MODELING OF ELECTROMAGNETIC WAVE SCATTERING THROUGH A WALL WITH ROUGH INTERFACES

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1. INTRODUCTION

In this paper, we present a theoretical model for the evaluation of electromagnetic transmission through a rough wall. Electromagnetic (EM) propagation through layered structures is a fundamental problem in several areas. As matter of fact, wave interaction with interfaces of actual layered structure, each one exhibiting some amount of roughness, gives rise to a radiation scattered in all directions, whose field pattern is influenced by the interferential phenomena taking place within the structure itself. To better clarify the importance of the relevant theoretical problem, we focus on a practical context. In order to simulate the detection capability of the emerging through-the-wall radar systems, wall structures represented by using (flat) half-space, slab or layered models has been commonly considered [1]-[2]. In these cases, the specular reflection and transmission formulation are commonly employed, with relevant characteristics calculated by the ordinary Fresnel coefficients. It should be underlined that the adoption of this rather simplistic model, in which only the dominant (specular and refracted) components are take into account, even if questionable, is motivated by two main reasons. First, its coding in computational tools is definitely less complicated with respect to considering the contributions in all directions. Second, and more in general, there is no simple, even approximate, close-form analytical model hitherto, in order to quantitatively assess the effect of the interfacial roughness on the propagation through layered structure. Nonetheless, the relevant oversimplification remains questionable and can turn out to be not appropriate, especially when the involved EM wave-lengths become comparable with the roughness scale, leading to

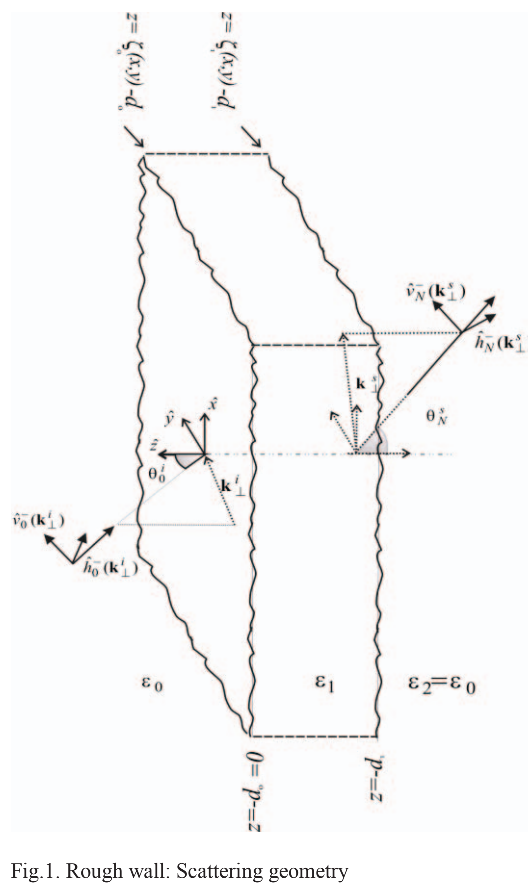


Fig.1. Rough wall: Scattering geometry

significant limitations in simulation of wave interaction with such structures. Indeed, more specifically, heterogeneity in real building walls can be treated as randomly rough structures, particularly when millimetric wavelengths are concerned [3]-[4]. As matter of fact, the investigation on scattering through rough interfaces of a layered media is indubitably less advanced with respect to classical rough surface scattering. In fact, due to the lacking in convenient analytical solutions, the simulation of wave propagation through such structures has been essentially conducted relaying on custom numerical approaches. We emphasize that numerical methods require a large amount of computation time, which limits the usefulness their practical applicability with respect to design of complex algorithms, such as ray-tracing-based or radar processing ones, where the scattering problem has to be solved many times.

In this paper, we present how the closed-form solution obtained in the theoretical framework of the *boundary perturbation theory (BPT)* [5]-[6] can be effectively applied (parametrically and polarimetrically) to the simulation of the transmission through a rough wall. In particular, for a rough slab we derive a specific compact solution directly expressed in terms of the (Fresnel) ordinary reflection coefficient only. Numerical results are carried out with reference to the canonical structure by considering two-dimensional Gaussian rough interfaces, and scattering patterns are presented. The effect of the several involved parameters on the scattering patterns is provided. This study provides a unique, innovative, valuable tool for the simulation and electromagnetic modeling of through-the-wall radar applications.

2. AN ELECTROMAGNETIC MODEL BASED ON PERTURBATION OF BOUNDARY CONDITIONS

The results of the *BPT* offer a valuable analytical tool to cope with the problem of EM propagation through layered media with rough interfaces [5] [6], providing a direct model, for which the underlying physics of the scattering processes is tangible, and whose analytical results can be easily applied to transmission through-the-rough wall. Each (mth) layer of the layered structure is assumed to be homogeneous and characterized by arbitrary and deterministic parameters: the dielectric relative (complex) permittivity ε_m, the magnetic relative permeability μ_m and the thickness $\Delta_m = d_m - d_{m-1}$. Each (mth) gently-rough interface is assumed to be characterized by a *zero-mean* two-dimensional process $\zeta_m = \zeta_m(\mathbf{r}_\perp) = \zeta_m(x, y)$. It can be demonstrated that *BPT* allow us to take into account the contribution of each nth corrugated interface, leading to the solution for the global *bistatic scattering cross-section* of the N-rough interface layered media, which results to be expressed as [6]:

$$\tilde{\sigma}_{qp}^{0} = \pi k_0^4 \,\mathrm{Re}\left\{\sqrt{\frac{\varepsilon_N}{\varepsilon_0}}\right\} \sum_{n=0}^{N-1} \left| {}_{N}^{0}\tilde{\beta}_{qp}^{n,n+1}(\mathbf{k}^s, \mathbf{k}^i) \right|^2 W_n(\mathbf{k}_\perp^s - \mathbf{k}_\perp^i) + \pi k_0^4 \,\mathrm{Re}\left\{\sqrt{\frac{\varepsilon_N}{\varepsilon_0}}\right\} \sum_{i \neq j} \mathrm{Re}\left\{ {}_{N}^{0}\tilde{\beta}_{qp}^{i,i+1} \left({}_{N}^{0}\tilde{\beta}_{qp}^{j,j+1} \right)^{*} \right\} W_{ij}(\mathbf{k}_\perp^s - \mathbf{k}_\perp^i) \quad (1)$$

with $\mathbf{k}_\perp^i$ and $\mathbf{k}_\perp^s$ denoting the projection on the *x-y* plane, respectively, of the incident and scattered vector wave-number (see Fig.1); where k_0 is the wave-number in the vacuum; the asterisk denotes the complex conjugated, the index q denotes the polarization of scattered field and the index p denotes the polarization of incident wave, with $p,q \in \{v,h\}$, $W_n(\boldsymbol{\kappa})$ is the (*spatial*) *power spectral density* of nth corrugated interface and $W_{ij}(\boldsymbol{\kappa})$ is the *cross power spectral density* between the interfaces i and j, and $\boldsymbol{\kappa}$ is the roughness spatial frequency. We here emphasize that the coefficient ${}_N^0\tilde{\beta}_{qp}^{m,m+1}$ is relative to the p-polarized incident wave impinging on the structure from half-space *0* and to q-polarized scattering contribution, originated from the rough interface between the layers m and $m+1$, through the structure into last N half-space. Full expression for ${}_N^0\tilde{\beta}_{qp}^{m,m+1}$ can be found in [6].

In addition, we point out that the closed-form solution (1) is also susceptible of a clear physical interpretation in terms ray series, so that the relevant scattering phenomenon can be clearly visualized as emerging from superposition of local single-scattering interactions taking place into the structure. This aspect is fundamental when the incident wave is a radar modulated pulse: each term of the ray series corresponds to an echo that will be received with a different time delay. Consequently, these results also open the way toward a time-domain formulation of the problem, which is of fundamental importance, for instance, if the considered models have to be embedded in signal processing algorithm to predict the characteristics of the radar response. When the canonical structure of Fig.1 is concerned, we derive the following, compact closed-form expressions:

$$ {}_2^0\tilde{\beta}_{hh}^{0,1}(\mathbf{k}^s,\mathbf{k}^i) = (\varepsilon_1-\varepsilon_0)(\hat{k}_\perp^s\cdot\hat{k}_\perp^i)\frac{[1+\Gamma_h(k_\perp^i)][1-\Gamma_h^2(k_\perp^s)][1-\Gamma_h(k_\perp^i)e^{j2k_{z1}^i\Delta_1}]}{[1-\Gamma_h^2(k_\perp^i)e^{j2k_{z1}^i\Delta_1}][1-\Gamma_h^2(k_\perp^s)e^{j2k_{z1}^s\Delta_1}]}e^{jk_{z1}^s\Delta_1} \tag{2} $$

$$ {}_2^0\tilde{\beta}_{hh}^{1,2}(\mathbf{k}^s,\mathbf{k}^i) = (\varepsilon_0-\varepsilon_1)(\hat{k}_\perp^s\cdot\hat{k}_\perp^i)\frac{[1-\Gamma_h^2(k_\perp^i)][1-\Gamma_h(k_\perp^s)][1+\Gamma_h(k_\perp^s)e^{j2k_{z1}^s\Delta_1}]}{[1-\Gamma_h^2(k_\perp^i)e^{j2k_{z1}^i\Delta_1}][1-\Gamma_h^2(k_\perp^s)e^{j2k_{z1}^s\Delta_1}]}e^{jk_{z1}^i\Delta_1} \tag{3} $$

where the (*Fresnel*) ordinary reflection coefficient for the h polarization Γ_h is given by.

$$ \Gamma_h(k_\perp) = \frac{k_{z0}-k_{z1}}{k_{z0}+k_{z1}} \tag{4} $$

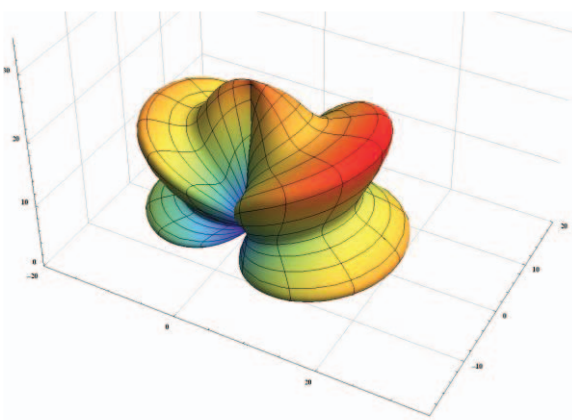
where $k_{zm} = \sqrt{k_m^2 - |\mathbf{k}_\perp|^2}$ with m=0,1; where $k_0 = \omega / c = 2\pi/\lambda$ and $k_1 = k_0\sqrt{\varepsilon_1}$ are the wave-number in the vacuum and in the slab, respectively. Similar expressions for the other polarization combinations can be obtained.
Accordingly, applying the expression (1) to the rough wall of Fig.1 and assuming no correlation between the roughnesses of the two interfaces, we obtain

$$ \tilde{\sigma}_{qp}^0 = \pi k_0^4 \left| {}_2^0\tilde{\beta}_{qp}^{0,1}(\mathbf{k}^s,\mathbf{k}^i)\right|^2 W_0(\mathbf{k}_\perp^s - \mathbf{k}_\perp^i) + \pi k_0^4 \left| {}_2^0\tilde{\beta}_{qp}^{1,2}(\mathbf{k}^s,\mathbf{k}^i)\right|^2 W_1(\mathbf{k}_\perp^s - \mathbf{k}_\perp^i) \tag{5} $$

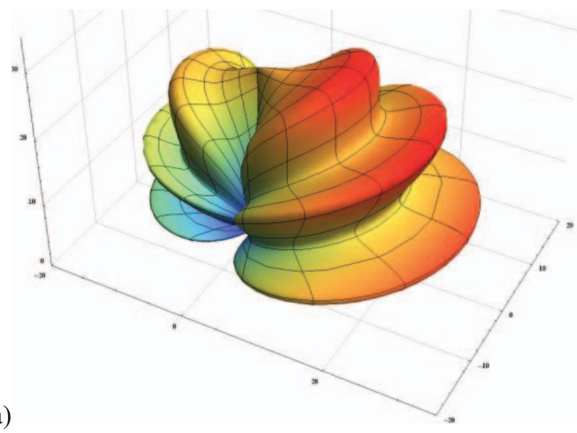
where p, $q \in \{v, h\}$, with h or v standing for *horizontal* and *vertical*, respectively; and where the expressions for the coefficients ${}_2^0\tilde{\beta}_{qp}^{0,1}$ and ${}_2^0\tilde{\beta}_{qp}^{1,2}$ for hh case are (2) and (3), respectively.

3. NUMERICAL SIMULATIONS

In this section, we present a few detailed cases aimed at studying scattering patterns with references to a canonical structure, which is representative of a typical wall building. In order to point out the capability of the proposed electromagnetic model for through-the-wall radar applications, we consider a structure with a single intermediate layer and two corrugated interfaces only. In agreement with classical theoretical studies of the scattering of waves from random surfaces, we model the interfaces as Gaussian random processes with *Gaussian correlations.* We study the scattering cross section through the structure as a function of the scattering direction in the last half-space, assuming fixed the incident direction. Therefore, to characterize in three-dimensional space the scattering pattern, scattering cross-sections are represented as function of zenithal and azimuthal angles and are treated as three dimensional surfaces. Two examples of such scattering patterns are reported in Fig.2. How the variation of the structure parameters affects the patterns shape will be also presented in the final paper, together with a detailed discussion on how the model can be conveniently applied to several through-the-wall radar application scenarios. The conducted analysis demonstrates the importance of including the effect of interfacial roughness when detection through an actual building wall is concerned.



(a)



(b)

Fig. 2. Scattering Pattern (dB) pol. *hh* with $\varepsilon_0=\varepsilon_2=1.0$, $\varepsilon_1=5.6+j0.01$, $\Delta_1=34cm$, $l_n=2.0$ *cm*, $\sigma_n=0.2$ *cm* for $n=0,1$, $\theta_0^i=45°$: (a) $f=3.5$ GHz, (b) $f=5.5$ GHz. It should be noted that to visualize the patterns an offset of +32dB has been considered for the radial amplitude.

4. REFERENCES

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