NUMERICAL SIMULATION OF WAVE BREAKING

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Wave breaking is important for understanding of both sea wave dynamics and microwave scattering/emission of the ocean. Experimental studies of breaking waves are mostly related to tank measurements and to a lesser extent to field observations [1, 2]. Theoretical results regarding wave breaking are mostly obtained by numerical simulation [3]. Comparison of those with tank experiments leads to the conclusion that 2D model of irrotational motion of ideal fluid provides accurate description of non-linear wave motion. The 3D effects are also known to contribute [4], however they do not seem to be a prime cause of breaking.

Main objective of the current work is to present results of numerical Monte-Carlo simulation of dynamics of 2D, potential, and random surface gravity waves which indicate that the dominant physical mechanism causing wave breaking appears to be the "concertina" effect, according to the terminology introduced by Longuet-Higgins [5].

Our study is based on two independent numerical codes developed for simulation of non-linear dynamics of 2D surface waves. The first one called "Laplacian" or L-code is based on calculation of Dirichlet-to-Neumann (DtN) operator, which maps values of surface potential into corresponding values of its normal derivative. We use cubic spline for interpolation of surface profile and surface potential between anchor points. This approach allowed us to reduce the number of points and, correspondingly, the amount of calculations without sacrificing its accuracy.

The second code (“Hamiltonian”, or H-code) accomplishes time integration of a discrete Hamiltonian system which approximates the original continuous equations for the potential surface waves which are well-known to be Hamiltonian. For a large number of degrees of freedom solutions of discrete and continuous equations appear to be very close up to the moment of breaking. We see two-fold advantage of such approach: 1) H-code appears to be very computationally effective and easily allows studying the systems with up to 1000 degrees of freedom; 2) solutions of the "approximating" discrete Hamiltonian system guarantees fulfillment of conservation laws and corresponding symmetries of its spectrum. The latter is not generally true for a discrete form of the primary equations which are not necessarily Hamiltonian *per se.* Also, studying the evolution of the modes corresponding to the linearized equations for perturbations of the discrete system helped us to better understand non-linear dynamics resulting in wave breaking.

H-code efficiency allowed us to perform Monte-Carlo simulations of evolution of random sea-like surfaces. Initial conditions were randomly generated according to K^{-3} and K^{-4} power spectrum with uniformly distributed phases of the harmonics. We found out that under certain conditions some modes of the discrete systems start growing "explosively" and this effect cannot be explained by numerical instabilities. Moreover, such unstable modes appeared to be well localized in space which indicates that their instability is caused by local conditions like surface slope or local currents. Figure 1 shows four different examples of the surface profile and velocity potential at final stage of wave evolution. Four cases correspond to four different initial conditions. One can notice a good correlation between instability location and zones of maximum current convergence (minimum of the second derivative of the velocity potential).

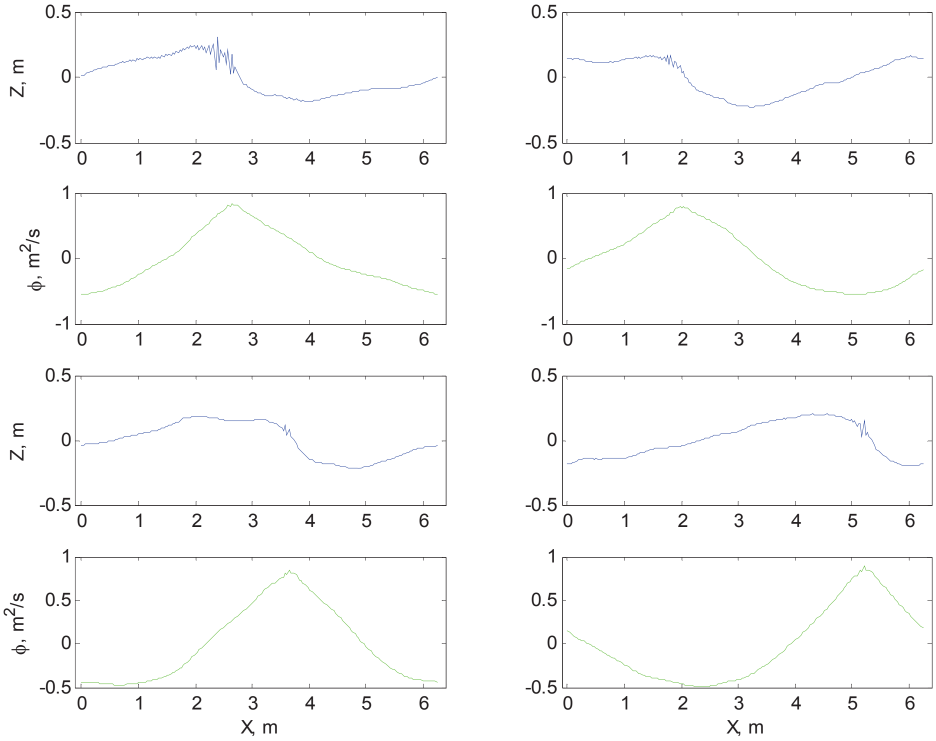

Figure 1. A few examples of the surface profile (blue) and velocity potential (green) at the final stage of instability development.

To make sure the observed instability is related to wave breaking, we combined two codes into one algorithm: we used H-code to follow wave evolution up to the "breaking" moment, then we stepped back in time by sufficiently long time interval which ensured that the unstable mode didn’t start to grow yet, and repeated the calculations from this moment using the L-code. Keeping in mind that L-code treats the surface as a continuous (although spline approximated), we can expect that pure numerical instability typical for discrete system will not exhibit itself in the evolution due to L-code. We observed that the location of the "Hamiltonian" instabilities generally quite well correlated with the point where the wave breaks according to the L-code. The most important finding of our simulations is that the locations of breaking events are well correlated with maximum of the surface current convergence due to large-scale waves. Figure 2 illustrates this correlation for the case of K^{-3} spectrum.

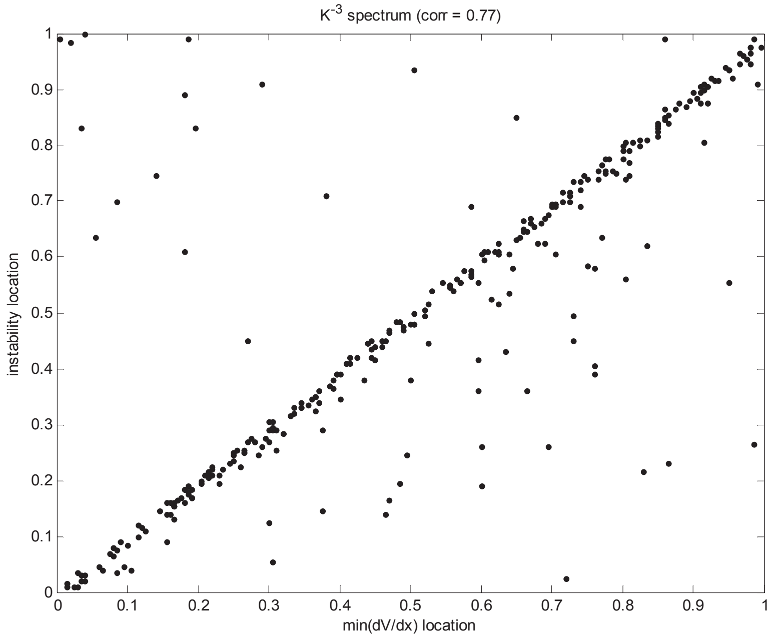

Figure 2. Correlation between "breaker" position and the point of the strongest surface current convergence.

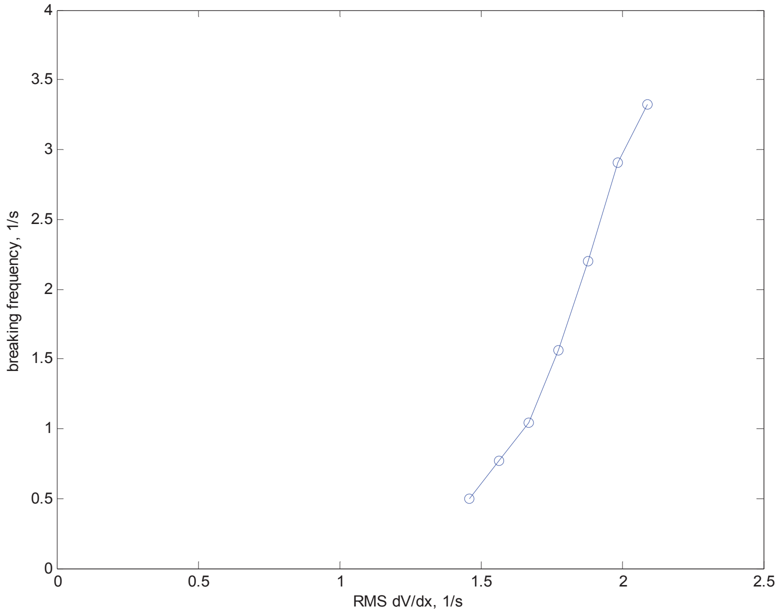

Figure 3. The dependence of the averaged breaking frequency on mean surface current (negative) gradient.

The importance of this effect for remote sensing is due to two factors. First, our Monte-Carlo simulations show that frequency of wave breaking events strongly depend on current gradients and, consequently, on amplitude of large-scale waves. Figure 3 shows the dependence of breaking frequency versus RMS current gradient of large waves. One can see that the dependence is steep and non-linear which means that even small variation of large waves will result in significant change in the number of breakers. Second, area of wave breaking is characterized by steep slopes and irregular profile. Our numerical calculations of microwave emissivity of such profiles show that they can contribute noticeably to the net brightness temperature of the sea surface. Wave-breaking effect is much more dramatic with respect to backscattered signal, because breakers are primary source of so-called spikes in backscattering.

We believe that the results of our numerical simulation will help to better understand the physical mechanisms responsible for wave breaking.

References

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