COMPLEX WAVELET REGULARIZATION FOR SOLVING INVERSE PROBLEMS IN REMOTE SENSING

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1. INTRODUCTION

Some problems in remote sensing consist in retrieving the image u acquired by a satellite, from a damaged observation. This can be modeled as follows :

$$g = \Delta_\Lambda(h * u) + n \tag{1}$$

where h is a blurring function (generally the *Point Spread Function* of the optical system of the satellite), n is a noise (Gaussian noise for example), g is the observed image and Δ_Λ is the sampling grid (regular or irregular). Variational approaches have been proposed [1, 2, 3] to solve this problem, using different norms on the data term and the regularizing term. The norm on the data term allows to adapt the restoration model to the noise model. For example the l^2-norm is adapted to Gaussian noise while the l^1-norm is more robust to impulse noise [3]. A variational approach consists in determining:

$$\min_{u \in \mathbb{R}^N} \left(\|Su - g\|_p^p + \lambda J(u) \right) \tag{2}$$

where $J(u)$ is the regularizing term, $\|.\|_p$ denotes the l^p-norm, λ is the regularizing parameter and S is a matrix notation which stands for the sampling and the blurring operators. Very efficient priors are of the form $J(u) = \|Bu\|_1$ where B is a linear transform.

2. APPLICATION AND RESULTS

The authors of [3] set $B = \nabla$. In this case, J is the total variation (TV) which is well known to remove noise while preserving the discontinuities of the image. However, this regularization does not allow to recover the textures well (this effect is known as "*cartoon*" effect). This is a problem in remote sensing as we want to retrieve the details. In order to restore all thin details, we set B to be the Dual-Tree Complex Wavelet Tranform (DTCW) [4]. The choice of the DTCW transform is motivated by the fact that this transform is quasi-invariant by translation and rotation, which is not the case of real non redundant wavelet transforms. The redundancy of the DTCW is 4 for 2D images. This *quasi*-invariance is a sufficient property to be used as a regularizing operator. Moreover, this wavelet transform gives less artefacts than real wavelets when the coefficients are thresholded (this threshold is here done by the l^1-norm regularizing term). As some errors in the sampling grid may generate huge errors on the intensity result we set $p = 1$. This gives robust estimation against outliers. Finally the model reduces to:

$$\min_{u \in \mathbb{R}^N} \left(\|Su - g\|_1 + \lambda \|Wu\|_1 \right) \tag{3}$$

where W is the DTCW Tranform. Due to the non-differentiability of the l^1-norms, this problem is very difficult to solve. The authors of [3] used smooth approximations and a gradient descent to solve their problem. We use, instead, a fast multi-step first order method originally proposed by Y. Nesterov [5] to solve this problem with notable improvements compared to other first order techniques. This techniques reduces the computing times by an order of magnitude. The reader should refer to [6] for more details on the implementation of this algorithm. Results are shown on figure 1. We can see that the image retrieved

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with the proposed method is better than the one obtained using the TV regularization as we do not have the ”*cartoon*” effect due to the TV (look at the diagonal zebra crossing on figures (c) and (d)). For strongly noisy images, we could check that this regularization gives some artefacts and slightly blurs the image. Small elements may thus lose intensity. Further results will be provided in the final paper showing, also, the comparison between the minimization of (3) and the minimization with respect to the wavelet coefficients, which is usually done, constraining the signal to be sparse in the wavelet domain.

(a) (b) (c) (d)

Fig. 1. Restoration of an irregularily sampled, blurred and noisy image. (a) Original image ©CNES, (b) distorted image (Gaussian noise, $SNR = 15.62\ dB$), (c) result with the TV regularization ($SNR = 24.09\ dB$), and finally (d) result with the DTCW regularization ($SNR = 24.42\ dB$).

3. CONCLUSION

We have proposed a new method for solving restoration problems in image processing using a variational approach regularized with the l^1-norm of a complex wavelet regularization. This method is very efficient to restore thin details and to remove noise compared to the TV regularization which smooths the oriented textures of the image. To our best knowledge, only few results are provided in image deconvolution with wavelet regularization (non orthogonal basis) as the minimization is very time consuming. In this paper, we use a fast algorithm to solve this problem.

4. REFERENCES

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