Rigorous Adaptive Resampling for High Resolution Image Warping

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Abstract

Techniques to precisely and accurately compare series of images are at the core of many remote sensing applications. For instance, stereoscopic image pairs may be analyzed to produce digital elevation models (DEM), time series of images may be compared to quantify some aspects of landscape changes such as vegetation cover evolution, glacier retreat or advance, damage or crisis assessment to monitor floods, fire, landslides, earthquakes, etc... In all these applications, images need to be projected in a common geometry to be jointly analyzed [1]. This particular geometry can be the epipolar geometry if a DEM is to be extracted from stereoscopic images pairs, it can be a ground projection if ortho-images are analyzed, or it can also be the viewing geometry of a particular image used as reference. The projection mapping from the raw image acquisition to the new common geometry can be highly irregular [1]. Although images can be assumed regularly sampled in sensor geometry (regular CCD matrix for digital frame camera, stable pusbhroom system, etc...), the mapped pixel grid often forms a highly irregular mapping due to the combination of high local topography variations, high incidence angles of the imaging systems, and large baselines between imaging systems. Mapping irregularities are further exacerbated in high resolution images when sharp 3D objects such as building corners or sharp topographic features can be resolved in the image.

In this study, we formalize the general resampling problem when the input image is critically sampled on a regular grid, and should be reconstructed without the introduction of aliasing, while preserving the local image frequency content. We will introduce a rigorous, locally varying resampling kernel, which is adapted to the local warping.

The theory of image resampling was introduced in [2], and it is commonly used in computer graphics applications to reconstruct sharp 3D objects onto 2D images [3]. We start by presenting a complete characterization of the resampling problem by looking at both direct and inverse image warping problems.

To simplify the notation and the discussion, we will assume that a regularly sampled image in sensor geometry, i_s, is to be resampled to an image on the ground, i_g, via the direct mapping $\mathbf{x}_g = m(\mathbf{x}_s)$, where $\mathbf{x}_s$ denote coordinates in the sensor geometry and $\mathbf{x}_g$ denote coordinates in the mapped geometry (e.g., on the ground). The general resampling theory can be described with four successive steps. The input image i_s is first continuously reconstructed into the image i_s^c, it is then mapped to i_g^c, then filtered to be adequately sampled onto the grid defined by the set of $\{\mathbf{x}_g\}$ to produce i_g^{cf}, and finally sampled at every $\mathbf{x}_g$. Formally, it is written as follows:

$$
\begin{aligned}
i_s^c(\mathbf{x}_s) &= \sum_{\mathbf{k}_s \in \mathbb{Z}^2} i_s(\mathbf{x}_s) r_s(\mathbf{x}_s - \mathbf{k}_s),\\
i_g^c(\mathbf{x}_g) &= i_s^c(m^{-1}(\mathbf{x}_g)),\\
i_g^{cf}(\mathbf{x}_g) &= \int_{\mathbb{R}^2} i_g^c(\mathbf{t}_g) h_g(\mathbf{x}_g - \mathbf{t}_g)\,\mathrm{d}\mathbf{t}_g,\\
i_g^{cf}(\mathbf{x}_g) &= \int_{\mathbb{R}^2} i_s^c(m^{-1}(\mathbf{t}_g)) h_g(\mathbf{x}_g - \mathbf{t}_g)\,\mathrm{d}\mathbf{t}_g,\\
i_g^{cf}(\mathbf{x}_g) &= \int_{\mathbb{R}^2} \sum_{\mathbf{k}_s \in \mathbb{Z}^2} i_s(\mathbf{k}_s) r_s(m^{-1}(\mathbf{t}_g) - \mathbf{k}_s) h_g(\mathbf{x}_g - \mathbf{t}_g)\,\mathrm{d}\mathbf{t}_g,\\
i_g^{cf}(\mathbf{x}_g) &= \sum_{\mathbf{k}_s \in \mathbb{Z}^2} i_s(\mathbf{k}_s) \int_{\mathbb{R}^2} r_s(m^{-1}(\mathbf{t}_g) - \mathbf{k}_s) h_g(\mathbf{x}_g - \mathbf{t}_g)\,\mathrm{d}\mathbf{t}_g,
\end{aligned}
\tag{1}
$$

with r_s the reconstruction kernel defined in sensor geometry, and h_g the anti-aliasing filter defined in the mapped geometry. Practically, Eq. 1 is then discretized by only computing it at the points of coordinates $\mathbf{x}_g$ that belong to the mapped grid. Assuming that m is invertible and locally linear, we can apply the change of variable $\mathbf{t}_g = m(\mathbf{u}_s)$, with associated Jacobian $\mathbf{J} = \frac{\partial \mathbf{m}}{\partial \mathbf{u}_S}$, and with $\mathrm{d}\mathbf{t} = |\frac{\partial \mathbf{m}}{\partial \mathbf{u}_S}|\,\mathrm{d}\mathbf{u}_s$. We then simplify the above equation as:

$$
\begin{aligned}
i_g^{cf}(\mathbf{x}_g) &= \sum_{\mathbf{k}_s \in \mathbb{Z}^2} i_s(\mathbf{k}_s) \int_{\mathbb{R}^2} r_s(\mathbf{u}_s - \mathbf{k}_s) h_g(\mathbf{x}_g - \mathbf{J}\mathbf{u}_s) |\mathbf{J}|\,\mathrm{d}\mathbf{u}_s,\\
i_g^{cf}(\mathbf{x}_g) = i_g^{cf}(\mathbf{J}\mathbf{x}_s) &= \sum_{\mathbf{k}_s \in \mathbb{Z}^2} i_s(\mathbf{k}_s) \int_{\mathbb{R}^2} r_s(\mathbf{u}_s) h_g(\mathbf{J}(\mathbf{x}_s - \mathbf{u}_s - \mathbf{k}_s)) |\mathbf{J}|\,\mathrm{d}\mathbf{u}_s,\\
i_g^{cf}(\mathbf{x}_g) = i_g^{cf}(\mathbf{J}\mathbf{x}_s) &= \sum_{\mathbf{k}_s \in \mathbb{Z}^2} i_s(\mathbf{k}_s) \rho_s(\mathbf{x}_s - \mathbf{k}_s),
\end{aligned}
\tag{2}
$$

with

$$\rho_s(\mathbf{x}) = \int_{\mathbb{R}^2} r_s(\mathbf{u}_s) h_g(\mathbf{J}(\mathbf{x}_s - \mathbf{u}_s))|\mathbf{J}| \,\mathrm{d}\mathbf{u}_s, \\ \rho_s(\mathbf{x}) = r_s(\mathbf{x}) \circledast h_g(\mathbf{J}\mathbf{x})|\mathbf{J}|. \tag{3}$$

We call ρ_s the equivalent resampling kernel defined in sensor geometry. Note that it is sampled by $\{\mathbf{k}_s\}$ (Eq. 2), which defines a regularly sampled grid as it scans all the pixels in the input image. This formulation implies that the image mapping is defined from the inverse image warping m^{-1}, and we call it the inverse resampling model. Indeed, although the direct mapping m, or it's linearized version $\mathbf{J}$, is used in this formulation, it has to be derived by the inversion of $\mathbf{J}^{-1}$ to locate without any coordinate search its location and extent in the sensor image. Practically, $|\mathbf{J}|$ is a normalization factor such that $\sum_{\mathbf{x}_s} \rho_s = 1$.

The direct resampling model formulates the image resampling in the destination image geometry instead of the sensor geometry. Starting from Eq. 1 we can use the alternative change of variable $\mathbf{l}_g = m(\mathbf{k}_s)$, linearize m, and write:

$$i_g^{cf}(\mathbf{x}_g) = \sum_{\mathbf{l}_g = \mathbf{J}\mathbf{k}_s, \mathbf{k}_s \in \mathbb{Z}^2} i_s(\mathbf{J}^{-1}\mathbf{l}_g) \int_{\mathbb{R}^2} r_s(\mathbf{J}^{-1}(\mathbf{t}_g - \mathbf{l}_g)) h_g(\mathbf{x}_g - \mathbf{t}_g)|\mathbf{J}^{-1}| \,\mathrm{d}\mathbf{t}_g, \\ i_g^{cf}(\mathbf{x}_g) = \sum_{\mathbf{l}_g = \mathbf{J}\mathbf{k}_s, \mathbf{k}_s \in \mathbb{Z}^2} i_s(\mathbf{J}^{-1}\mathbf{l}_g) \rho_g(\mathbf{x}_g - \mathbf{l}_g), \tag{4}$$

with

$$\rho_g(\mathbf{x}) = \int_{\mathbb{R}^2} r_s((J)^{-1}\mathbf{t}_g) h_g(\mathbf{x}_g - \mathbf{t}_g)|\mathbf{J}^{-1}| \,\mathrm{d}\mathbf{t}_g, \\ \rho_g(\mathbf{x}) = r_s(\mathbf{J}^{-1}\mathbf{x}) \circledast h_g(\mathbf{x})|\mathbf{J}^{-1}|. \tag{5}$$

We call ρ_g the equivalent resampling kernel defined in mapped geometry. Note that in Eq. 4 ρ_g is sampled by $\mathbf{l}_g$, which is irregularly sampled as it is the irregular projection of the regular grid sampled by $\mathbf{k}_s$. The practical implementation of this resampling model is extremely tedious as it requires, for each $\mathbf{x}_g$, the determination of all $\mathbf{l}_g$ falling within the extent of ρ_g, and the irregular sampling of ρ_g. For this reason, we only consider in this study the practical implementation of the inverse resampling model defined by Eq. 2.

We propose an implementation of ρ_s where both r_s and h_g are approximated sinc kernels. In particular, we show how to predict the shape of the Fourier reciprocal cells of each kernel based on the local mapping distortion given by the mapping Jacobian $\mathbf{J}$. This leads to an algorithm where three cases are possible to construct ρ_s. If the reciprocal cell of r_s is contained within the reciprocal cell of h_g, then $\rho_s = r_s$. If the reciprocal cell of h_g is contained within the reciprocal cell of r_s, then $\rho_s = h_g$. If none of the previous cases happen, ρ_s is the convolution of a separable kernel r_s with a non-separable kernel h_g. When this case happens, h_g needs to be up-sampled to avoid aliasing, then down-sampling and convolution with r_s are both achieved directly in the Fourier domain for efficiency. A new, locally adapted, kernel ρ_s is determined for each point $\mathbf{x}_g$ of the resampling grid defined in the mapped geometry. Note that in general, ρ_s is not a separable kernel nor a linear transformation of a separable kernel because of the up-sampling and down-sampling operations.

Examples and illustrations are shown where this formulation allows us to resample high resolution satellite and aerial images taken in mountainous steep topographic areas. High image frequencies are preserved in all areas of the images, although local image resolution has been found to vary by a factor of more than 10. Without a locally adaptive kernel, extreme image filtering or extreme aliasing artifacts would have occurred. The use of an approximated sinc kernel is also key in preserving the images sub-pixel information. This property is critical when subpixel correlation algorithms are used on the resampled set of images to derive ground deformation maps from mutli-temporal images [1], or when accurate disparity fields must be estimated to reconstruct accurate topography models from epipolar images.

The methodology described in this study was implemented in the Co-registration of Optically Sensed Images and Correlation (COSI-Corr)[1] software package, which is freely distributed for non-commercial and research purposes.

References

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