A COMPARISON OF TWO STREAM APPROXIMATION FOR THE DISCRETE ORDINATE METHOD AND THE SOS METHOD

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1. INTRODUCTION

The radiative transfer process is key issue for the fundamentals of atmospheric radiation. The physical process of radiative transfer is described by a differential-integral equation. The azimuthally independent radiative transfer equation is

$$\mu \frac{dI(\tau,\mu)}{d\tau} = I(\tau,\mu) - \frac{\omega}{2}\int_{-1}^{1} I(\tau,\mu')P(\mu,\mu')d\mu' + \frac{\omega}{4\pi}F_0 P(\mu,\mu_0)\exp(-\frac{\tau}{\mu_0}) \quad (1)$$

where $I(\tau,\mu)$ is the azimuthally averaged intensity at optical depth τ and for the direction of the cosine of the zenith angle μ, μ_0 is the cosine of the solar zenith angle, ω is the single scattering albedo, F_0 is the incident solar irrandiance, and $P(\mu,\mu')$ is the azimuthally averaged phase function. Due to the complexity of this equation, it is generally solved numerically rather than analytically. Among the available approximation methods, the two stream approximation is the most feasible in a computationally efficient manner [1].

2. THE DISCRETE ORDINATES METHOD

The discrete ordinates method is one of the most frequently used techniques to solve the radiative transfer equation [2]. It was originally developed by Chandrasekhar to study the transfer of radiation in planetary atmospheres, and has become a useful tool in calculating radiation in atmospheres containing aerosols and clouds. This method solves the raditive transfer equation via the discretization of the basic equation, representing it as a set of 2N coupled differential equations. They are solved using Eigen analysis. The discrete ordinate method for radiative transfer in vertically inhomogeneous layered media, as embodied in a FORTRAN computer code, is called DISORT. In particular, a multiple scattering code which uses only two streams (one up, one down), to be henceforth called TWOSTR.

3. THE SOS METHOD

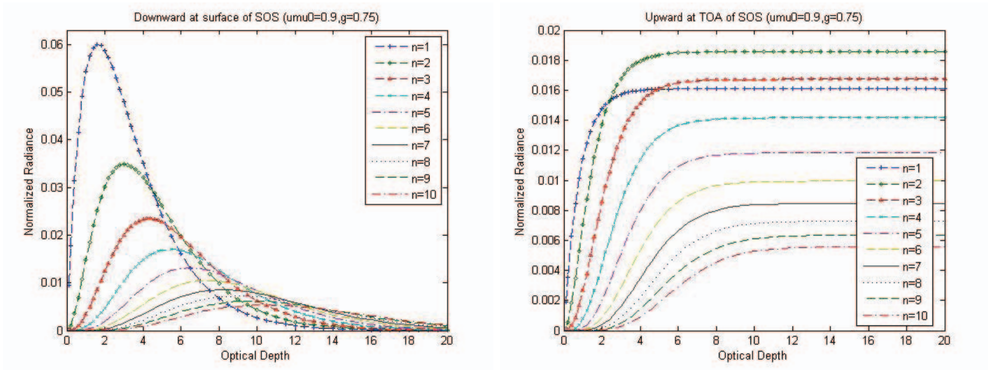
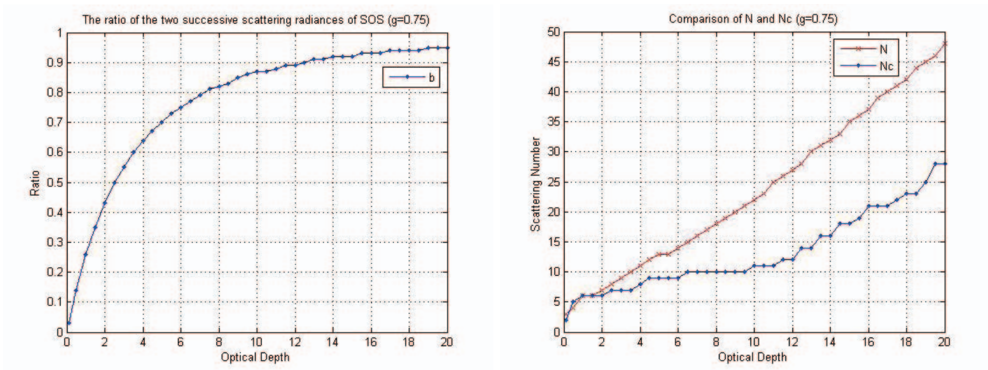


Fig. 1. Intensity distribution along optical depth of each scattering ($\mu_0 = 0.9$, $g = 0.75$, $\omega = 1$)



(a) Value of b (b) Comparison of N and Nc

Fig. 2. Semi-empirical model of the two-stream SOS ($g = 0.75$, $\omega = 1$)

The successive order of scattering (SOS) method, by computing the contribution of each scattering order, is physically straightforward and is easier to analyze the importance and characteristics of scattering-absorption processes. The solution of radiative transfer for the SOS method is expressed as a summation of contributions of all successive orders of scattering

$$I(\tau, \mu) = \sum_{n=1}^{N} I_n(\tau, \mu) \tag{2}$$

Here, N is the scattering number and $I_n / I < \varepsilon$, ε is a given precision value. In the two-stream approximation, $\mu = \pm\sqrt{3} / 3$, as the direction of the two stream radiative intensities in up and down hemispheres.

However, the solution convergence of the SOS method becomes very slow when the single scattering albedo of an optically thick layer tends to 1. To accelerate the convergence to reduce the computational burden of the SOS method, a semi-empirical model can be used [3]. The summation of Eq. (2) can be truncated at the nth order of scattering when the ratio approaches a constant. The reminder can be replaced by a geometric series as

$$I(\tau, \mu) = \sum_{n=1}^{Nc-1} I_n(\tau, \mu) + I_{Nc}(\tau, \mu) / (1 - b) \tag{3}$$

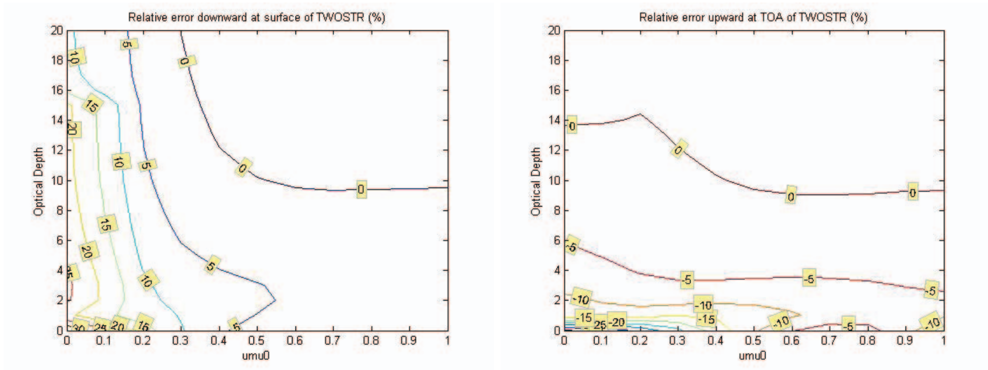


Fig. 3. Relative errors of two-stream discrete ordinates method for the flux ($g = 0.75$, $\omega = 1$)

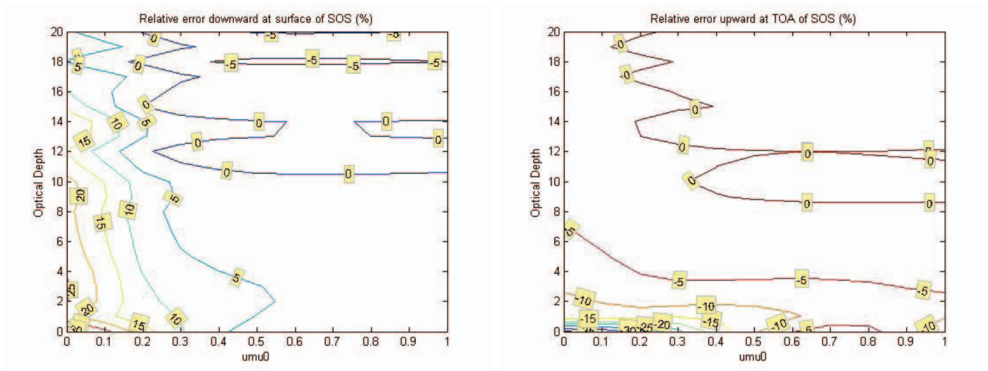


Fig. 4. Relative errors of two-stream SOS method for the flux ($g = 0.75$, $\omega = 1$)

where $b = I_{Nc} / I_{Nc-1}$. Therefore, by using the semi-empirical model, the computational time can be significantly reduced without calculating higher order of scattering.

For the non-absorbing case of $\omega = 1$, we investigate the effect of the two-stream SOS method by the delta-M method, using a Henyey-Greenstein phase function with $g = 0.75$. For each scattering order, we compute the results of intensity distributions with a small incremental optical depth of 0.025. Fig. 1 shows the intensity distribution along optical depth of each scattering when $\mu_0 = 0.9$. Fig. 2(a) shows value of b as a function of optical depths. Fig. 2(b) shows the comparison of N and Nc , where N is the scattering number of $\varepsilon = 1\%$.

4. COMPARISON RESULTS

To analyze the accuracy of the two two-stream methods, we consider the case of anisotropic scattering by the delta-M method [4], using a Henyey-Greenstein phase function with the asymmetry factor $g = 0.75$. We use the code TWOSTR to compute the flux as the results of the two-stream discrete ordinates method. Meanwhile, we use 32-stream DISORT as the benchmark model to compute the flux upward at TOA and downward at surface.

For the non-absorbing case of $\omega = 1$, Fig. 3 shows the relative errors of two-stream discrete ordinates method, and Fig. 4 shows the relative errors of two-stream SOS method. Compare Fig. 4 with Fig. 3, we can see that though the results shown in Fig. 4 differ slightly from those in Fig.3, the results of the two two-stream methods are almost the same in general.

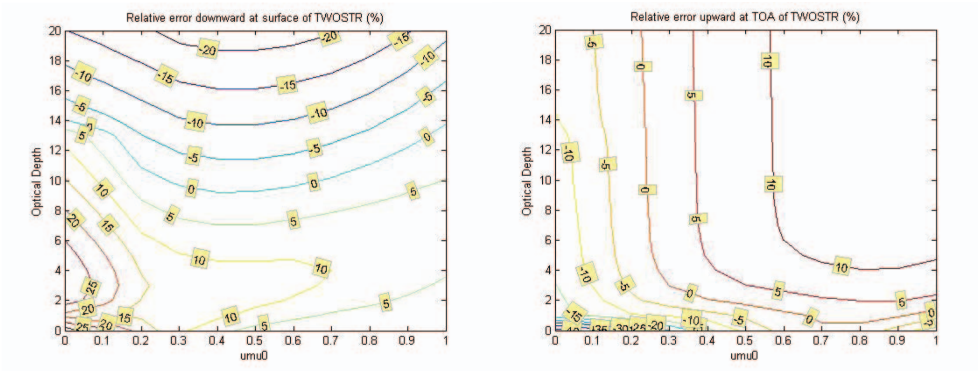


Fig. 5. Relative errors of two-stream discrete ordinates method for the flux ($g = 0.75$, $\omega = 0.9$)

$\omega = 0.9$ is used as an example of absorbing case, as well as $g = 0.75$. The relative errors of the two-stream discrete ordinates method are illustrated in Fig. 5. The relative errors of two-stream SOS method are not shown. Just as in the non-absorbing case, the results of the two two-stream methods are almost the same in general. Fig. 5 shows that the absorbing media lead to larger errors of flux in the most of the region, compared with that of the non-absorbing case.

5. CONCLUSION

In this study, we make a comparison of two stream approximation for the discrete ordinate method and the successive orders of scattering method. Based on the convergence characteristics of successive scattering, we use a semi-empirical model to improve the computing efficiency in the SOS method. The delta-M method is also used in the two methods. With the comparisons for the accuracy of flux, the results of the two two-stream methods are almost the same in general and the absorbing media lead to larger errors of flux compared with that of the non-absorbing case.

6. REFERENCES

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