ESTIMATION OF RAINFALL RATE FROM TERRESTRIAL MICROWAVE LINK MEASUREMENTS

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1. SUMMARY

The large number of millimetre and microwave terrestrial links that are in operational use has lead to the development of techniques to estimate two-dimensional rainfall rate fields e.g., [1]. This paper outlines a reconstruction algorithm that can be used to estimate the rainfall field from a number of measurements of path attenuation on terrestrial links. Previous methods described in the literature have used either statistical methods, interpolation techniques or direct tomographic approaches. Given the generally ill-conditioned problem the methods are often not robust [2, 3]. In this paper we adopt a different approach to the regularization of this inverse problem. Rather than attempt to reconstruct the entire rainfall field directly from what may be a sparse set of links, we assume that the rainfall field can be estimated from a weighted sum of orthonormal two-dimensional basis functions. The paper describes a robust algorithm to determine the weights. Consideration is given to the generation of basis functions Gaussian functions and using a data-derived approach using radar data. The robustness of the approach, demonstrated through simulation. The approach is shown to be robust and relatively insensitive to errors and quantisation in the link measurements.

2. INVERSE METHOD

For a reconstruction grid of n pixels and a network of m links, a matrix $\mathbf{L}$ can be calculated whose elements are the path length of the ith link through the jth pixel denoted l_{ij}. The product of the link geometries with the (unknown) specific attenuations, $\mathbf{k}$, yields the (known) link attenuations $\mathbf{a}$,

$$\mathbf{L}\mathbf{k}^T = \mathbf{a}^T. \tag{1}$$

Since it is unlikely that the links entirely cover the domain it is likely that the matrix $\mathbf{L}$ is sparse with a large condition number. This implies an ill-posed inverse problem whose solution is highly sensitive to measurement errors. It follows that it is unlikely that a satisfactory solution can be obtained by attempting to seek the direct inverse. Instead, we first assume that it is possible to approximate the solution by a weighted finite sum of f orthogonal basis functions. Thus the approximate solution to $\mathbf{k}$ can be written as;

$$\mathbf{k} \approx [\mathbf{K}\mathbf{w}]^T, \tag{2}$$

where $\mathbf{w}$ is a column vector of weights and $\mathbf{K}$ is an $n \times f$ matrix of basis functions. The basis function weight vector $\mathbf{w}$ can now be obtained from the solution of $\mathbf{L}\mathbf{K}\mathbf{w} \approx \mathbf{a}^T$. The ordinary least-squares solution of can be found by solving the minimisation problem:

$$\min_{\mathbf{w}} \left\{ \left\| \mathbf{L}\mathbf{K}\mathbf{w} - \mathbf{a}^T \right\|_2 \right\}, \tag{3}$$

where $\|\cdots\|_2$ represents the Euclidian norm. However, in general the solution will not be unique and will still be succeptable to measurement errors. Further regularisation of the inverse problem is required to filter out the influence of noise by imposing smoothness constraints on the solution. Various solutions to this problem will be discussed. Finally, the rainfall rate R, can be retrived using an optimised k-R relationship.

3. RESULTS

As an example of the technique the original field and reconstuctions using data-derived and analytic basis functions are shown in Fig. 1. The RMS errors in the derived attenuation fields are 0.49 dB and 0.43 dB for the Gaussian approach and the data-derived approach respectively. Linear regression between the two datasets shows gradient values of 0.98 and 0.92 and intercept values of 0.07 dB and 0.08 dB respectively. By continuing the same data analysis on different grid scales an understanding of the interaction between the various parameters can be obtained. As an example, Fig. 2a shows that there is an optimum number of basis functions for a given number of links and a given reconstruction which

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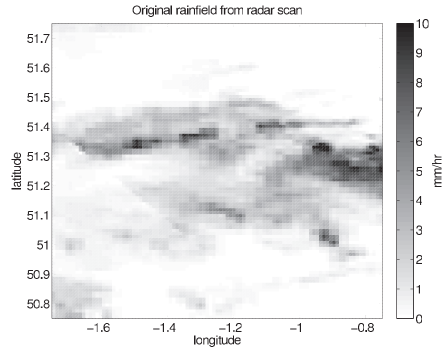
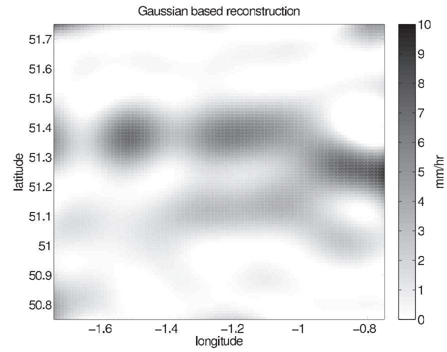
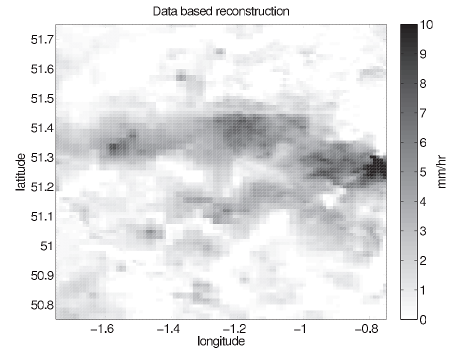


(a) Original



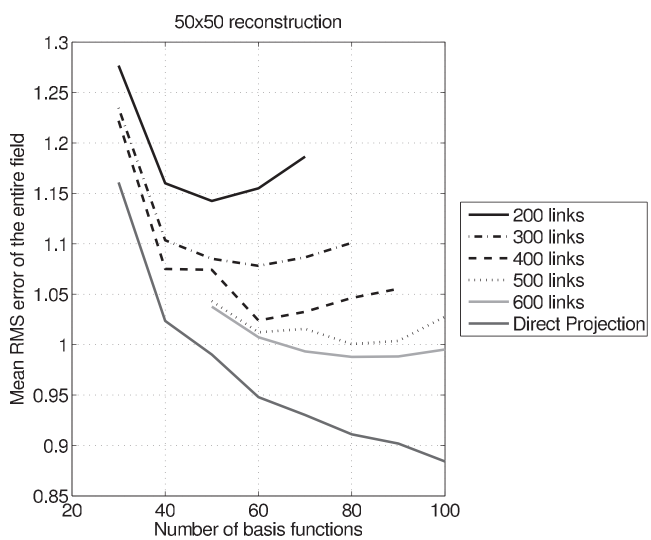
(b) Gaussian basis functions



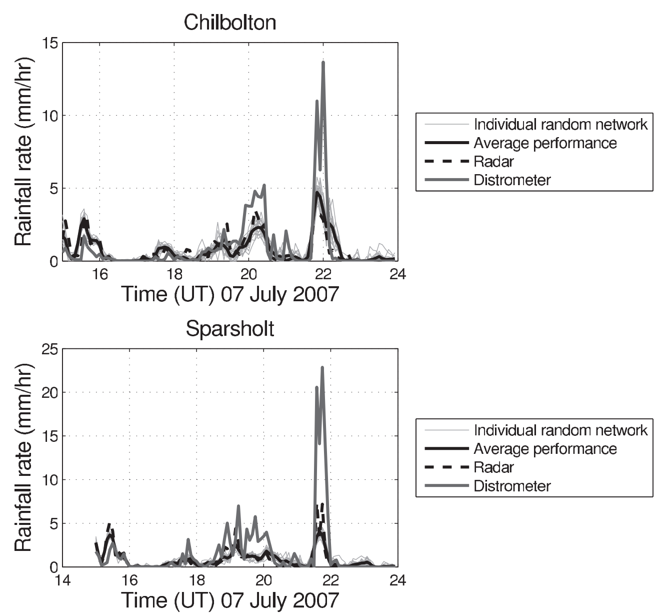
(c) Data-derived basis functions

Fig. 1. Example reconstructions using the analytic and data-derived basis functions.

best matches the amount of information available from the link measurements. The reconstruction method is able to provide a time-series of inversions. To illustrate the performance of the technique Fig. 2b shows a comparison between rainfall rate derived from the technique and rainfall rate determined from distrometers operated by the UK Rutherford Appleton Laboratory at two sites in the UK (Chilbolton and Sparsholt). It can be seen that there is generally good agreement between the distrometer and the inverse method.



(a) The mean value of the RMS error (mm hr^{-1})



(b) Comparison of rainfall rate timseries

Fig. 2. (a) Calculated from 25 random networks with varying numbers of basis function and linkdensity with a 50×50 pixel domain, grid length ≈2.2 km. (b) timeseries from two UK sites link: inversion, distrometer and weather radar

4. REFERENCES

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