SPATIAL SPECTRUM OF BISTATIC SAR WITH ONE FIXED STATION

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1. INTRODUCTION

Bistatic synthetic aperture radar (BSAR) is receiving more and more attention and research around the world recently. Bistatic SAR with one fixed station (OF-BSAR), also called bistatic parasitic SAR [1], refers to the bistatic SAR where the aperture is synthesized by only one moving station. In this mode, the fixed transmitter or receiver could be mounted on a geostationary satellite [2, 3], a ground based high platform [1, 4, 5] or a stratosphere low speed airship.

Spatial spectrum, which is also called k-set or wave-number pattern, is an important tool to analyze the imaging performance of an imaging system [6], because the Point Spread Function (PSF) can be written as the inverse Fourier transform of the Spatial Spectrum Support (SSS). Compared with the traditional monostatic SAR and general bistatic SAR with two moving platforms, OF-BSAR has different and special properties in spatial spectrum. In this paper, we will analyze the spatial spectrum of OF-BSAR in detail.

2. SPATIAL SPECTRUM THEORY OF OF-BSAR

The analysis of the spatial spectrum for general bistatic SAR can be found in [7–9]. Here we summarize the results briefly first. The spatial spectrum are formed by the wavenumber vectors which are effective quantities combined from the transmit and receive **k**-vectors. They are always determined by the total signal bandwidth and the relative motion of the sensors during the synthetic aperture interval.

We assume the looking angles of the transmitter and the receiver to the point target at a certain time are ϕ_T and ϕ_R, respectively. They are measured by the angles rotating about the point target counterclockwise from the x axis in Fig.1(a). So the unit direction vectors $\mathbf{u}_T$ and $\mathbf{u}_R$ which look in the direction from the transmitter and receiver to the point target can be expressed by

$$\mathbf{u}_T = (\cos\phi_T, \sin\phi_T)^T, \mathbf{u}_R = (\cos\phi_R, \sin\phi_R)^T \tag{1}$$

The effective direction vector is

$$\begin{aligned}\mathbf{u}_{eff} &= \frac{1}{2}\left(\mathbf{u}_R + \mathbf{u}_T\right) \\ &= \frac{1}{2}\left[\left(\cos\phi_R, \sin\phi_R\right)^T + \left(\cos\phi_T, \sin\phi_T\right)^T\right] \\ &= \cos\frac{\phi_R - \phi_T}{2}\left(\cos\frac{\phi_R + \phi_T}{2}, \sin\frac{\phi_R + \phi_T}{2}\right)^T \end{aligned} \tag{2}$$

where $(\phi_R - \phi_T)/2$ is the half bistatic angle, $(\phi_R + \phi_T)/2$ is the equivalent looking angle.

The radius **k**-vector of bistatic SAR is composed by two parts:

$$\mathbf{k}_R = \frac{2\pi f \mathbf{u}_R}{c}, \mathbf{k}_T = \frac{2\pi f \mathbf{u}_T}{c}, f \in [f_c - B/2, f_c + B/2] \tag{3}$$

Then the synthetic **k**-vector is:

$$\mathbf{k}_{tot} = \mathbf{k}_R + \mathbf{k}_T = \frac{4\pi}{c} f \mathbf{u}_{eff} \tag{4}$$

So we have

$$\mathbf{k}_{tot} \in \mathbf{K}_{tot} = \frac{4\pi}{c}\cos\frac{\phi_R - \phi_T}{2}\left[\left(f_c - \frac{B}{2}\right), \left(f_c + \frac{B}{2}\right)\right] \tag{5}$$

For a group of ϕ_T and ϕ_R, $\mathbf{K_{tot}}$ is a polar line segment along the sum LOS vector. As for general bistatic SAR, ϕ_T and ϕ_R vary in the whole synthetic interval. So it is complicated to analyze the spatial spectrum property for general bistatic SAR.

As for OF-BSAR, we always assume the transmitter is stationary generally, then $\phi_T = const$. Thus, from (2), we can find the trajectory of the end of $4\pi f \mathbf{u}_{eff}/c$ is always a circle, whose radius is $2\pi f/c$ and the center is located at $2\pi f\left(\cos\phi_T, \sin\phi_T\right)/c$. In addition, there is always a ϕ_R satisfying the condition that $\phi_R = \phi_T + \pi$. So, the $4\pi f \mathbf{u}_{eff}/c$ trajectory circle always passes the original point $(0, 0)$. From (2), we can also find that ϕ_R corresponds to the counterclockwise rotation angle about the circle center. If the receiver beam LOS angle $\phi_R \in [\phi_{R\min}, \phi_{R\max}]$, the resulted trajectory of the $4\pi f \mathbf{u}_{eff}/c$ end is an arc section of the circle. In Fig.1(b), $\overset{\frown}{CD}$ shows the wavenumber section of $4\pi f_c \mathbf{u}_{eff}/c$, while $\overset{\frown}{GH}$ and $\overset{\frown}{EF}$ represent the wavenumber section of $4\pi(f_c + B/2)\mathbf{u}_{eff}/c$ and $4\pi(f_c - B/2)\mathbf{u}_{eff}/c$ respectively. So the SSS of OF-BSAR is shown by the shading area $GHFE$ in Fig.1(b).

3. NUMERICAL SIMULATION

From Section 2, we know that if we determined the boundary circles of $4\pi(f_c - B/2)\mathbf{u}_{eff}/c$ and $4\pi(f_c + B/2)\mathbf{u}_{eff}/c$ and the angle change scope, we can obtain the SSS of OF-BSAR. We should note that the shading area only represents the support region or the bandwidth of the spatial domain target function; the phase and amplitude modulations within this band do not need to be considered. So we can construct two two-dimensional circular window and a fan-shaped window to form the SSS of OF-BSAR:

$$W_{circ1}(k_x, k_y) = \begin{cases} 1, & \sqrt{\left[k_x - \frac{2\pi}{c}\left(f_c + \frac{B}{2}\right)\cos\phi_T\right]^2 + \left[k_y - \frac{2\pi}{c}\left(f_c + \frac{B}{2}\right)\sin\phi_T\right]^2} \leqslant \frac{2\pi}{c}\left(f_c + \frac{B}{2}\right) \\ 0, & otherwise \end{cases} \tag{6}$$

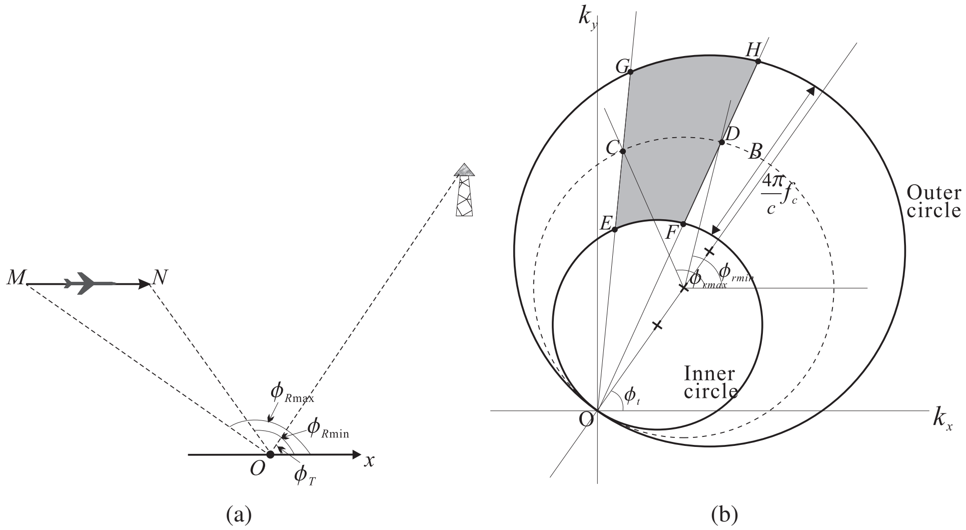

Fig. 1. (a) Imaging Geometry of General OF-BSAR. (b) Spatial Spectrum Support of OF-BSAR.

$$W_{circ2}(k_x,k_y)=\begin{cases}1, & \sqrt{\left[k_x-\frac{2\pi}{c}\left(f_c-\frac{B}{2}\right)\cos\phi_T\right]^2+\left[k_y-\frac{2\pi}{c}\left(f_c-\frac{B}{2}\right)\sin\phi_T\right]^2}\geqslant\frac{2\pi}{c}\left(f_c-\frac{B}{2}\right)\\ 0, & otherwise\end{cases} \tag{7}$$

$$W_{fan}(k_x,k_y)=\begin{cases}1, & \frac{\phi_T+\phi_{R\min}}{2}\leqslant\arctan\left(\frac{k_y}{k_x}\right)\leqslant\frac{\phi_T+\phi_{R\max}}{2}\\ 0, & otherwise\end{cases} \tag{8}$$

Then the SSS of OF-BSAR can be constructed as:

$$SSS=W_{circ1}(k_x,k_y)W_{circ2}(k_x,k_y)W_{fan}(k_x,k_y) \tag{9}$$

The resolution of an imaging system is given by the two-dimensional extent of the PSF. Roughly, in each dimension it is inverse to the extent of the SSS in the concerning dimension. So the resolution along x and y are: $\rho_x=2\pi/\Delta k_x$ and $\rho_y=2\pi/\Delta k_y$. In the simulation the bandwidth of the transmitted signal is $B=200$MHz. The parameters for the first simulation are: $f_c=10$GHz, $\phi_T=90°$, $\phi_{Rmin}=80°$ and $\phi_{Rmax}=100°$. The second set of parameters is: $f_c=1$GHz, $\phi_T=60°$, $\phi_{Rmin}=120°$ and $\phi_{Rmax}=130°$. The resulted SSS and PSF of these two configurations are shown in Fig.2 and Fig.3. We can get the 3dB width of the PSF along the directions x and y. For the first case: $\rho_x=0.11m$ and $\rho_y=0.85m$. For the other case: $\rho_x=1.2m$ and $\rho_y=0.95m$.

4. CONCLUSION

In this paper, we have presented a method to determine the spatial spectrum of OF-BSAR. The results can be used to analyze the spatial resolution of an OF-BSAR system. Simulations presented confirm the validity of method in this work.

5. REFERENCES

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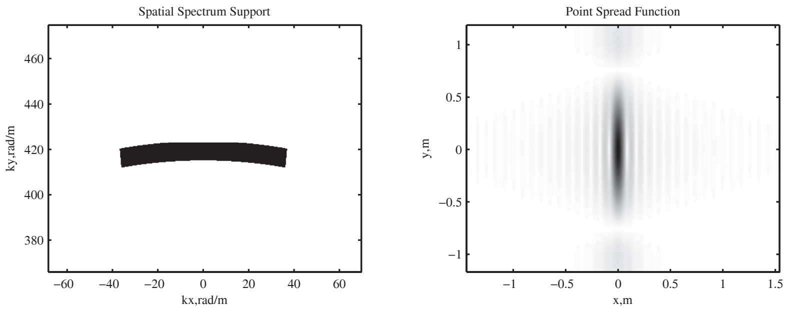
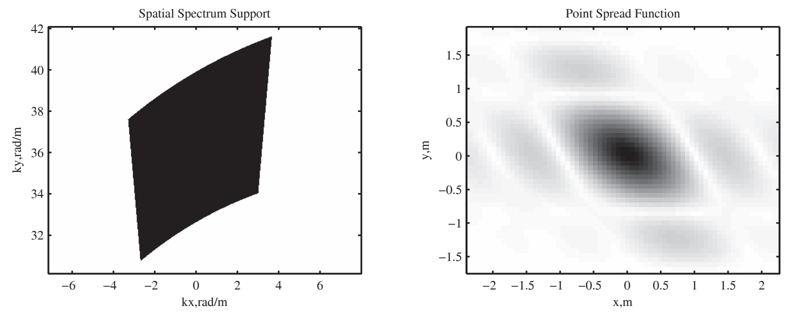


(a) (b)

Fig. 2. Mode I. (a) Spatial Spectrum Support of OF-BSAR. (b) Point Spread Function of OF-BSAR.



(a) (b)

Fig. 3. Mode II. (a) Spatial Spectrum Support of OF-BSAR. (b) Point Spread Function of OF-BSAR.

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