \tilde{\gamma}(\vec{\omega}_{M}) - \tilde{\gamma}(\vec{\omega}_{N}) \right| = \left| \frac{\vec{\omega}_{M} \Omega_{12} \vec{\omega}_{M}}{\sqrt{(\vec{\omega}_{M} \mathbf{T}_{11} \vec{\omega}_{M})(\vec{\omega}_{M} \mathbf{T}_{22} \vec{\omega}_{M})}} - \frac{\vec{\omega}_{N} \Omega_{12} \vec{\omega}_{N}}{\sqrt{(\vec{\omega}_{N} \mathbf{T}_{11} \vec{\omega}_{N})(\vec{\omega}_{N} \mathbf{T}_{22} \vec{\omega}_{N})}} \right| \quad while \quad \mathbf{T}_{11} = \left\langle \vec{k}_{1} \vec{k}_{1} \right\rangle, \\ \mathbf{T}_{22} = \left\langle \vec{k}_{2} \vec{k}_{2} \right\rangle \quad and \quad M, N = \{A, B, C, D\}.$$
(3)

Since the longest axis coincides with the orientation of the elliptical coherence region, the straight line fitted from the coherences given by the propose method are used to estimate topographic phase.





Fig. 1 Orientation Direction Close to Horizontal Direction Fig. 2 Orientation Direction Close to Vertical Direction

#### 4. EXPERIMENTS

To analyze the performance of the proposed method, experiments are applied on DLR E-SAR L-band data acquired from Traunstein. In the experiments, L=10 is used to estimate the longest axis of coherence region and topographic phase. Compared with the longest axis from the shape of coherence region, the estimation accuracy of proposed algorithm is analyzed.

#### 6. REFERENCES

- S.R.Cloude and K.P.Papathanassiou, "Polarimetric SAR interferometry," IEEE Trans. Geosci. Remote Sens., vol. 36, no. 5, pp.1551 – 1565, Sep. 1998.
- [2] Mark Tabb, Thomas Flynn, and Richard Carande. "Phase Diversity: A Decomposition for Vegetation Parameter Estimation Using Polarimetric SAR Interferometry," EUSAR'2002.
- [3] Thomas Flynn, Mark Tabb, and Richard Carande. "Coherence Region Shape Extraction for Vegetation Parameter Estimation in Polarimetric SAR Interferometry," IGARSS'2002.
- [4] S.R.Cloude and K.P.Papathanassiou, "Three-stage inversion process for polarimetric SAR interferometry," IEE Proceedings Radar, Sonar and Navigation, vol. 150, no. 3, pp.125 – 134, Jun. 2003.