MODEL FREE GRAVIMETRIC DETECTION

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In recent work [1], we introduced a new, linear-programming based method, for detecting underground features based on gravity measurements. The current paper takes this method one important step further, in that it makes it work under realistic conditions, where most of the physical characteristics of the environment are not known. In particular, the ADCS algorithm we introduce works when the following physical parameters are *unknown*: the spatial variability of the underground density, the amplitude of this variability, the number of the holes (assuming that there are only a few), the shape of the hole(s). The algorithm will construct a map of the unknown holes' locations and shapes.

We achieve this by reformulating our original compressed sensing (CS) which assumes knowledge of the mass density variability. In the new formulation the unknown variable is rescaled for a better balanced optimization problem, and the noise parameter of the problem is calculated based on the input data.

1. ADAPTIVE THRESHOLDING AND NEW REPRESENTATION

It is known that solving the gravitational inverse problem (see e.g [1],[2] amounts to solving a highly underdetermined linear system $t = M\theta + \epsilon$, where $t \in \mathbb{R}^n$ is the (residual) measurement, $\theta \in \mathbb{R}^p$ contains the unknown density, $M \in \mathbb{R}^{n \times p}$ is the measurement matrix, ϵ is the vector of residuals and $n < p$. Now our task is to find the *large components* of θ. The compressed sensing theory of [3] and Candes, shows that under certain mathematical conditions (largely *unfulfilled* by the gravitational inverse problem [2]), the elements of the true solution θ^* that are above the noise level can be recovered by the simple linear program (LP) [1]

$$(\mathcal{P}_0) \quad \begin{array}{ll} \min_\rho & ||\rho||_1 \\ \text{s.t.} & ||t - M\rho||_\infty \;\leq\; \sigma\sqrt{2\log p} \end{array} \tag{1}$$

This allows one to recover just the holes, which can be seen as large anomalies in the overall underground densities[1].

In the above σ denotes the noise variance, which depends both on the measurement noise and on the spatial covariance of the underground density. With improved gradiometric measurements becoming available, it is the latter term that dominates the noise ϵ. This term is unknown in practice, and it is hard to determine theoretically because of the paucity of existing underground density models which describe the spatial correlation in the density of the various rocks. Therefore we propose to replace problem $(\mathcal{P}_0)$ with the new linear program $(\mathcal{P})$

$$(\mathcal{P}) \quad \begin{array}{lll} \min_{\theta \in \mathbb{R}^p} & \mathbf{1}^T\theta + ps + \sum_{j=1}^n s_j) & \\ \text{s.t.} & 0 \;\leq\; \theta_\xi \leq 1 & \text{for } \xi = 1:p \\ & |\rho_0 M_{j:}\theta - t_j| \;\leq\; (w + s + s_j)\eta_j & \text{for } j = 1:n \end{array} \quad \text{with} \quad \begin{array}{l} \eta_j \;=\; 0.5(t_j + ||t||_1) \\ w \;=\; 0.3,\; \rho_0 \;=\; \operatorname{avg}\rho \end{array} \quad \begin{array}{ll} s, s_j \;\equiv\; 0 & \text{Simple slack} \\ s_j \;\equiv\; 0 & \text{Global slack} \\ s \;=\; 0 & \text{Local slack} \end{array} \tag{2}$$

In words, we use the hole indicator variables θ_ξ to describe the hole. For a "full" voxel, $\theta_\xi = 0$ and for a hole voxel $\theta_\xi = 1$. For a hole that does not completely overlap with a voxel, θ_ξ can take fractional values. The slack parameters s_s, s_a allow us to adapt the threshold, by tightening it as long as the problem $\mathcal{P}$ remains feasible. The actual tolerance for reconstructing the input is a function of the measurement (we tolerate more noise for larger measurement values) and of the slack variables.

2. EXPERIMENTS

We simulated two geometries, shown in figure 1, one containing an L-shaped hole ($h = 95$ hole voxels), the other one three vertical holes($h = 88$). The total number of voxels $p = 2048$, and the geometry of the environment were the same for both. The mass density noise had realistic values, variying from low to high. Table 2 shows the precision and recall in retrieving the hole

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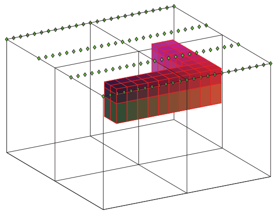
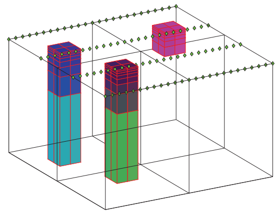


L shaped, $h = 95$



3 holes, $h = 88$

underground volume $16m \times 16m \times 10m$
$n = 2 \times 100$ measurements
Holes don't align with reconstruction grid
$p = 2048$ voxels for reconstruction, of which
h intersect with the true hole(s)

Fig. 1. Simulated problems. $n/h < \log p$) hence these are theoretically very hard problems for CS.

Table 1. Precision and recall of ADCS with different slack types w.r.t original solution from [1] (denoted here CS+Constant); CS is the LP in the original variables ρ, while ADCS is the LP in $\mathcal{P}$. Precision and Recall are evaluated w.r.t to numbers of voxels, but the results w.r.t volumes are qualitatively similar. NaN marks the cases where the LP was infeasible or didn't converge. The table represents medians over 10 runs.

Recall=#TP voxels/#hole voxels

Hole	**Noise**	**LP**	**Slack type**			
			Global	Local	Simple	Const
L hole	0.010	CS	0.17	0.13	0.15	0.32
	0.025	CS	0.16	0.11	0.16	0.33
	0.050	CS	0.15	0.11	0.19	NaN
	0.010	ADCS	0.64	0.51	0.65	
	0.025	ADCS	0.66	0.47	0.66	
	0.050	ADCS	0.61	0.41	NaN	
3 holes	0.010	CS	0.19	0.20	0.17	0.33
	0.025	CS	0.14	0.17	0.15	0.27
	0.050	CS	0.12	0.13	0.09	NaN
	0.010	ADCS	0.52	0.60	0.61	
	0.025	ADCS	0.39	0.55	0.39	
	0.050	ADCS	0.34	0.42	NaN	

Precision=#TP voxels/(#TP voxels +#FP voxels)

Hole	**Noise**	**LP**	**Slack type**			
			Global	Local	Simple	Const
L hole	0.010	CS	0.63	0.51	0.64	1.00
	0.025	CS	0.64	0.40	0.59	0.93
	0.050	CS	0.60	0.32	0.53	NaN
	0.010	ADCS	0.51	0.33	0.49	
	0.025	ADCS	0.50	0.28	0.50	
	0.050	ADCS	0.42	0.20	1.00	
3 holes	0.010	CS	0.68	0.73	0.64	0.90
	0.025	CS	0.64	0.94	0.50	0.83
	0.050	CS	0.53	0.85	0.31	NaN
	0.010	ADCS	0.62	0.60	0.66	
	0.025	ADCS	0.52	0.55	NaN	
	0.050	ADCS	0.52	0.46	NaN	

voxels for the standard CS representation denoted by CS , the new representation in ($\mathcal{P}_0$) denoted by ADCS , with a variety of adaptive methods for setting the reconstruction tolerance. The original CS algorithm is denoted by CS+Const, reflecting the fact that the tolerance is a constant. This constant was set by hand to 10^{-3}, after a small set of trials. Hence, the original method has an unfair advantage in the comparisons, especially w.r.t the Precision parameter.

The most striking feature in table 2 is that the recall of CS is overall very low, indepedently of the slack type or noise. Using ADCS improves recall consistently and significantly, at a small loss in precision. Hence, using the new representation, results in a less sparse estimated hole; most voxels which are assigned by ADCS to the hole are correct hole voxels, while a few remaining add to the false positives (thus slightly reducing precision). The slack type has only a weak influence on the precision/recall of ADCS, but using the novel slack methods seems to assure convergence in cases when the Simple slack fails to converge.

Moreover, ADCS ran about 50 times faster than CS on average on our examples (5-10 seconds vs. 1-2 minutes CPU time).

3. REFERENCES

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[3] David Donoho, "Compressed sensing," *IEEE Trans. on Information Theory*, vol. 52, no. 4, pp. 1289–1306, 2006.